Steen Eiler Rasmussen. *Experiencing Architecture*

CHAPTER V

Scale and Proportion

Legend has it that one day when Pythagoras passed a smithy he heard the clang of three hammers and found the sound pleasing. He went in to investigate and discovered that the lengths of the three hammer-heads were related to each other in the ratio of 6:4:3. The largest produced the keynote; the pitch of the shorter was a fifth and that of the shortest an octave above it. This led him to experiment with tautly stretched strings of different lengths and he ascertained that when the lengths were related to each other in the ratios of small numbers the strings produced harmonious sounds.

This is only a legend and in my opinion it is too good to be true. But it tells us something essential about harmony and how it is produced.

The Greeks tried to find some explanation for the phenomena they observed. They said something like this: It makes the soul happy to work with clear mathematical ratios and therefore the tones produced by strings of simple proportions affect our ears with delight.

The truth is, however, that a person listening to music has no idea of the lengths of the strings that produce it. They have to be seen and measured. But whatever the Greeks' reasoning, they found that there was some relation between simple mathematical proportions in the visual world and consonance in the audible. As long as no one was able to explain what happens when a tone is produced and how it affects the listener, the relationship continued to be a mystery. But it was obvious that man was in possession of a special intuition which made it possible for him to perceive simple mathematical proportions in the physical world. This could be demonstrated as regards music and it was believed that it must be true of *visible* dimensions also.

Architecture, which often employs simple dimensions, was then as well as later frequently compared with music. It has been called frozen music. That scale and proportion play a very important role in architecture is unquestionable. But there are no visual proportions which have the same spontaneous effect on us as those which we ordinarily call harmonies and disharmonies in music.

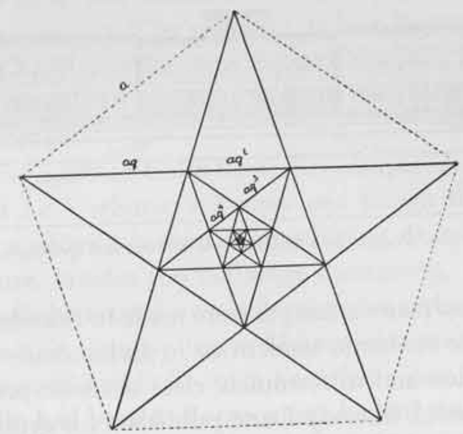
The tones of music differ from other, more accidental noises by being sounds produced by regular periodic vibrations and having fixed pitch. Vibrations which result from striking a chord constitute a keynote with a definite rate of frequency and a series of overtones with frequency rates that are double, triple, etc., the keynote rate. Tones with simple frequency ratios have the same overtones and when they are sounded simultaneously a new, absolutely regular period of vibrations will result and it will still be heard as a musical tone. But if sound waves of slightly different periods of vibration are set in motion the sound produced is incoherent and often directly unpleasant. If two sound waves with a frequency ratio of 15:16 arise simultaneously, they will reinforce each other every time the one has vibrated fifteen and the other sixteen times. This will produce extra large oscillations and between these strong blasts there will be points where the vibrations annihilate each other so that they become practically inaudible. The result will be a tone of a weird, quavering, uneven sequence which can be very unpleasant. A sensitive listener may actually get a stomachache from hearing such discords. But there is nothing analogous to this in the visual world, for while we are immediately aware of false tones, small irregularities in architecture can be discovered only by careful measuring. If two strings with lengths in the relation of 15:16 are struck simultaneously the resulting sound will be distinctly unpleasant. But if in a building that is divided in regular bays a difference in proportions of this same ratio were introduced probably no one would notice it. The truth is that all comparison of architectural proportions with musical consonances can only be regarded as metaphor. Nevertheless innumerable attempts have been made to work out principles of architectural proportioning analogous to the mathematical principles of musical scales.

There is one proportion (incidentally without parallel in music) which has attracted great attention ever since the days of antiquity. This is the so-called golden section. Pythagoras and

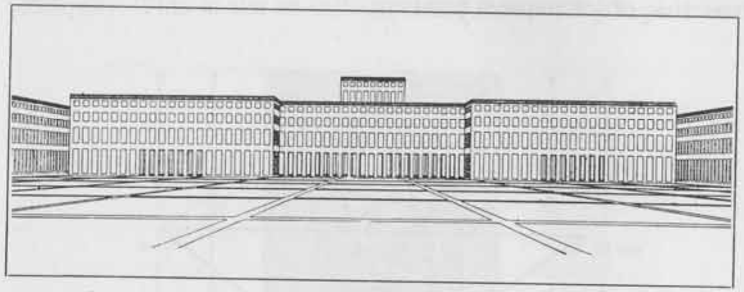
his disciples were interested in it, theorists of the Renaissance took it up again, and in our day Le Corbusier has based his principle of proportion, "Le Modulor," on it. A line segment is said to be divided according to the golden section when it is composed of two unequal parts of which the first is to the second as the second is to the whole. If we call the two parts a and b, respectively, then the ratio of a to b is equal to the ratio of b to $a+b$. This may sound somewhat complicated but is easily grasped when seen in diagram.

Until recently an ordinary Danish match box, bearing a picture of Admiral Tordenskjold, measured 36×58 mm. If we subtract the shorter side from the longer we get $58 - 36 = 22$. It is approximately true that 22 is to 36 as 36 is to 58. In other words, the mutual relation of the sides is that of the golden section.

Unfortunately for Denmark the economic situation of the country made it necessary to reduce the length of matchsticks and therefore Tordenskjold's portrait is now placed in a rectangle, which is regarded as less æsthetic. Formerly the various sizes of paper were also often based on the golden section and the same was true of letter-press printing.



To Pythagoras the pentagram was a mystical and holy symbol. A pentagram is a five-pointed star which is formed by lengthening the sides of a pentagon both ways to their points of intersection. The relation between the length of one of the sides of a pentagram's point and the side of a pentagon is the same as the golden section. By connecting the five points of the pentagram a new pentagon is formed, from that again a new pentagram, etc. In this way you get an infinite series of line segments which grow according to the rule of the golden section. This can be drawn in a diagram but these lengths cannot be expressed as rational numbers. On the other hand, it is possible to draw up a series of integers, the ratios of which come close to that of the golden section. These are 1, 2, 3, 5, 8, 13, 21, 34, 55, etc., each new unit being formed by adding together the two immediately preceding. The remarkable thing about this series is that the higher it goes, the closer it approaches the golden section ratio. Thus, the ratio 2:3 is far from it, 3:5 is closer, and 5:8 almost there. Incidentally, 5:8 is the approximation in rational numbers most often used.



Ivar Bentsen: Project for a philharmonic building in Copenhagen. 1918

Around 1920 many attempts were made in Scandinavia to get away from the romantic tendencies in architecture of the previous generation and to formulate clear æsthetic principles. In Norway Frederick Macody Lund published his great work "Ad Quadratum" in which he sought to prove that the great historical works of architecture were based on the proportions of the golden section. He suggested therefore that those proportions should be used in the reconstruction of Trondhjem Cathedral. In Denmark the architect Ivar Bentsen designed a large project for a philharmonic building in which the proportions were based on the above-mentioned series. It was to be built on a square grid in plan and in elevation to be proportioned according to the golden section rule. The distance between the balusters on the flat roof was the smallest unit, or module. The width of the pillars was set at three of these units and the window width at five. The top row of windows were square, that is 5×5, the next below 8×5, then 13×5, and finally the bottom row (which actually comprised two stories—a ground floor of shops and a mezzanine) was to be 21×5.

Even when this has been explained, as here, you cannot experience the interrelationship in the proportions of the philharmonic building in the same way that you experience it in certain natural phenomena in which there is a rhythmic progression in proportions. Many snail shells, for example, have whorls which grow steadily larger in regular progression from the innermost to the outermost, and this is immediately perceptible. But the whorls grow in several dimensions so that they continue to have the same proportions. The windows in Ivar Bentsen's building, on the other hand, increase only in one dimension and therefore change successively from square to more than four times as high as they are broad.

An American author, Colin Rowe, has compared a Palladio villa with one of Le Corbusier's houses and shown that there is a remarkable similarity in their proportioning. It is an interesting study because, besides the buildings themselves, we have both the plans and the artists' own reflections on architecture.

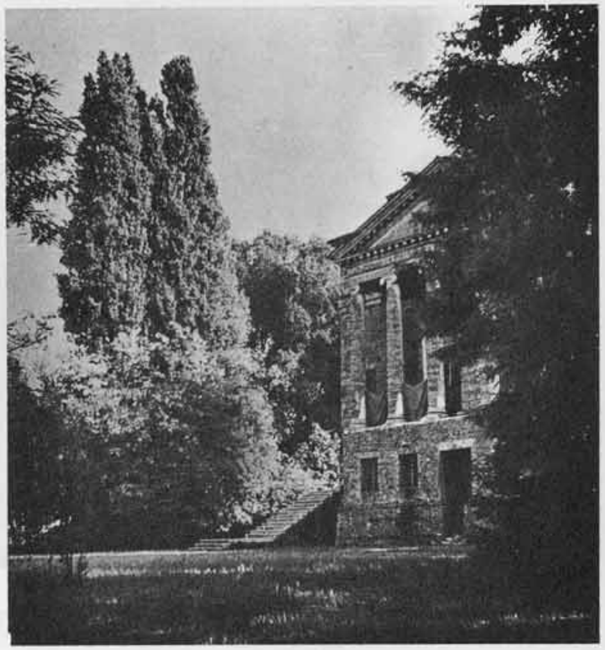
Palladio's villa, Foscari, lies in Malcontenta on the mainland, near Venice, and was built for a Venetian about 1560. By that time Palladio had been to Rome where he had studied the great ruins of antiquity and he now saw it as his mission to create architecture that was just as sublime in composition and simple in proportions. From the architectural world of pure harmonies one should be able to experience Nature in all its phases.

