The Father of Algebra

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Muhammad Ibn Musa al-Khwarizmi was a Persian Muslim who lived in Baghdad, Iraq, during the Golden Age of Islam. Born in 783 A.D, Al-Khwarizmi seems to have lived until some time between 830 and 850 A.D. (Arndt, 2015; Nabirahni, Evans, & Persaud, 2019). He worked at the Wisdom House, which was founded by caliph al-Mamun and "contained a research library and observatory where many scholars studied" (Habirahni, Evans and Persaud, 2019, p. 14) He also worked with many mathematicians and scientists in the region, such as the Banu Musa brothers, known as the Sons of Musa, and Al-Farghani (Britannica; Nabirahni, Evans, & Persaud, 2019, p. 14). Through his work, Al-Khwarizmi created rules to find the area of the circle and the volume of the sphere, cone, and pyramid (Abdulrahman, 2016; Nabirahni, Evans, & Persaud 2019). He also developed the zero and decimal system, and invented sines, cosines, tangents, and the preparations of their tables (Abdulrahman, 2016). One of the most innovative thinkers to ever work in mathematics, astronomy, and geography, al-Khwarizmi also invented the concept of algorithm, a word derived from his name. His contributions and accomplishments in all these subjects were well-known and helped to develop them (Abdulrahman, 2016; Britannica).

Al-Khwarizmi's accomplishments include what many scientists consider one of the most influential books in the history of mathematics: *al-Kitab al-mukhtasar fi hasab al-jabr wa-l-muqabala,* "The Compendious Book on Calculation by Completion and Balancing" (Nabirahni, Evans, & Persaud, 2019; Britannica). Used as "the standard mathematical text at European universities until the 16th century" (Abdulrahman, 2016, p3), it "is considered to be the first book on algebra" (Nabirahni, Evans, & Persaud, 2019, p.2). This highly influential book includes steps for finding the area of the circle, as well as for finding the volume of solid shapes, such as spheres and pyramids. It also includes solved problems on legacies using arithmetic and simple linear equations, as well as methods for solving quadratic equations algebraically (in words) and geometrically (Nabirahni, Evans, & Persaud, 2019; Abdulrahman, 2016). Al-Khwarizmi explained that he wrote his book for people's needs in resolving issues like inheritance, partition, lawsuits, trade, finding land area, and engineering, (Ahmed, 2005; Mashrafah & Ahmed, 1937).

Al-Khwarizmi introduced the three fundamental algebraic methods. The first, reduction, is the process of writing and simplifying mathematical expression; the second, completion, is the process of moving the negative number quantity from one side of the equation to the other and changing the sign to positive; the third, balancing, is subtracting the same quantity from both sides of the equation and eliminating like terms on opposite sides of the equation (Abdulrahman, 2016).

According to Abdulrahman, al-Khwarizmi did not use any symbols or algebraic notation in his algebra book, as we do nowadays. Instead, his way of solving quadratic and linear equations was all in words and diagrams to illustrate his work.

For example, when he dealt with quadratic and linear equations, he wrote equations in six different standard forms, as follows:

1. Squares equal to roots → $ax^2 = bx$.
2. Squares equal to numbers → $ax^2 = c$.
3. Roots equal to numbers → $bx = c$.
4. Squares and roots equal to numbers → $ax^2 + bx = c$.
5. Squares and numbers equal to roots → $ax^2 + c = bx$.
6. Roots and numbers equal to squares → $bx + c = ax^2$.

(Nabirani, Evans, and Persaud 2019; Abdulrahman, 2016; Mashrafah and Ahmed, 1937)

However, instead of using letter symbols, Al-Khwarizmi called x^2 a square, x a root, and c a number (Mashrafah & Ahmed, 1937).

Elsewhere, al-Khwarizmi explained equations that contain two terms, then mentioned the quadratic equations that include three terms and positive numbers. He classified the quadratic equations to three types (Mashrafah & Ahmed, 1937):

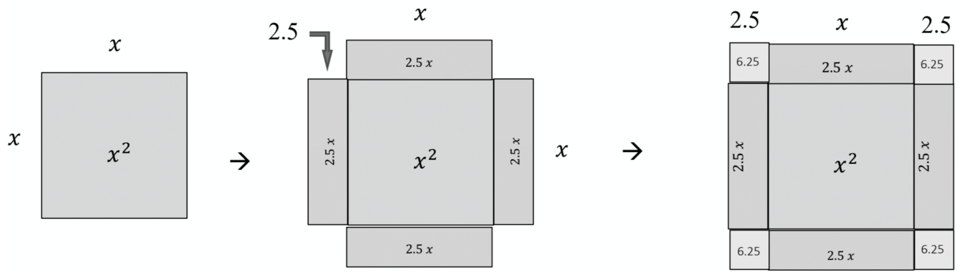
Two Terms:

1. $ax^2 = bx$
2. $ax^2 =c$
3. $bx=c$

Three Terms:

1. $ax^2 + bx = c$
2. $ax^2 + c = bx$
3. $bx + c = ax^2$

When solved $x^2 + 10x = 39$, he explained the process in words and drew a diagram to illustrate his work. He drew a square to represent x and separately divided $10x$ to 4 parts. Each rectangular quarter equals $2.5x$. Then, he put each part ($2.5x$ or $5/2x$) in each side of the square, which all equal 39. After that, he completed the square by constructing small squares in each corner of the diagram, with each square's area equal to 6.25; 4 of them equal a total of 25. That creates a big square with an area of 64 (39+25) and the root of 64 is 8. Now, each side of the big square is 8 which also equal x + 2.5 +2.5. That creates a linear equation, $x + 2.5 + 2.5 = 8 \rightarrow x+5=8 \rightarrow x=3$.



