A One-Sided Perspective with a Twist

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When I heard my instructor introduce a discussion on the Mobius band, my initial reaction was to think of physics and my previous knowledge of the bending of space. These areas include relativity and multiverse theory, with information gathered from books such as *The Elegant Universe* by Brian Greene, in addition to various documentaries and educational videos. However, as the presentation began, I thought less about statements I had previously heard and focused more on the new information presented to me.

Our first objective is to define the Mobius strip, also known as the Mobius band. The Mobius strip was discovered by mathematicians August Ferdinand Mobius and Johann Benedict Listing in 1858. It is defined by *Merriam-Webster* as "a one-sided surface that is constructed from a rectangle by holding one end fixed, rotating the opposite end through 180 degrees, and joining it to the first end." However, this definition lacks substance. What is meant by a one-sided surface? We live in three-dimensional space consisting of height, width, and depth. We can think of incredibly complex shapes with a near infinite amount of sides. Yet the fewest sides we can conceive of are two. Now imagine an infinitely flat object with only a front and back. It is possible for our minds to grasp this concept, but it is near impossible to imagine an object with only one side. How can something exist having a front but not a back? We can picture an infinitely thin line, but a line itself has no sides. We will delve into what exactly it means for the Mobius strip to have one side, the resulting properties, and areas of application in mathematics and science.

In our exercise we began by creating a Mobius strip. We took a long rectangular strip of paper and brought the two ends together, forming a loop. We then twisted one end of the paper by 180 degrees and taped the ends together, forming a Mobius strip. This was followed by a guided experiment led by Professor Singh. First we made an incision at one third of the width parallel to the length of the strip. We continued to cut parallel to the edge until we went all the way around the Mobius strip and met with the original incision. This resulted in two interlocking rings with several interesting relationships between them, the most obvious of which is that one ring is twice as long as the other. We shall call one strip gamma and the other strip beta. Upon further inspection, I discovered strip gamma, the longer of the two, is actually not a Mobius strip while strip beta, the shorter ring, remained the same. I then constructed an additional Mobius strip the same size as the original. I found strip beta to be the same size as the original Mobius strip before any tampering.

It is obvious that strip gamma had changed while strip beta essentially remained the same. Both strips resulted from the incision made along the Mobius band. Strip gamma resulted from the part of the incision closer to the edge. As the cut was made it was necessary to go around the Mobius band twice in order to reach the original incision. This caused strip gamma to be twice the original length. In addition, while going twice around, the half twist of the Mobius strip was doubled and became one full twist of 360 degrees. This resulted in strip beta having two sides, undoing the single side phenomenon of the Mobius strip.

Let us take a long rectangular sheet of paper, and label one of the longer sides Edge A and the opposing side Edge B. We will now label the corners, or vertices, A1, A2, B1, and B2 accordingly. Vertex A1 is next to B1 and vertex A2 is next to B2. If we bend the paper into a ring without any twist, Vertex A1 will meet A2 and vertex B1 will meet vertex B2. Now, if we twist the paper 360 degrees (what I will call a full twist) once again vertex A1 will meet vertex A2 and vertex B1 will meet vertex B2. If we then start from the beginning but use a 180 degree twist (what I will call a half twist) as we bring the ends together, vertex A1 will meet vertex B2 and vertex A2 will meet vertex B1. This has made a connection between Edge A and Edge B. What were two previous edges are now only a single unified edge. We thus find that not only does the Mobius strip only have one side, but it also has only one edge.

When we look at the Mobius strip we see a long band in between two edges. However, our optical interpretation of this shape could not be more wrong. There is only one curved plane, and that plane is created with a single line. I believe that it is this trait of having a single edge that gives the Mobius strip its fascinating properties. This single side phenomenon of the Mobius band occurs when the degree of the twist in the paper must be in the form 180+360x, where x is an integer. A twist of degree 360x, where x is an integer, would give a strip with two sides. Twists by any other degree and the four vertices would not meet.