The main story of the Villa Foscari is raised high above the ground over a basement which resembles a broad, low pedestal. From the garden, staircases on either side lead up to the free-standing portico of the main floor. From here you enter the main room of the villa, a great barrel-vaulted hall, cruciform in plan, which runs through the entire building, affording a view of the garden at the back and of the approach with its large, symmetrically arranged avenues at the front. On either side of this central hall lie three absolutely symmetrical lesser rooms. This was in keeping with the Venetian custom of grouping the bedrooms and living rooms round a large, airy hall in the central axis. But in-

Palladio: Villa Foscari, Malcontenta near Venice. Main entrance façade. The façade design reflects the interior disposition in which a large barrel-vaulted central hall rises to the height of the pediment. The pediment in front corresponds to the loggia on the garden façade shown on the opposite page

stead of the Venetian loggia, which is pushed back into the block of the building, Palladio grafted a classic temple front onto the façade of the villa. Behind it, the house appears solid and monumental. Above the basement the outer walls present a pattern of large blocks in dimensions corresponding to the thickness of the walls—both outer and inner. Within the house, too, you are aware of the thickness of the walls that separate the rooms, each of which has been given definitive and precise form. At either end of the cross-arm of the central hall is a square room measuring 16 × 16 feet. It lies between a larger and a smaller rectangular room, the one 12 × 16, the other 16 × 24 feet, or twice as large. The smaller has its longer wall, the larger its shorter, in common with the square room. Palladio placed great emphasis on these simple ratios: 3:4, 4:4, 4:6, which are those found in

Palladio: Villa Foscari, Malcontenta. Garden front with loggia with enormous columns standing out from the body of the building

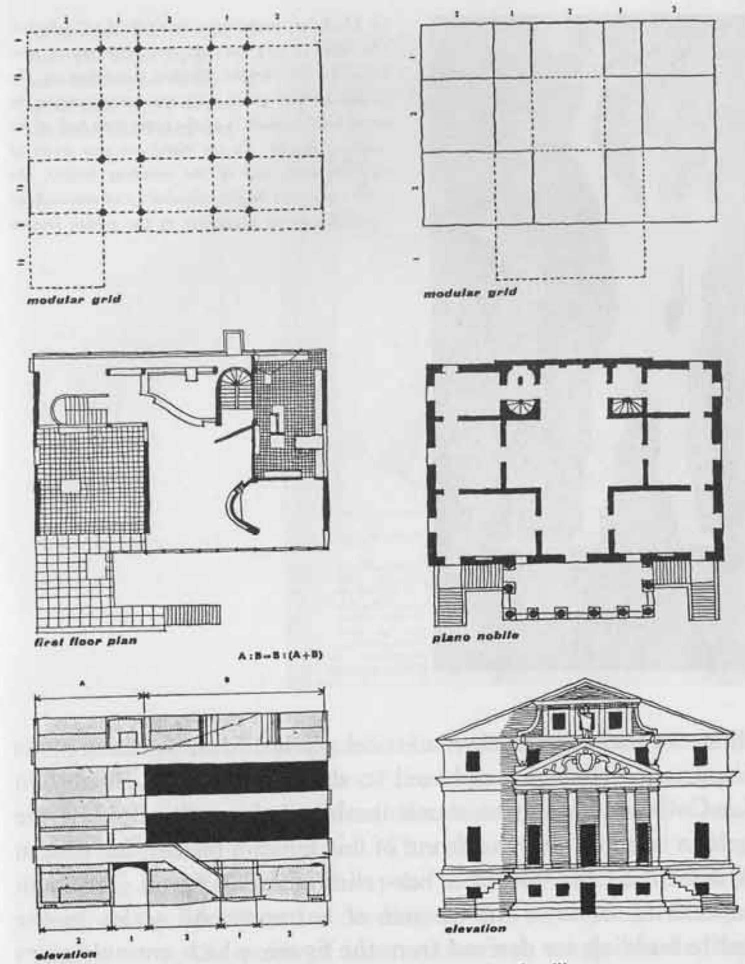


musical harmony. The width of the central hall is also based on sixteen. Its length is less exact because the thickness of the walls must be added to the simple dimensions of the rooms. The special effect of the hall in this firmly interlocked composition is produced by its great height, the barrel-vaulted ceiling towering high above the side rooms into the mezzanine. But, you may ask, does the visitor actually experience these proportions? The answer is yes—not the exact measurements but the fundamental idea behind them. You receive an impression of a noble, firmly integrated composition in which each room presents an ideal form within a greater whole. You also feel that the rooms are related in size. Nothing is trivial—all is great and whole.

In Le Corbusier's house in Garches, built for de Monzie in 1930, the main rooms are also raised above the ground but here

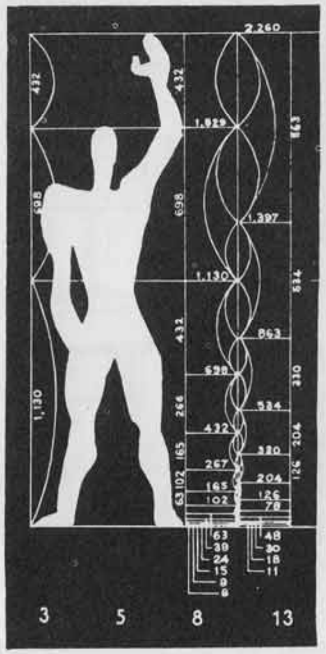
Le Corbusier: Villa in Garches

the outer walls hide the pillars on which it stands. Colin Rowe points out that these pillars form nodal points in a geometric net which is divided in a system very similar to the one that could be drawn of the Villa Foscari's supporting walls. In width the proportions in both cases are 2, 1, 2, 1, 2. But while Palladio used his system to give the rooms fixed and immutable shapes and harmonic interrelation in proportions, Le Corbusier has, if anything, suppressed his supporting elements so that you are not aware of them and have not the slightest feeling of any system in their placement. That which is felt to form the fixed and immutable system in the Garches house is the horizontal planes separating the floors. The location of the vertical partitions is quite incidental and, as already mentioned, the pillars are not noticed at all. Le Corbusier himself has stressed the fact that the house is divided in the ratio 5:8, that is, approximating the golden section, but he has hidden it so well that probably no one who has



Colin Rowe's comparison of proportions in villas designed respectively by Le Corbusier and Palladio

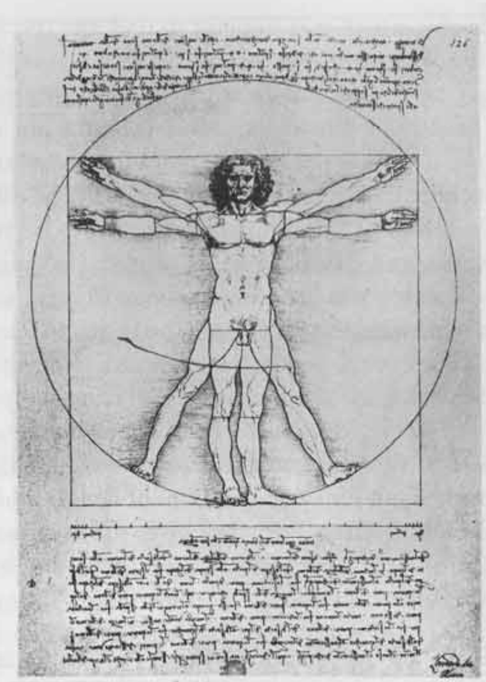
seen the building had any inkling of it. There is no similarity in the principles of composition in the two buildings. Palladio worked with simple mathematical ratios corresponding to the harmonic ratios of music and he probably never thought of the golden section. Le Corbusier worked with rooms of widely



Le Modulor, proportion study by Le Corbusier. The man is 183 cm tall and with raised arm 226 cm. His height divided according to the golden section gives 113 cm corresponding to navel height which is at the same time half of his reaching height. To the right are two series of measurements, one of the reaching height, the other the man's height, divided up in diminishing measurements according to the golden section

different shapes in an asymmetrical whole and the location of his important divisions was based on the golden section. Since then Le Corbusier has gone much further in his cultivation of the golden section. On the front of his famous residential unit in Marseille he has placed a bas-relief of a male figure. This man represents, he says, the essence of harmony. All scales in the entire building are derived from the figure, which not only gives the proportions of the human body but a number of smaller measurements based on the golden section.

