Review Problems, Exam I
Math 2440 – Discrete Structures and Algorithms, I

1. Let $P(x)$ be the statement $x = x^4$. If the domain consists of integers, what are these truth values?

   (a) $P(0)$, (b) $P(2)$, (c) $P(-1)$, (d)$\forall x P(x)$ and (e)$\exists x P(x)$.

2. Construct a truth table for each of these compound propositions

$$p \vee \neg q, \quad (p \vee \neg q) \rightarrow q, \quad (p \vee q) \rightarrow (p \wedge q).$$

3. Consider the statement "The sum of any two perfect squares is a perfect square. "

   Prove or disprove this statement.

4. Show that

   (a) $p \vee (p \wedge q)$ and p

   (b) $p \rightarrow q$ and $(\neg q) \rightarrow (\neg p)$

   are logically equivalent.

5. Write the negation of the following statements:

   (a) $\forall x \exists y (2x + 11 > y^2)$.

   (b) $\exists x \forall y (x + y = 0)$.

6. Prove that if x^{100} is irrational, then x is irrational.

7. Translate the statement "The sum of any two positive integers is a positive integer" into a logical expression using quantifiers.

8. Let n be an integer. Prove that if $3n + 5$ is odd, then n is even.

9. Let n be an integer.

   (a) Prove that if n is odd then $2n + 3$ is odd.

   (b) Prove that if $7n + 8$ is odd then n is odd.

10. Domain of $x = \{-2, -1, 0, 1, 2\}$. Write out each of these propositions using disjunctions, conjunctions and negations.

    (a) $\exists x P(x)$

    (b) $\forall x P(x)$

    (c) $\neg (\forall x P(x))$.