A Thought on "The Struggle for Existence from the Point of View of the Mathematicians," Chapter III in G.F. Gause's 1934 Book

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Gause says that unlike biologists who have had limited accomplishments analyzing the struggle for existence of various populations, mathematicians seem to have implemented theories to investigate this phenomenon. Sir Ronald Ross in 1908 and 1911 made the first attempt to study and quantify this phenomenon with mathematical models. Ross focused his study on analyzing the spread of malaria in a local community and built a mathematical model under the assumption that the populations of both human beings and mosquitoes are constant during the period of time of his study. He asserted that the number of newly infected people depends on the number of infective and non-infective mosquitoes and vice versa. The number of bites made by infective mosquitoes on healthy persons per unit of time changes the number of sick people. Simultaneously the change in the number of infective mosquitoes depends on the number of infected and non-infected people.

Ross expressed in mathematical terms this phenomenon by using the following two differential equations that need to be solved simultaneously:

$$dz/dt = b_1 f_1 z_1 (p-z)/p - rz$$
$$dz_1/dt = b_1 fz (p_1 - z_1)/p - M_1 z_1$$

where
p = total number of human individuals in a given locality.
p_1 = total number of mosquitoes in a given locality.
z = total number of people infected with malaria.
z_1 = total number of mosquitoes containing malaria parasites.
fz = total number of infected malarians.
$f_1 z_1$ = total number of infective mosquitoes with matured parasites.
r = recovery rate or fraction of infected population that reverts to healthy state per unit of time.
M_1 = mosquito mortality or death rate per head unit of time.
b_1= the number of bites.
t = time.

Lotka analyzed Ross's model and mentioned that since assumptions are made that the populations of both human beings and mosquitoes are constant, the results of Ross's equations will not be completely accurate and are prone to errors. He argued that the model developed by Ross portrayed an idealized situation and is inconsistent with reality. Biologists have also argued that representing this phenomenon by exact mathematical equations is pointless since this process is usually sensitive to the slightest change of the environmental conditions.

However, Ross's idea in formulating this process seems interesting. Ross came to the conclusion that (a) if the number of anophelines and the other parameters are constant all the time, the ratio of malaria converges to an equilibrium point; (b) if the initial number of anophelines is quite high, the ratio of malaria approaches a rest point; and (c) if the number of anophelines is low, the ratio of malaria eventually converges to zero, that is the spread of the disease will stop and the infection will die out.

Comment

Ross's formulation of the mathematical model and conclusions about the course and the results of the competition between the mosquitoes and human population are informative. However, the results would be different if the environment became more complex. In other words, the assumptions made by Ross are simplistic and other factors need to be considered. For example, like Lotka, I find the assumptions made by Ross unrealistic in considering both the human and mosquito populations as constant. Ross's assumption that the rate of birth is equal to the rate of death is questionable and made his case an idealized one. Ross ignored the fact that many factors can affect both populations such as: people/mosquitoes leaving the locality, people/mosquitoes arriving at the locality, imported cases of malaria from other countries, the rate of growth of people/mosquitoes, the rate of death of people/mosquitoes, the environmental conditions of both populations, and finally the rate of contamination within species. Taking into consideration all of these factors might improve Ross's mathematical model.

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