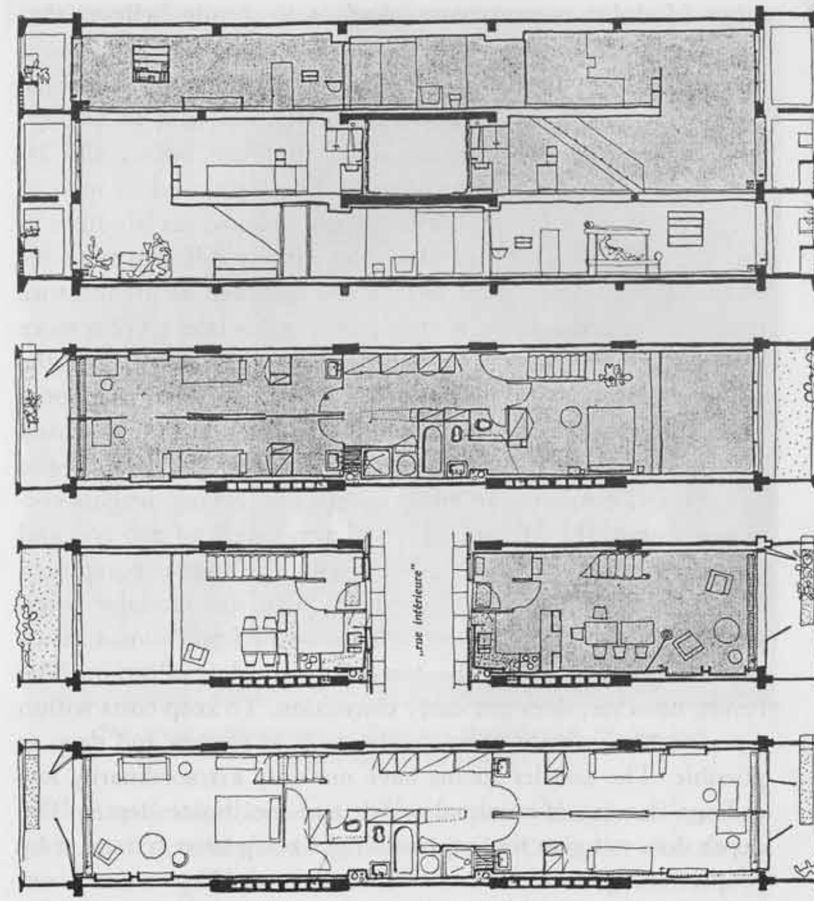
How he has arrived at these results makes interesting reading. You feel that antiquity with its combination of religious mysticism and artistic intuition lives on in this man who, for many people, stands as the representative of rational clarity and modern thought. Originally Le Corbusier placed the average man's



Leonardo da Vinci's ideal man. The man's navel marks the center. With hands outstretched he can reach the circle's periphery

height at 175 cm. This figure he divided according to the golden section rule and got 108 cm. Like Leonardo da Vinci and other Renaissance theorists he found that this corresponds to the height from the floor to man's navel. There was believed to be a deeper meaning in the fact that man, the most perfect creation of Nature, was proportioned according to this noble ratio and that, furthermore, the point of intersection was neatly marked by a little circle. Le Corbusier then divided his navel height in the same way and continued with sub-divisions until he obtained a whole harmonic series of diminishing measurements. He also found—likewise in accordance with the masters of the Renaissance—that man's height with upraised arm was double the navel height, i. e. 216 cm. It must be admitted that this measurement seems of greater importance to the architect than navel height, which it is difficult to find any use for at all in architecture. However, the awkward thing about the raised arm height is that it does not form part of the newly established scale of "beautiful" dimensions. But this did not deter Le Corbusier, who used it as the starting point for a whole new series of golden section measurements. In this way he obtained two sets of figures to work with, which proved to be very fortunate.

But one day he learned that the average height of English policemen was six feet, or about 183 cm, and as average height is increasing the world over, he began to fear that the dimensions of his houses would be too small if he utilized measurements derived from the height of the average Frenchman. Therefore he resolutely established 183 cm as the definitive quantity from which all other measurements were to be derived. He then worked out his two final series of figures which give a great many variations, from very small up to the very largest. What he cannot find in one he is almost sure to find in the other. But still you would seek vainly for a measurement for anything so simple as the height of a door or the length of a bed. Man's height of 183 cm is too small; a door should preferably be somewhat higher than the people who will go through it. And the raised arm height of 226 cm, which Le Corbusier uses as the ceiling height for the smallest rooms in the Marseille block, is too high for a door. In a diagram he has shown how the various measurements, from man's height down, can be employed for different purposes and functions, such as the high desk or platform, table heights, various seat heights, etc. In other words, he has not followed the scientific method of measuring things to determine the extreme limits for their dimensions, but with the help of his two series (in which only man's height and upraised arm height have been determined by measuring) he has arrived at two sets of measurements which he believes in and which therefore *must* suit all purposes. Even if you attached great æsthetic value to the proportions of the golden section it still would not justify the results because the measurements which follow each other in his



Le Corbusier: The Marseille block. Cross-section and plans of flats. Scale 1:200

tables, and which will often be seen together, have not that ratio (e.g. man's height and upraised arm height). Le Corbusier himself feels that the two series are of great service to him. As pointed out earlier, we are not spontaneously aware of simple proportions in dimensions as we are of harmonic proportions in music. Le Corbusier, therefore, corrects every one of the measurements that he arrives at intuitively so that it will correspond to one or the other Modulor measurement. And as he firmly believes that "Le Modulor" satisfies both the demands of beauty—because it is derived from the golden section—and functional demands. Le Modulor is for him a universal instrument, easy to employ, which can be used all over the world to obtain beauty and rationality in the proportions of everything produced by man.

Let us examine how he himself has employed his Modulor in the Marseille block. This building is entirely different from his earlier works. While they were to be regarded as architecture based on the principles of Cubist painting, his later work is more like gigantic sculpture. The buildings are still raised above the ground but now on enormous substructures. The residential unit in Marseille is like a mammoth box placed on an enormous trestle. The box is divided into innumerable small cells—the apartments, consisting of small rooms with ceiling heights corresponding to the Modulor's raised arm height of 226 cm, and larger living rooms of double that height. The built-in equipment has been dimensioned in accordance with the Modulor rules. Here, the method of proportioning derived from human measurements was to stand the test of practical application. The result, however, does not carry conviction. To keep costs within a reasonable limit the rooms were made as narrow and deep as possible. The smaller rooms have not only extraordinarily low ceilings but are of minimal width and inordinate depth. The depth does not give the impression of having been arrived at by proportioning work. And in relation to it, the large room is not as large as it should be to give a sense of spaciousness in the otherwise cramped conditions.

Nevertheless the building makes a strong impression on the visitor. When you have gone through it, walked about among its gigantic pillars, gone up to the roof and seen the weird landscape of enormous chimneys and other large cast concrete features arranged effectively in relation to the surroundings, ordinary buildings seem strangely petty in comparison. There are several other high apartment buildings in Marseille but not only are they

The colossal underpinning of Le Corbusier's Marseille block, four men high
See also illustrations page 172

slicker in detail, they seem to be composed simply of innumerable small details added together while Le Corbusier's house has real greatness. Why is this so?

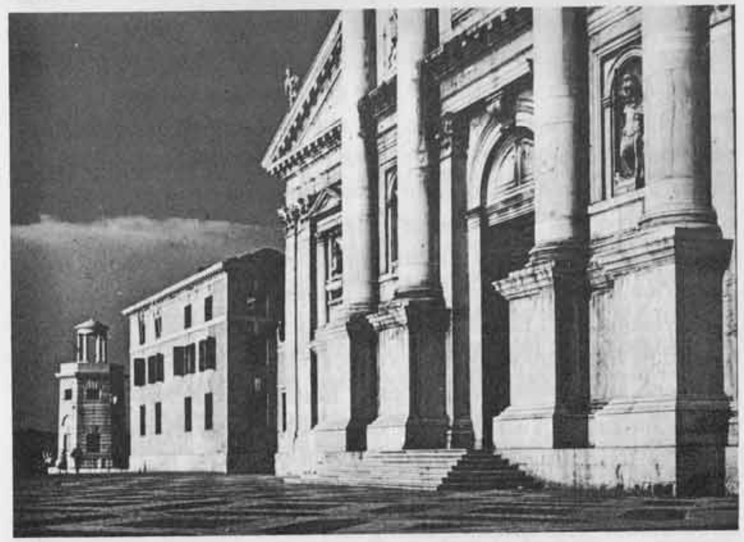
Above all, it is due to the fact that the understructure was *not* proportioned according to human measurements—that is in relation to the small apartments—but on a gigantic scale; a fitting substructure for a mammoth box. When you stand down there among these fantastic pillars you are made vividly aware that they were created to support a gigantic building.

Here you find something of the grandeur of Palladio's architecture. In the villa in Malcontenta the old wall decorations still exist and in one of the square rooms the frescoes depict titanic figures in various attitudes. You feel that the house was originally built for such giants and that later ordinary people moved in with their household goods, which seem rather lost in the vaulted stone rooms.

In reality, the ratios of Palladio's villa were derived from the classical columns he used. The columns, taken over from antiquity, were regarded as perfect expressions of beauty and harmony. There were rules for their proportioning down to the smallest details. The basic unit was the diameter of the column and from that were derived the dimensions not only of shaft, base and capital but also of all the details of the entablature above the columns and the distances between them. These ratios were laid down and illustrated in handy pattern-books of the "five orders." Where small columns were used everything was correspondingly small; when the columns were large, everything else was large too. During the early Renaissance buildings were constructed in layers with a new set of columns and entablatures for each story. But Michelangelo and Palladio introduced columns in "large orders" comprising several stories, and from then on there was no limit to how large they could be made or how monumental the buildings. Instead of a small cornice corresponding to the proportions in one story, there now came huge crowning cornices proportioned in relation to the entire building, like the top and bottom parts of Le Corbusier's Marseille block. The pilgrim who came to S. Peter's in Rome must have felt like Gulliver in the land of the giants. Everything was in harmony but adapted to ultralarge columns.

From then on there was an essential difference between the proportioning of monumental architecture and that of domestic buildings. The monumental edifice became even more effective when it was placed in a row of ordinary structures, as Italian churches often were during the Baroque period. The domestic buildings also had their definite rules of proportioning but they were less elastic, not based on column modules but on human dimensions, determined in a purely practical manner.

When we consider how a building is produced we realize that it is fairly necessary to work with standard units. The timber which the carpenter prepares in his lumberyard must fit the brickwork which the mason has built up on the site. The stonecutter's work, which may have been carried out in a distant



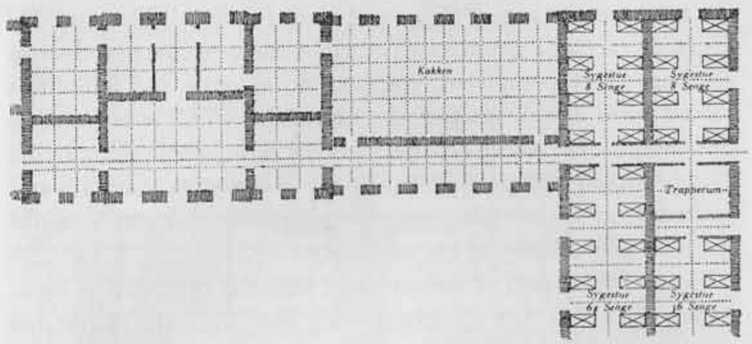
Church of San Giorgio Maggiore in Venice, by Palladio
When the colossal columns are seen together with the more normal-sized side-buildings it becomes apparent how immense the church is

quarry, must square with all the rest when it arrives. Windows and doors must be easy to order so that they will exactly fit the openings that have been prepared for them.