My hypothesis guided me towards the field of topology. Topology is defined as “the study of qualitative properties of certain objects (called topological spaces) that are invariant under a certain kind of transformation (called a continuous map), especially those properties that are invariant under a certain kind of transformation (called homeomorphism)” (*Merriam-Webster Online,* n.d.). While this definition is very dense, it is easy to understand if we break down its separate components. The “qualitative properties of certain objects” are properties which are not measured, or quantitative. These measurements include the object’s length, angle, etc. Qualitative properties include the number of holes an object has, or number of legs, etc. An invariant transformation would alter this shape without changing its qualitative properties. Essentially, topology studies show we can transform an object without losing certain mathematical properties. A homeomorphism transformation occurs when given sufficient pliability you can mold one shape to another without any cutting or attaching.

The Mobius strip is unique to topology given that it is non-orientable. An object is orientable if you can complete the test for right or left-handedness. That is, if we choose to follow the strip along an edge with either right or left hand, we will return to our original point on the same side. However, as we were shown in the initial experiment, we must travel around the Mobius strip twice in order to reach our initial starting point on the same side. This is why we had to go twice around in order to reach the initial incision. However, the first time we go around the Mobius strip, we wind up on the opposite side, hence the Mobius strip is non-orientable. The Mobius strip is unique in this regard for two reasons: it is a two dimensional object and it is a subspace of every other non-orientable object. Two-dimensional objects are the lowest possible dimensional objects that can be non-orientable. Otherwise, we are merely dealing with either a line or objects of a lower dimension. Object X is a subspace of object Y if all parts of object X are contained in object Y. For example, a two-dimensional square is a subspace of a cube. If an object is non-orientable it must have the Mobius band as a subspace.

The Mobius strip also has several applications in physics, one of which is multiverse theory. We are used to living in three-dimensional space made of height width and depth—four dimensions, if you include time or duration. Multiverse theory states that there are higher dimensions, with most theories stopping at ten. This is not to be confused with dimensions in string theory. String theory has to deal with incredibly complex spatial dimensions on a subatomic level too small for us to be aware of—just as when we walk on a beach, we don't feel each individual grain of sand, but the beach as a whole. Multiverse theory concerns itself with higher dimensions. These dimensions, at first, are beyond our capability to understand. What is four-dimensional space? How can something like that possibly exist? Well, as we've been discussing, dimensions don't always behave as we think they would.

The Mobius band is a two-dimensional object moving through three-dimensional space. If a two-dimensional being traveled along the Mobius strip, it would think the Mobius strip were a normal, flat plane. The two-dimensional being would be completely ignorant to the fact that it's traveling through three-dimensional space. Similarly, we may be able to travel through a higher dimension of space without being fully aware of it. It might require similar bending or warping of space as occurs with the Mobius strip. One basis for this conjecture is due to the Mobius strip being a subspace for every non-orientable object. Theoretically, travel between timelines and alternate universes might be possible based on an invention involving the Mobius strip.

Another theoretical application of the Mobius strip is in the study of wormholes. A traditional wormhole is made by bending space and connecting two separate points. This allows an object to instantly travel from point A to point B. However, during this process the object traveling between these two points becomes inverted, a mirror image. In order to counter balance this, it is suggested that space can be bent in the wormhole similarly to a Mobius strip. This second inversion will return the traveling object to its original orientation. However, when bending space for this wormhole, we create multiple paths in the universe, both the regular path C and a path D similar to a Mobius strip. Here, anything traveling along path D will become inverted: charges, rotation of subatomic particles, etc. Yet, this would only be observable by objects that had not taken this same path.

Charges and rotations of particles become relative and only worth note in a restricted area; there would be no absolute definition. This theoretical universe is known as an Alice Universe.

In the presentation made by Professor Satyanand Singh, we were given the opportunity to embrace and interact with the Mobius strip. We were given the opportunity to ponder and rediscover the Mobius strip as it continues to challenge our way of thinking over a hundred years after its discovery. As we continue to progress in the fields of mathematics, physics, and technology, the Mobius band will become an even more useful tool to help us understand and progress towards the future. I would like to thank Professor Singh for introducing me to this fascinating topic and guiding me through this process of discovery and intrigue.

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