$$\frac{d}{dx}\sin(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x)(1-\cos(h)) + \cos(x)\sin(h)}{h}$$

$$= \lim_{h\to 0} \sin(x)\left(\frac{1-\cos(h)}{h}\right) + \lim_{h\to 0} \cos(x)\left(\frac{\sin(h)}{h}\right)$$

$$= \sin(x) \lim_{h\to 0}\left(\frac{1-\cos(h)}{h}\right) + \cos(x) \lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)$$

$$= \cancel{\sin(x)(0)} + \cos(x)(1)$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$f(x) = \sin(x) \qquad \frac{d}{dx}\left(\cos(x)\right) = -\sin(x)$$

$$f'(x) = \cos(x)$$

$$f''(x) = -\sin(x) \leftarrow \text{2nd derivative}$$

$$f'''(x) = -\cos(x) \leftarrow \text{3rd derivative}$$

$$f^{IV}(x) = \sin(x)$$

$$f^{V}(x) = \cos(x)$$

$$\vdots$$

$$y = \cos(x)$$

$$\frac{dy}{dx} = -\sin(x)$$

$$\frac{d^2y}{dx^2} = -\cos(x)$$

$$\frac{d^3y}{dx^3} = \sin(x)$$

$$\frac{d^4y}{dx^3} = \cos(x)$$

$$\frac{d}{dx}\left(\tan(x)\right) = \frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right)$$

Quotient Rule

$$\frac{f = \sin(x)}{g = \cos(x)} \quad \bigg| \quad \begin{matrix} f' = \cos(x) \\ g' = -\sin(x) \end{matrix}$$

$$= \frac{f'g - fg'}{g^2}$$

$$= \frac{\cos(x)\cdot\cos(x) - (\sin(x))(-\sin(x))}{(\cos(x))^2}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{(\cos(x))^2} \qquad\qquad = \frac{\cos^2(x)}{\cos^2(x)} + \frac{\sin^2(x)}{\cos^2(x)}$$

$$= \left(\frac{1}{\cos(x)}\right)^2 \qquad\qquad = 1 + \tan^2(x)$$

$$\frac{d}{dx}\left(\tan(x)\right) = \boxed{\sec^2(x)} \qquad\qquad = \sec^2(x)$$