The very designation of the most common measuring unit employed in the past—and still used in Great Britain and America—the foot, refers to part of the human body. We also speak of measuring by rule of thumb, the thumb being taken as equal to one inch. A foot can be divided by eye into two, three, four, six, or twelve parts, and these easily gauged divisions are designated by simple numbers in inches. Earlier, there were standard specifications for bricks, timber, distances between beams and rafters in a house, windows and doors—all expressed in simple numbers in feet and inches. And they all fitted together without requiring any further adjustment at the building site. In Denmark half-timber construction particularly had attained a high degree of standardization though it varied in different parts of the country. In some provinces bays were five feet wide, in others six. Each half-timber bay comprised a window, a door, or a section of solid wall. In the stable the width of a bay corresponded to a stall; in the house to the narrowest room—either a pantry or a corridor. Two bays equalled an ordinary room, three the "best room." Heights were also standardized and in some provinces all roofs had the same pitch. In other countries with other methods of construction there were other subdivisions. In England, for instance, they built two-story dwellings for farm-workers in rows, on the beam-ridge principle, with one supporting wall to each house. The subdivision here was in houses—of sixteen feet each—instead of in bays.

In the Baroque period it was not only churches that were built on a monumental scale; palaces too were often given gigantic dimensions. The columns and pilasters of exterior architecture now entered the rooms and dominated them. We are generally told that these palaces were built on such a huge scale to gratify the vanity of princes. Actually, the grandiose dimensions were taken over from classical structures which all architects of that period strove to imitate, and the palaces were neither comfortable nor easy to live in. But with the Rococo period the small room came into its own. Even for official residences the proportioning principles of domestic architecture were employed and in castles and palaces privacy and comfort were now preferred to pageantry and splendor.

Frederik's Hospital in Copenhagen (now Museum of Decorative Art), built by the great Danish architect Nicolai Eigtved about 1750, is a good example of how realistically the architect



Kaare Klint's proportion study of the rooms in Frederik's Hospital, Copenhagen
To the right, beds measuring 3 × 6 feet and with 6-foot spaces between them

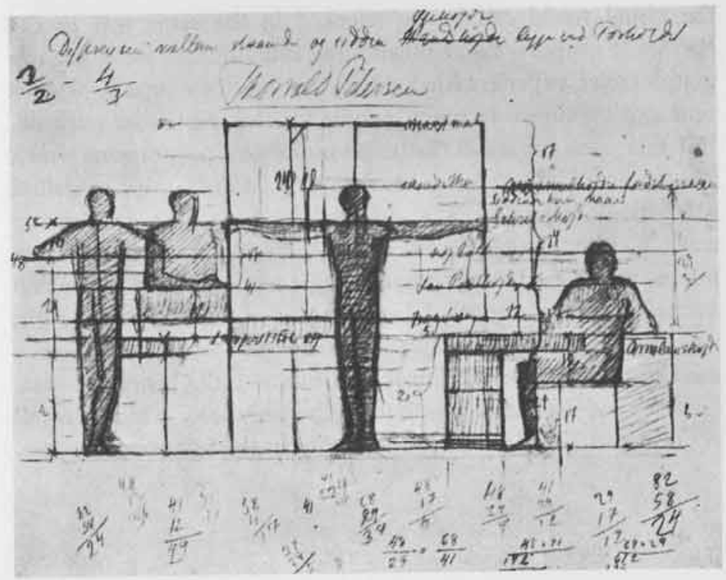
could approach his problem—and of the good result obtained thereby. The entire design, as was only natural, was based on the wards, which were formed as long galleries. Their dimensions were determined by the basic element of a hospital ward: the bed. This was placed at 6 × 3 feet. The beds were to stand with the head-ends against a wall so that it would be possible to approach them from either side and from the foot with one row standing out from the window wall and one from the opposite wall. There was to be six feet between beds in both directions. This gave a room depth of eighteen feet (a bed plus a passage space plus a bed) and a distance of nine feet from bed center to bed center. At every other intervening space a window was placed so that the distance from window center to window center was eighteen feet, i.e. equal to the depth of the room.

In this building, as we see, the dimensions were not determined by columns, or golden sections, or any other "beautiful" proportions, but by the beds which the hospital was built to hold.

This is only one example of the way Eigtved worked. In the course of four years—from 1750 until his death in 1754—he drew up the plans for an entire neighborhood, the Amaliegade district where now the Royal Family lives. He subdivided the ground, made model drawings for individual houses, designed the four Amalienborg Palaces and built Frederik's Hospital. He also made arrangements for all other buildings in the new district so that, when completed, the streets, squares and buildings would form a well integrated composition. This was possible only because he, as the architect who held the whole thing in his grasp, worked with proportions he was entirely familiar with and related them to each other in such a simple manner that he could see it all very clearly in his mind's eye.

Here, comparison of the architect with the composer is completely justified—the composer who must be able to put his composite work into notes by means of which others will be able to perform his music. He can do this because the tones that are available have been firmly established and each note corresponds to a tone with which he is completely familiar.

By a happy accident in the twentieth century Kaare Klint was chosen to restore the hospital building designed by Eigtved in the eighteenth. Earlier, Klint had made exhaustive studies of the dimensions of all sorts of domestic articles as a basis for general architectural proportioning. In his work on the hospital he discovered that when the buildings were measured in meters and centimeters it was impossible to find any coherent system in their proportioning. But measured in feet and inches the whole thing became lucid and simple. In his earlier studies he had found that many of the things we use in daily life were already standardized without our being aware of it. These included bed sheets, table cloths, napkins, plates, glasses, forks, spoons, etc. You can design a new pattern for the handles of spoons but a tablespoonful and a teaspoonful must remain an invariable quantity as long as liquid medicine is given in spoonfuls. Not only were the dimensions standardized but in feet and inches they could be expressed in integral numbers. Many kinds of furniture, too, have standard dimensions based on the proportions of the human body—such as seat heights and the heights of tables for various purposes, etc. Klint was not trying to find a magic formula that would solve all problems; his only desire was to

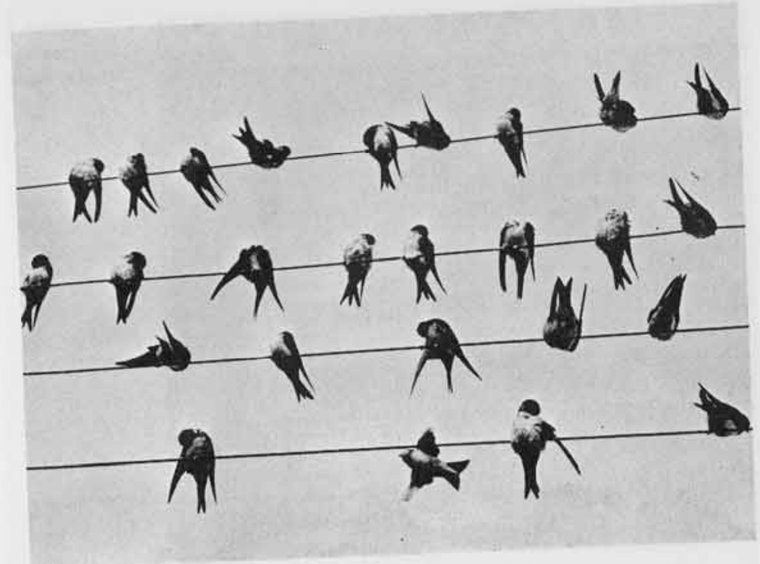


Kaare Klint: Proportion studies for factory-made furniture, 1918

determine, by scientific method, the natural dimensions of architecture and to find out how they could be made to harmonize with each other again—not according to any predetermined ratio but by simple division with nothing left over.

As early as 1918 he designed a whole series of commercial furniture adapted to human measurements and human needs, and until his death in 1954 he continued to improve and supplement it. Today many other designers are working along the same lines. In a world in which mass-production is such a dominating factor it is absolutely necessary to work out standards based on human proportions. But this is nothing new. It is simply the further development of the proportioning rules that were so universally accepted in older days.

In other words, architecture has its own, natural methods of proportioning and it is a mistake to believe that proportions in the visual world can be experienced in the same way as the harmonic proportions of music. For individual objects, such as match boxes, experience has shown that there are certain proportions which appeal to many people for that particular purpose. But this does not mean that there are certain proportions which are the only right ones for architecture. In the Gothic cathedral a breath-taking effect was obtained by bays that were many times higher than they were broad, dimensions which probably no one would find attractive in a single section of wall. But when such abnormally elongated bays are joined together in the right way the result, as shown in the illustration on page 140, may convey an impression of musical harmony to the beholder—not, however, of musical tones but of the regularity which we call rhythm and which we shall investigate in the following chapter.



CHAPTER VI

Rhythm in Architecture

The photograph of the swallows on the wires makes a charming picture with its combination of life and geometry. It is a simple composition of four parallel lines on which a number of birds are perched against a white ground. But within the rigid rectilinear pattern the continuous flashing and fluttering of the birds are variations on a theme which give a completely cinematographic impression of the little flock in vivacious activity. You can almost hear their joyful chirps.

In the world of architecture you can also experience delightful examples of subtle variation within strict regularity. It may be a row of houses in an old street where dwellings of the same type and period were built individually within the framework of a general plan. These houses, too, are variations on a theme within a rectilinear pattern.

