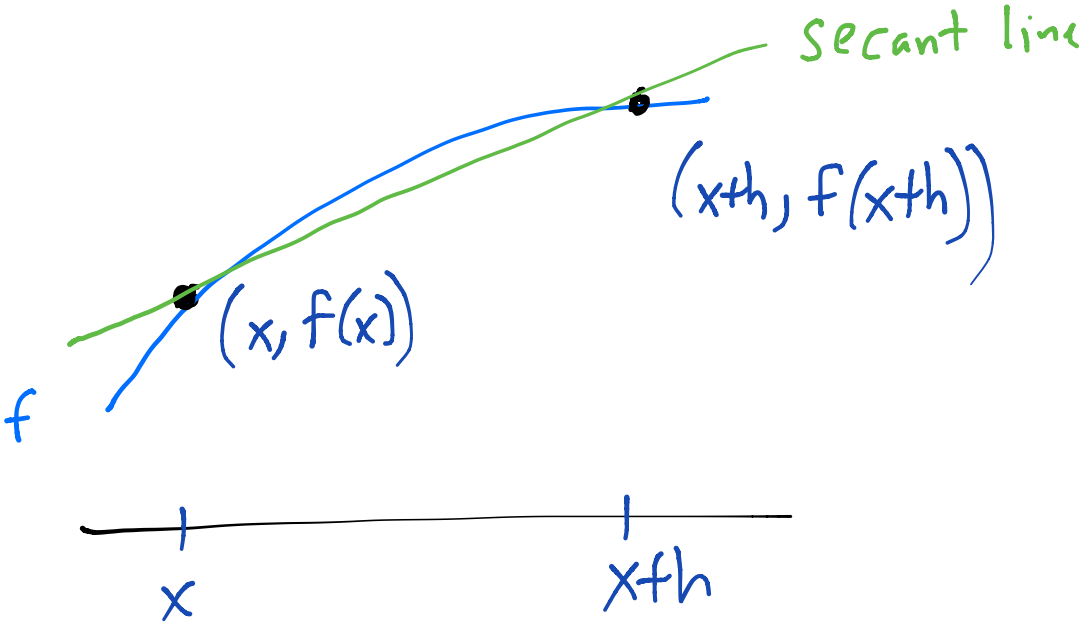
$$\frac{f(x+h)-f(x)}{h}$$

- PreCalculus
  - difference Quotient

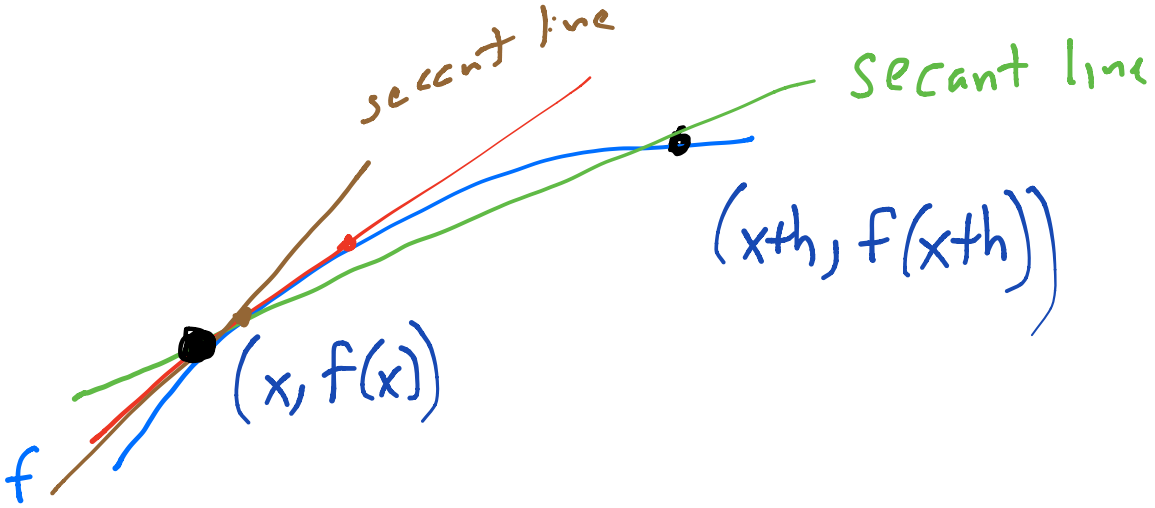


slope : $\frac{\Delta y}{\Delta x} = \frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$

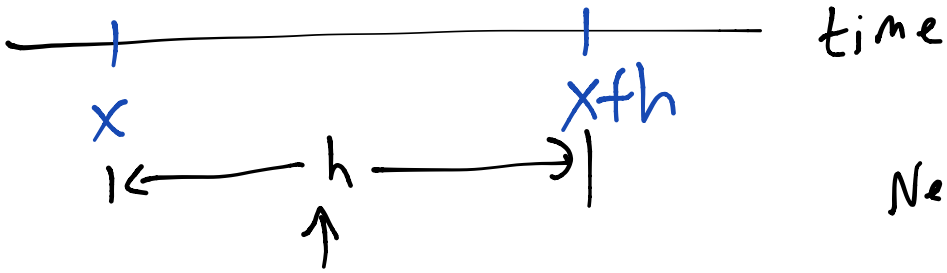
difference quotient:
slope of secant line

* Calculus deals in instantaneous
not over time

point in time
↳ no length in time



$$\frac{f(x+h)-f(x)}{h}$$



h is a difference in time

Need $h=0$?

$\rightarrow f(x+h) = f(x)$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$ ← slope of tangent line to a function

← instantaneous change

← limit definition of derivative function.

* if given $y=f(x)$

derivative function: y' , $\frac{dy}{dx}$, $f'(x)$, $D_x f$

Lagrange

Newton

Leibnitz notation

Euler

dy

Linear Approximation

Objective: $(-2.01)^3$ on calculator: $-8.1201\ldots$

Use tangent line equation to approximate the value.

$f(x) = x^3$

* Need a nice place for tangent point (CLOSE)

Nice value: $(-2)^3 = -8$

$f(-2) = -8$

$(a, f(a)) = (-2, 8)$

$\vdots$

Using tangent line: $y = 12x + 16$

$y = 12(-2.01) + 16$

$y = -8.12$

Derivative of constant functions

Let $f(x) = c$

$f(x+h) = c$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c - c}{h}$$

$$= \lim_{h \to 0} \frac{0}{h}$$

"$\frac{0}{0}$"?

h is close to 0
not really 0.

$$= 0$$

so, we good.

Derivative of the sum of two functions

$$m(x) = f(x) \pm g(x)$$
$$m(x+h) = f(x+h) + g(x+h)$$

$$\lim_{h \to 0} \frac{m(x+h) - m(x)}{h}$$

$$\lim_{h \to 0} \frac{(f(x+h) + g(x+h)) - (f(x) + g(x))}{h}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x) - g(x+h) - g(x)}{h}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$\frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x)$$