Even though al-Khwarizmi did not use symbols in his work, he was credited with inventing algebra because "he focused on more on the generalities of manipulating algebraic expressions than on the numbers they represent," which distinguished his work from arithmetic (p32).

Interestingly, there is a book in Arabic called *Kitab Algaber Walmuqabalah le Mohamed Abn Musa Al-Khwarizmi,* "The book of Algebra and Balancing for Mohamed Abn Musa al-Khwarizmi." This book was written in 1937 by Ali Moustafa Mashrafah and Mohamed Musa Ahmed at the Egyptian University. The authors mentioned that they used a manuscript that was written about five hundred years after the death of al-Khwarizmi, which is a copy of Al-Khwarizmi's work. This manuscript was written in Cairo, Egypt and it was in Bodleian Library, Oxford, UK at the time of writing this book. This manuscript is the only one that survived from Al-Khwarizmi's work in Arabic. The Arabic version was published only once in 1831 by Fredeick Rosen, which was printed and translated to English. Mashrafah and Ahmed explained, analyzed, and commented on the part of the manuscript that dealt with algebra. For other topics, the authors only copied them without comments. They found some written comments on the sides of the manuscript, but did not think it important to describe them (Mashrafah & Ahmed, 1937).

Other major contributions of al-Khwarizmi's included introducing the Hindu-Arabic numerals to European math, and adopting the concept of zero, which was then used by European universities, industries, and trading (Britannica; Abdulrahman, 3). His book *The Book of Addition and Subtraction According to the Hindu Calculation* was translated into Latin (*Algorithmi de Numero Indorum*); however, the original version in Arabic has unfortunately been lost. In this book, Al-Khwarizmi introduced the decimal system, which was created by Hindu mathematics in the 6th century, also introducing zero to complete the decimal system. He explained how to apply different operations (addition, subtraction, multiplication, and division) on the decimal system. In fact, the concept of algorithm we know now was created and developed based on an idea of al-Khwarizmi's, through which he provided sequenced steps solutions (Nabirahni, Evans, & Persaud, 2019).

Al-Khwarizmi's work in astronomy involved calculating the position of the sun, moon, and planet. He explained his work in astronomy books, which since have been translated into worldwide languages. He was the first who created an astronomical table (Abdulrahman, 2016; Nabirahni, Evans, & Persaud, 2019). Al-Khwarizmi also developed sundials, which were used to determine the time, and also astrolabes (Ahmed, 2015; Arndt, 2015). In addition, he wrote a compendium called *Kitab al-ta'rikh* (Chronicle), which recorded events that happened in Southwest Asia. "It has been noted that al-Khwarizmi used the astrological methods of this text to figure out the hour in which the prophet Mohammed was born according to the astrological events of his life" (Nabirahni, Evans, & Persaud, 2019, P4). This book, however, does not survive (Brentjes). *Zīj al–Sindhind,* an astronomical handbook according to the *Sindhind,* was another part of his writing and it survives only in Latian (Brentjes). He also wrote two works on the astrolabe (Brentjes), and a description of the Jewish calendar (Brentjes). His work *Kitab al-rukhnama* (On the Sundial) did not survive (Nabirahni, Evans, & Persaud, 2019).

In terms of geography, al-Khwarizmi wrote a book called *Surat Al-ared,* which means "image of the earth," in 833. In it, he drew and described a world map including a list of important places. He also mentioned the latitudes and longitude for around 2,400 places and landmarks worldwide, which then became the basis for the first World map (Abdulrahman, 2016; Nabirahni, Evans, & Persaud, 2019). He divided these places into six sections: seas, mountains, islands, rivers, cities, and regions (Abdulrahman, 2016; Nabirahni, Evans, & Persaud, 2019). Furthermore, he revised and corrected Ptolemy's views on geography (Ahmed, 2005; Arndt, 2015). According to an article by Nabirahni, Evans, and Persaud, al-Khwarizmi identified information from Ptolemy's geographical work, corrected that information, and made it more accurate (Nabirahni, Evans, & Persaud, 2019). Additionally, al-Khwarizmi worked with other scholars to calculate earth's circumference and determine that the Earth was spherical, an idea that was later accepted by many scientists (Nabirahni, Evans, & Persaud, 2019).

Al-Khwarizmi's scientific work was widely known and had significant contributions in Mathematics, Geography, and Astronomy. He was the author of many books and treatises in these subjects. Centuries later, we can see that al-Khwarizmi's scientific work had significant contributions in Mathematics, Geography, and Astronomy. He was the author of many books and treatises in these subjects, some of which were sponsored by Caliph Al-Ma'mun (Arndt, 2015). Indeed, his achievements continue to have significant influence on the development of mathematics, geography, astronomy nowadays. No wonder that "history remembered him as the father of Algebra, the grandfather of computers and one of the most creative scientific minds of early Middle East culture" (Abdulrahman, 2016, p2).

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