Kimberly Elam. *Geometry of Design.*

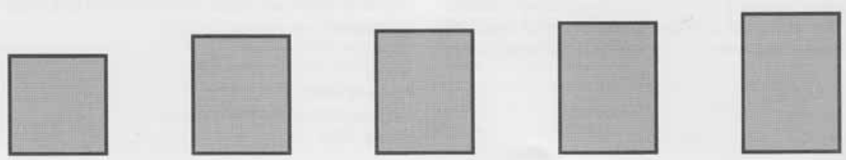
Cognitive Proportion Preferences

Within the context of the man-made environment and the natural world there is a documented human cognitive preference for golden section proportions throughout recorded history. Some of the earliest evidence of the use of the golden section rectangle, with a proportion of 1:1.618, is documented in the architecture of Stonehenge built in the twentieth to sixteenth centuries, B.C. Further documented evidence is found in the writing, art, and architecture of the ancient Greeks in the fifth century, B.C.. Later, Renaissance artists and architects also studied, documented, and employed golden section proportions in remarkable works of sculpture, painting, and architecture. In addition to man-made works, golden section proportions can also be found in the natural world through human proportions and the growth patterns of many living plants, animals, and insects.

Curious about the golden section a German psychologist, Gustav Fechner, late in the late nineteenth cen-

Table of Rectangle Proportion Preference

Ratio: Width/Length	Most Preferred Rectangle % Fechner	% Lalo	Least Preferred Rectangle % Fechner	% Lalo	
1:1	3.0	11.7	27.8	22.5	square
5:6	0.2	1.0	19.7	16.6	
4:5	2.0	1.3	9.4	9.1	
3:4	2.5	9.5	2.5	9.1	
7:10	7.7	5.6	1.2	2.5	
2:3	20.6	11.0	0.4	0.6	
5:8	35.0	30.3	0.0	0.0	Golden Section Proportion
13:23	20.0	6.3	0.8	0.6	
1:2	7.5	8.0	2.5	12.5	double square
2:5	1.5	15.3	35.7	26.6	
Totals:	100.0	100.0	100.0	100.1	



1:1 square   5:6   4:5   3:4   7:10

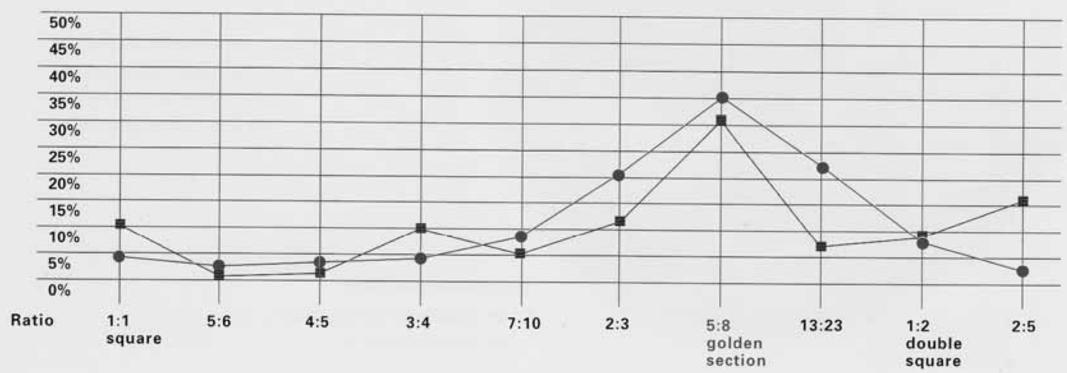
tury, investigated the human response to the special aesthetic qualities of the golden section rectangle. Fechner's curiosity was due to the documented evidence of a cross-cultural archetypal aesthetic preference for golden section proportions.

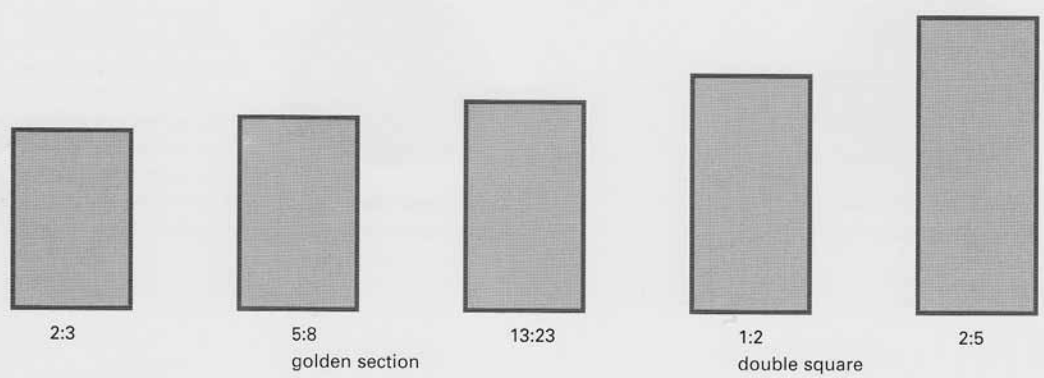
Fechner limited his experiment to the man-made world and began by taking the measurements of thousands of rectangular objects, such as books, boxes, buildings, matchbooks, newspapers, etc. He found that the average rectangle ratio was close to a ratio known as the golden section, 1:1.618, and that the majority of people prefer a rectangle whose proportions are close to the golden section. Fechner's thorough yet casual experiments were repeated later in a more scientific manner by Lalo in 1908 and still later by others, and the results were remarkably similar.

Comparison Graph of Rectangle Preference

Fechner's Graph of Best Rectangle Preference, 1876 ●

Lalo's Graph, 1908 ■

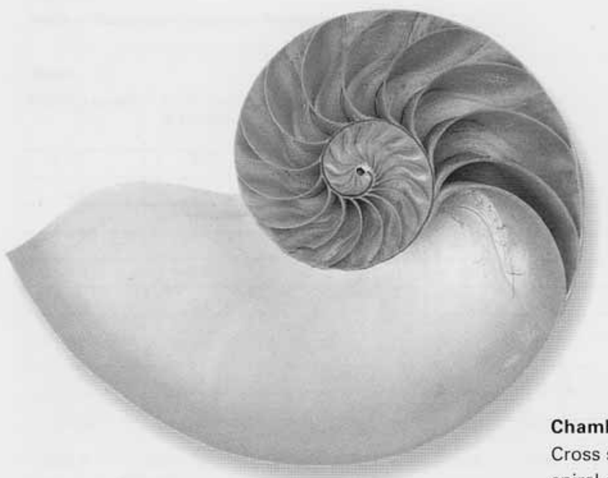


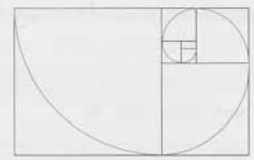


"The power of the golden section to create harmony arises from its unique capacity to unite different parts of a whole so that each preserves its own identity, and yet blends into the greater pattern of a single whole."
György Doczi, *The Power of Limits, 1994*

Golden section preferences are not limited to human aesthetics but are also a part of the remarkable relationships between the proportions of patterns of growth in living things such as plants and animals.

The contour spiral shapes of shells reveal a cumulative pattern of growth and these growth patterns have been the subject of many scientific and artistic studies. The growth patterns of shells are logarithmic spirals of golden section proportions, and what is known as the theory of a perfect growth pattern. Theodore Andreas





Golden Section Spiral
Construction diagram of the golden section rectangle and resulting spiral.

Chambered Nautilus
Cross section of the Nautilus' spiral growth pattern.

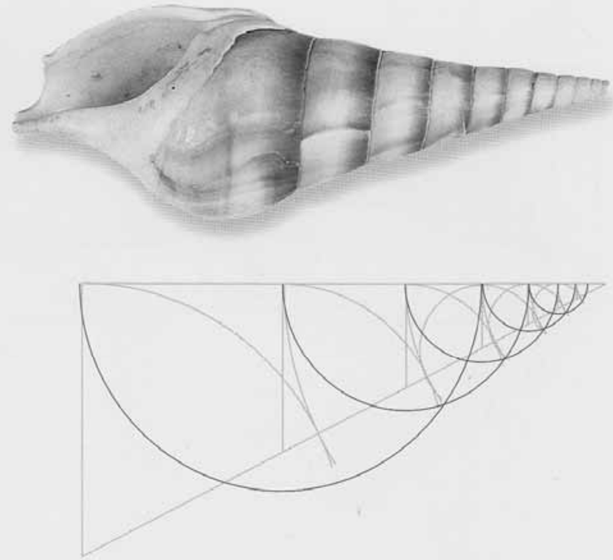
Atlantic Sundial Shell
Spiral growth pattern.

Moon Snail Shell
Spiral growth pattern.

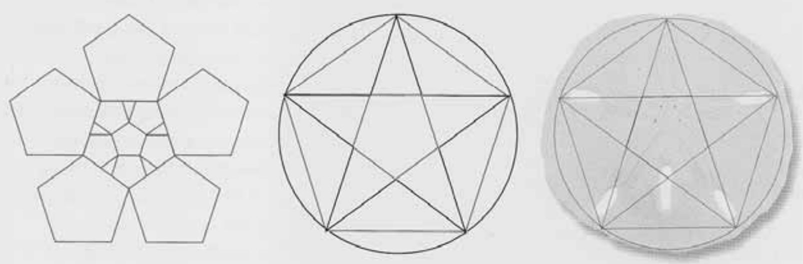
Cook in his book *The Curves of Life* describes these growth patterns as "the essential processes of life...." In each growth phase characterized by a spiral, the new spiral is very close to the proportion of a golden section square larger than the previous one. The growth patterns of the nautilus and other shells are never exact golden section proportions. Rather, there is an attempt in biological growth pattern proportion to approach but never reach exact golden spiral proportions.

