The Rise of the Primes As Their Gaps Succumb

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A college-level course in "Number Theory" may not seem like an exciting way to pass the semester. Indeed, the course name alone is enough to send chills up some people's spines. As a Math Education major, even I was tense about taking a course in a subject I had little knowledge about. The course is filled with such brow-raising topics as Diophantine equations, the Sieve of Eratosthenes, and the Euler Phi function, to name a few. By the middle of the semester, I thought I'd just about had it with these brain twisters. Then one day, the professor brought in an article titled, "The Beauty of Bounded Gaps." That was the day number theory became fun.

The Hype: the bounded gaps conjecture states that, although the gap (or difference) between consecutive prime numbers seems to increase, there are actually infinitely many prime numbers where this gap is bounded, or constant. In a recent mathematical breakthrough, mathematician Tom (Yitang) Zhang proved that this was indeed the case, showing that there are infinitely many prime numbers separated by at most 70,000,000 (Ellenberg). Since this breakthrough discovery, mathematicians the world over have scrambled to lower this number, with recent results showing a gap as small as 633 (Web). One reason to be excited about this foray into the search for smaller and smaller gaps is that it may succeed in proving another important conjecture in number theory: the twin prime conjecture which states that there are infinitely many twin primes p such that p+2 is also prime. Examples of twin primes are {3,5}, {5,7}, {11,13} and {17,19}. Another reason, as pointed out by Jordan Ellenberg, is "Zhang's success points to a prospect even more exciting than any individual result about primes—that we might, in the end, be on our way to developing a richer theory of randomness" (7).

The twin prime conjecture attracted the attention of mathematicians in the 19th century, when mathematician Alphonse de Polignax noticed that twin primes become less common as numbers get bigger (Ghose). Still, he believed that such twin primes are infinite. For the past two hundred years, mathematicians have tried to solve this difficult problem. Today, with the help of Zhang's breakthrough, they are one step closer to solving the twin prime conjecture, or so they hope.

So who is Yitang Zhang, and how did he achieve such a breakthrough? It's important to note that Zhang did not reach his result on his own. Rather, he relied on many years of published research in the area of number theory. In particular, Zhang relied on an important paper known as the GPY paper, named after mathematicians Goldston, Pintz and Yaldirim. The GPY paper showed that "there would always be pairs of primes closer than the average distance between two primes" (Chang). Although important in its own right, the paper was shy of the result mathematicians had been hoping for. Besides relying on the GPY results, Zhang had also used math tools developed by mathematicians Bombieri and Friedlander in the 1980s (Chang).

So why should we care about an obscure result in number theory? Although formulating theories about prime numbers was once considered a purely academic pursuit, modern times have found uses for prime numbers that make them relevant in a variety of fields, from theoretical physics and cryptography to internet commerce and, of course, mathematics. An article in phys.org, a technology-based website, describes a research finding that illustrates the relevance of prime numbers in modern times:

> In a recent study, Bartolo Luque and Lucas Lacasa of the Universidad Politécnica de Madrid in Spain have discovered a new pattern in primes that has surprisingly gone unnoticed until now. They found that the distribution of the leading digit in the prime number sequence can be described by a generalization of Benford's law. In addition, this same pattern also appears in another number sequence, that of the leading digits of nontrivial Riemann zeta zeros, which is known to be related to the distribution of primes. Besides providing insight into the nature of primes, the finding could also have applications in areas such as fraud detection and stock market analysis. (Zyga)

These new and exciting uses for prime numbers give our knowledge of prime numbers a new dimension of significance, making it more important than ever that we not only dig deeper into the nature of the primes, but also that we use our knowledge of prime numbers responsibly.

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