The pentagon and pentagram star also share golden section proportions and can be found in many living things such as the sand dollar. The interior subdivisions of a pentagon create a star pentagram, and the ratio of any two lines within a star pentagram is the golden section proportion of 1:1.618.

Comparison of Tibia Shell Spiral Growth Pattern and Golden Section Proportion

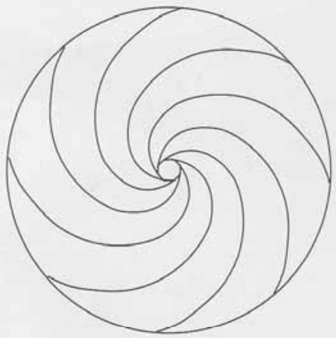


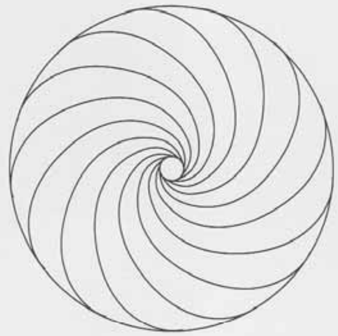
Pentagon Pattern
The pentagon and star pentagram have golden section proportions, as the ratios of the sides of the triangles in a star pentagram is 1:1.618. The same pentagon/pentagram relationships can be found in the sand dollar and in snowflakes.



The spiral growth patterns of the pine cone and the sunflower share similar growth patterns. The seeds of each grow along two intersecting spirals which move in opposite directions, and each seed belongs to both sets of intersecting spirals. Upon examining the pine cone seed spirals, 8 of the spirals move in a clockwise direction and 13 in a counterclockwise direction, closely approximating golden section proportions. In the case of the sunflower spirals there are 21 clockwise spirals and 34 counter clockwise spirals, which again approximate golden section proportions.

The numbers 8 and 13 as found in the pine cone spiral and 21 and 34 as found in the sunflower spiral are very familiar to mathematicians. They are adjacent pairs in the mathematical sequence called the Fibonacci sequence. Each number in the sequence is determined by adding together the previous two: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34,





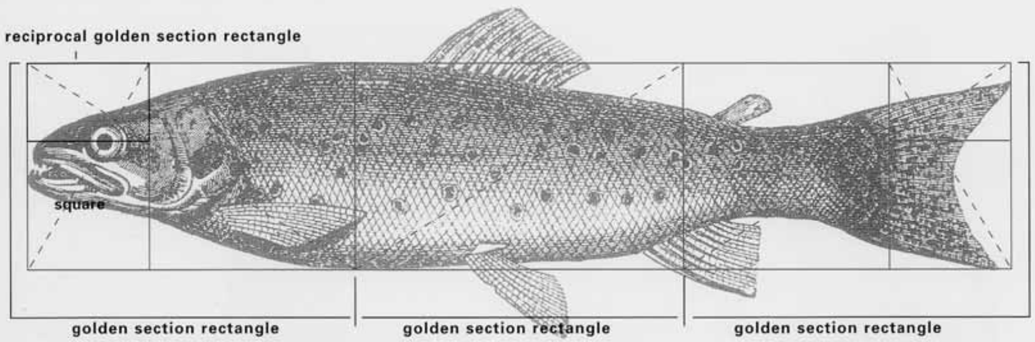
Spiral Growth Patterns of Pine Cones
Each seed in the pine cone belongs to both sets of spirals. 8 of the spirals move clockwise and 13 of the spirals move counterclockwise. The proportion of 8:13 is 1:1.625 which is very close to the golden section proportion of 1:1.618

Spiral Growth Patterns of Sunflowers
Similar to the pine cone each seed in the sunflower belongs to both sets of spirals. 21 spirals move clockwise, and 34 spirals move counterclockwise. The proportion of 21:34 is 1:1.619 which is very close to the golden section proportion of 1:1.618

55.... The ratio of adjacent numbers in the sequence progressively approaches golden section proportions of 1:1.618.

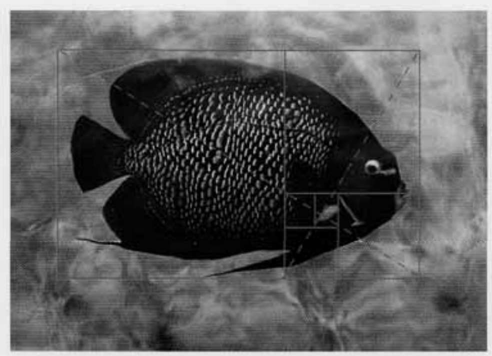
Many fish also share relationships with the golden section. Three golden section construction diagrams placed on the body of the rainbow trout show the relationships of the eye and the tail fin in the reciprocal golden rectangles and square. Further, the individual fins also have a golden section proportions. The blue angle tropical fish fits perfectly into a golden section rectangle and the mouth and gills are on the reciprocal golden section point of the body's height.

Perhaps a part of our human fascination with the natural environment and living things such as shells, flowers, and fish is due to our subconscious preference for golden section proportions, shapes, and patterns.



Golden Section Analysis of a Trout
The body of the trout is enclosed by three golden section rectangles. The eye is at the level of the reciprocal golden rectangle and the tail fin is defined by a reciprocal golden rectangle.

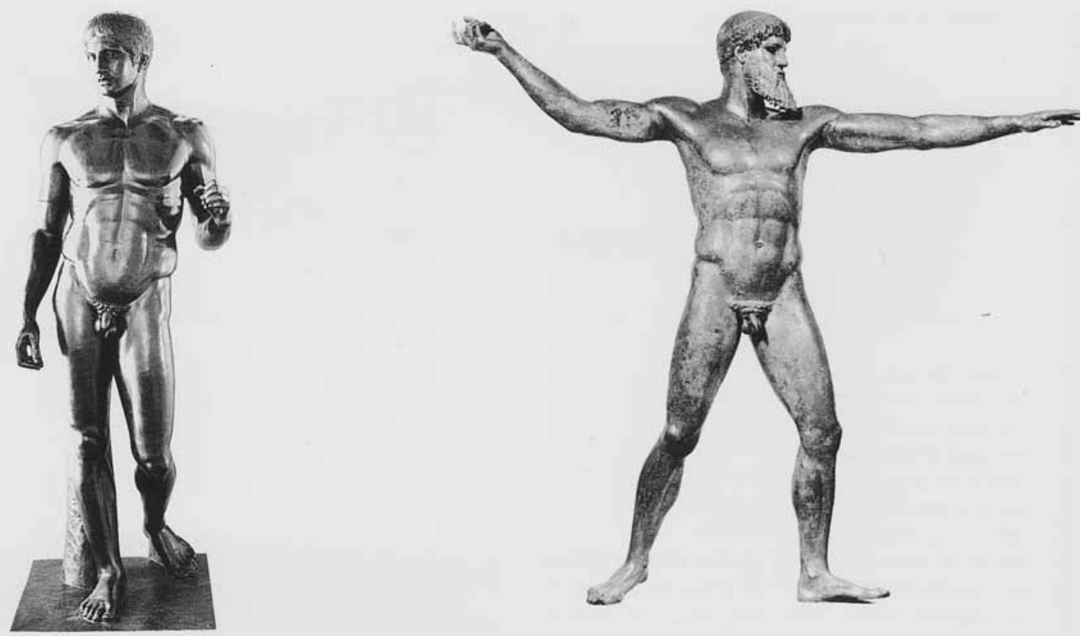
Golden Section Analysis of a Blue Angle Fish
The entire body of the fish fits into a golden section rectangle. The mouth and gill position is at the reciprocal golden section rectangle.



Human Body Proportions in Classical Sculpture

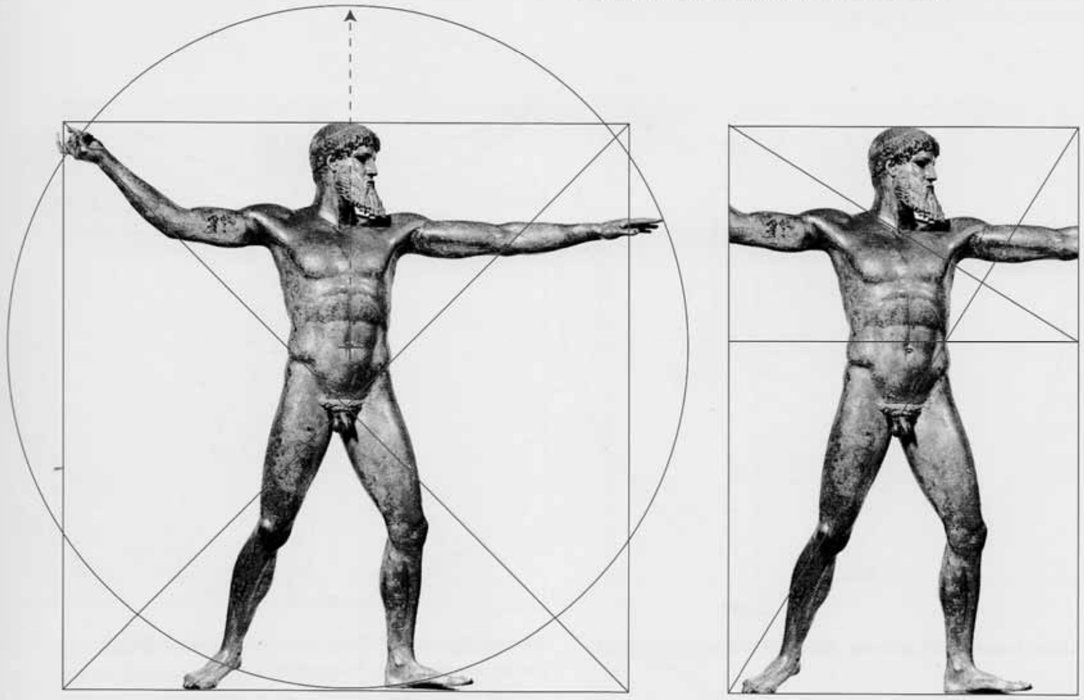
Just as many plants and animals share golden section proportions, humans do as well. Perhaps another reason for the cognitive preference for golden section proportions is that the human face and body share the similar mathematical proportional relationships found in all living things.

Some of the earliest surviving written investigations into human proportion and architecture are in the writings of the ancient Greek scholar and architect Marcus Vitruvius Pollio, who is widely referred to as Vitruvius. Vitruvius advised that the architecture of temples should be based on the likeness of the perfectly proportioned human body where a harmony exists among all parts. Vitruvius described this proportion and explained that the height of a well proportioned man is equal to the length of his outstretched arms. The body height and length of the outstretched arms create a square that enclose the human body, while the hands and feet touch a circle



Golden Section Proportions of Greek Sculpture
Doryphoros, the Spear Bearer (left). *Statue of Zeus from Cape Artemision* (right). Each golden section rectangle is represented by a rectangle with a dashed diagonal line. Multiple golden section rectangles share the dashed diagonal. The proportions of the two figures are almost identical.

with the navel as the center. Within this system the human form is divided in half at the groin, and by the golden section at the navel. The statues of the *Spear Bearer* and *Zeus* are both from the fifth century B.C. Although created by different sculptors, the proportions of the *Spear Bearer* and *Zeus* are both clearly based on the canon of Vitruvius and the analysis of the proportions used is almost identical.



***Zeus* Analyzed According to the Vitruvius' Canon**
A square encloses the body while the hands and feet touch a circle with the navel as center. The figure is divided in half at the groin, and (far right) by the golden section at the navel.

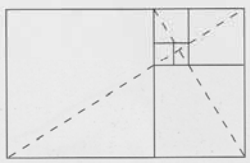
Architectural Proportions

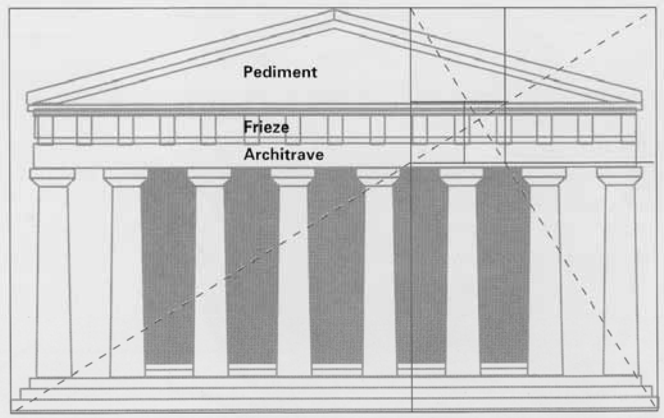
In addition to documenting human body proportions Vitruvius was also an architect and documented harmonious architectural proportions. He advocated that the architecture of temples should be based on the perfectly proportioned human body where there exists a harmony between all parts. He is credited with introducing the concept of the module, in the same way as the human proportions were expressed in a module representing the length of the head or feet. This concept became an important idea throughout the history of architecture.

The *Parthenon* in Athens is an example of the Greek system of proportioning. In a simple analysis the façade of the parthenon is embraced by a subdivided golden rectangle. A reciprocal rectangle forms the height of the architrave, frieze, and pediment. The square of the main rectangle gives the height of the pediment, and the smallest rectangle in the diagram yields the placement of the frieze and architrave.

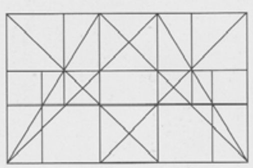
Centuries later the "divine proportion," or golden section, was consciously employed in the architecture of

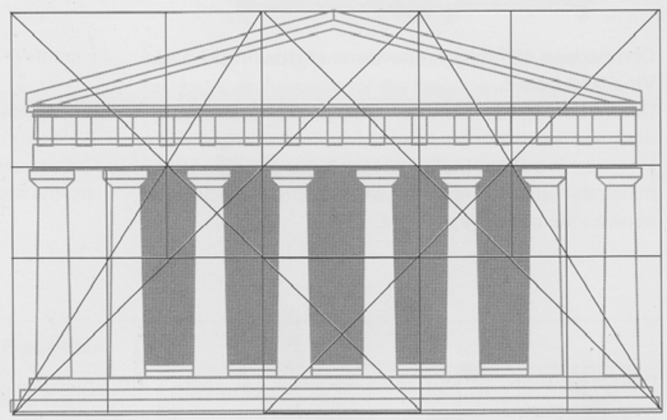
Drawing of the *Parthenon*, Athens, ca. 447–432 B.C., and the Architectural Relationship to the Golden Section
Analysis of golden section proportions according to the golden section construction diagram.





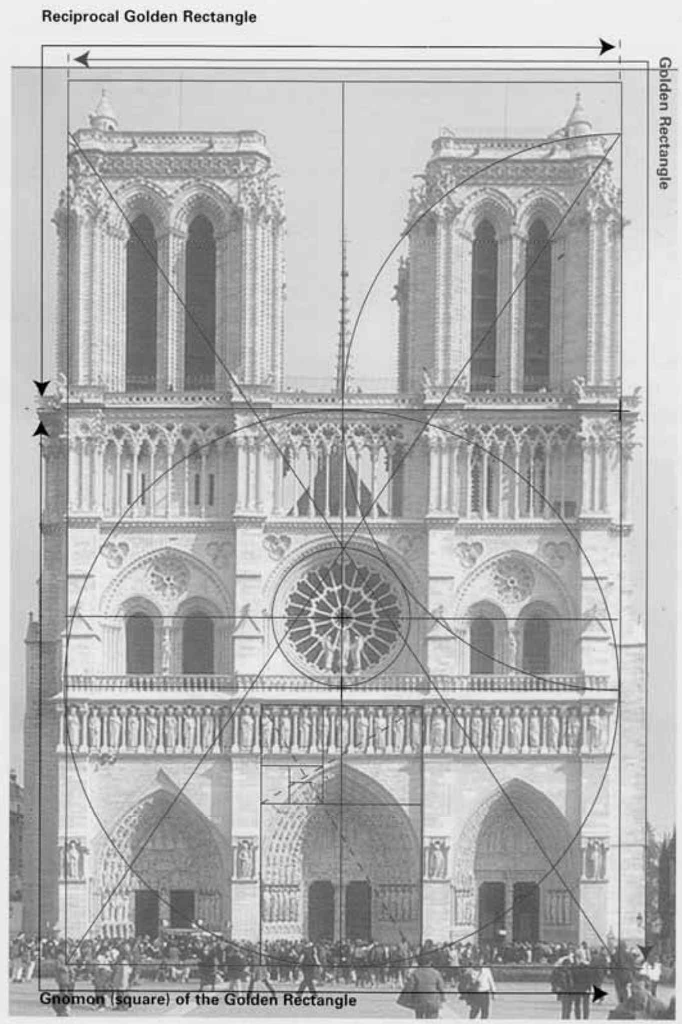
Golden Section Harmonic Analysis
Analysis of golden section proportions according to a diagram of a harmonic analysis of the golden section.

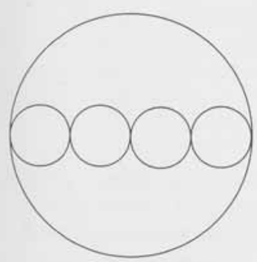




Gothic cathedrals. In *Towards A New Architecture*, Le Corbusier cites the role of the square and the circle in the proportions of the façade of the Cathedral of Notre Dame, Paris. The rectangle around the cathedral façade is in golden section proportion. The square of this golden section rectangle encloses the major portion of the façade, and the reciprocal golden section rectangle encloses the two towers. The regulating lines are the diagonals that meet just above the clerestory window, crossing the corners of the major variations in the surface of the cathedral. The center front doorway is also in golden section proportion as shown by the construction diagram. The proportion of the clerestory window is one-fourth the diameter of the circle inscribed in the square.

***Notre Dame* Cathedral, Paris, 1163 –1235**
Analysis of proportions and regulating lines according to the golden section rectangle. The entire façade is in golden rectangle proportion. The lower portion of the façade is enclosed by the square of the golden rectangle and the towers are enclosed by the reciprocal golden section rectangle. Further, the lower portion of the façade can be divided into six units, each another golden rectangle.



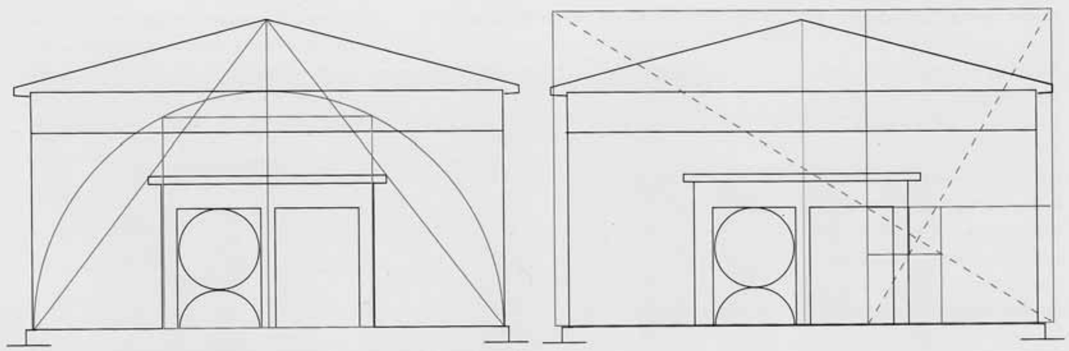


Proportion Comparison
The clerestory window is in proportion of 1:4 to the major circle of the façade.

Le Corbusier's Regulating Lines

Le Corbusier
Towards a New Architecture, 1931
"An inevitable element of Architecture. The necessity for order. The regulating line is a guarantee against willfulness. It brings satisfaction to the understanding. The regulating line is a means to an end; it is not a recipe. Its choice and the modalities of expression given to it are an integral part of architectural creation"

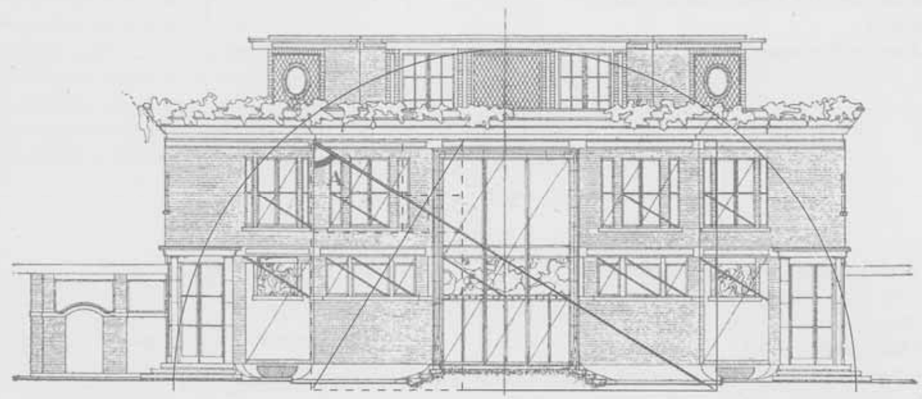
Corbusier's interest in the application of the geometry of structure and mathematics is recorded in his book *Towards a New Architecture.* Here he discusses the need for regulating lines as a means to create order and beauty in architecture and answers the criticism, "With your regulating lines you kill imagination, you make a god of a recipe." He responds, "But the past has left us proofs, iconographical documents, eteles, slabs, incised stones, parchments, manuscripts, printed matter.... Even the earliest and



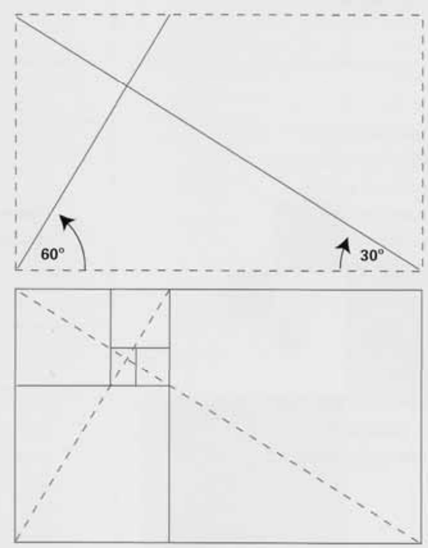
Redrawn from the Marble Slab Found in 1882, *Facade of the Arsenal of the Piraes*. Le Corbusier, *Towards a New Architecture,* 1931
Corbusier cites the regulated lines of simple divisions that determine the proportion of the height to the width, and guide the placement of the doors and their proportion to the facade. The facade fits into a golden section rectangle and the placement and height of the doorway corresponds to that proportion.

most primitive architect developed the use of a regulating unit of measure such as a hand, or foot, or forearm in order to systemize and bring order to the task. At the same time the proportions of the structure corresponded to human scale."

Corbusier discusses the regulating line as "...one of the decisive moments of inspiration, it is one of the vital operations of architecture." Later, in 1942, Le Corbusier published *The Modulor: A Harmonious Measure to the Human Scale Universally Applicable to Architecture and Mechanics. The Modulor* chronicled his proportioning system on the mathematics of the golden section and the proportion of the human body.



Le Corbusier, 1916. *A Villa,* From *Towards a New Architecture,* 1931
(above) This drawing by Le Corbusier diagrams the series of regulating lines that were used in the building design. Red lines placed on top of the drawing show the golden section rectangle and construction diagonals.



Golden Section Construction (right)
The relationship of Corbusier's regulating lines to the two construction diagrams of the golden section rectangle.

Construction of the Golden Section Rectangle

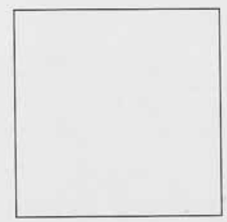
The golden section rectangle is a ratio of the Divine Proportion. The Divine Proportion is derived from the division of a line segment into two segments such that the ratio of the whole segment, AB, to the longer part, AC, is the same as the ratio of the longer part, AC, to the shorter part, CB. This gives a ratio of approximately 1.61803 to 1, which can also be expressed $\frac{1+\sqrt{5}}{2}$.

The Divine Proportion:

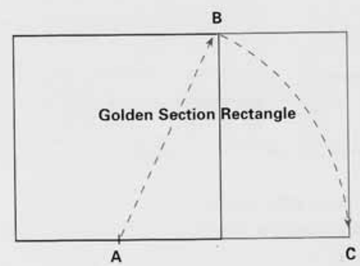
$$\frac{AB}{AC} = \frac{AC}{CB}$$

Golden Section, Square Construction Method

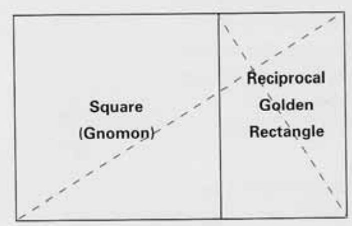
1. Begin with a square.



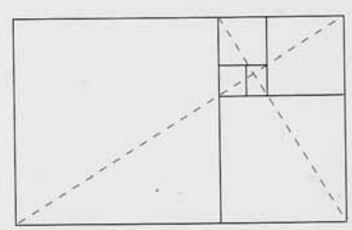
2. Draw a diagonal from the midpoint A of one of the sides to an opposite corner B. This diagonal becomes the radius of an arc that extends beyond the square to C. The smaller rectangle and the square become a golden section rectangle.



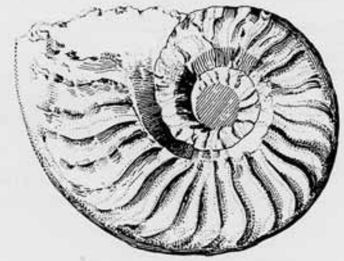
3. The golden section rectangle can be subdivided. When subdivided the rectangle produces a smaller proportional golden section rectangle which is the reciprocal, and a square area remains after subdivision. This square area can also be called a gnomon.



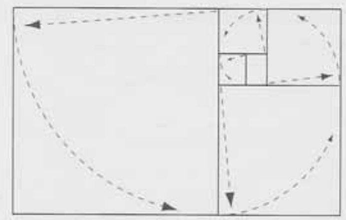
4. The process of subdivision can endlessly continue, again and again, producing smaller proportional rectangles and squares.

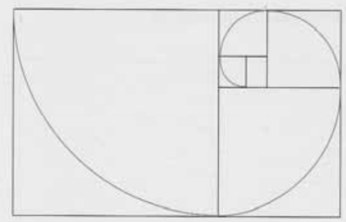


The golden section rectangle is unique in that when subdivided its reciprocal is a smaller proportional rectangle and the area remaining after subdivision is a square. Because of the special property of subdividing into a reciprocal rectangle and a square, the golden section rectangle is known as the whirling square rectangle. The proportionally decreasing squares can produce a spiral by using a radius the length of the sides of the square.

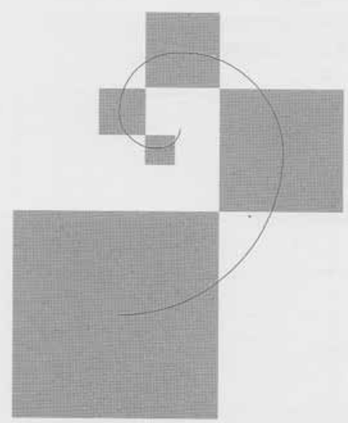


Golden Section Spiral Construction
By using the golden section subdivision diagram a golden section spiral can be constructed. Use the length of the sides of the squares of the subdivisions as a radius of a circle. Strike and connect arcs for each square in the diagram.



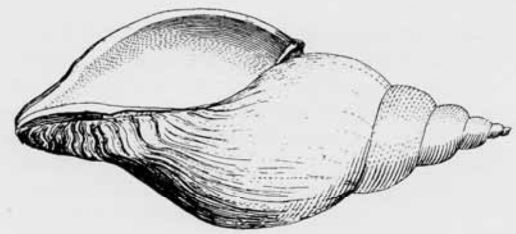


Proportional Squares
The squares from the golden section subdivision diagram are in golden section proportion to each other.

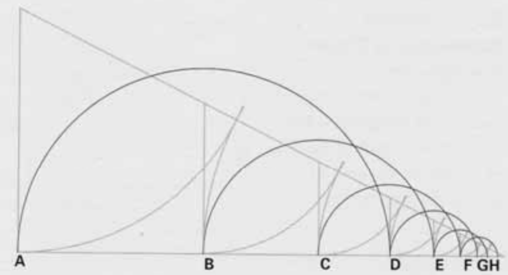


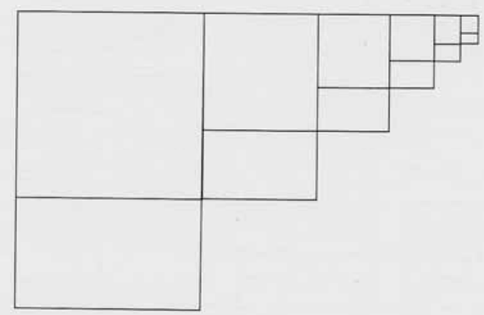
Golden Section Proportions

The divisions and proportion of the triangle method of the golden section construction produce the sides of a golden section rectangle, and in addition, the method can produce a series of circles or squares that are in golden section proportion to each other as in the examples below.

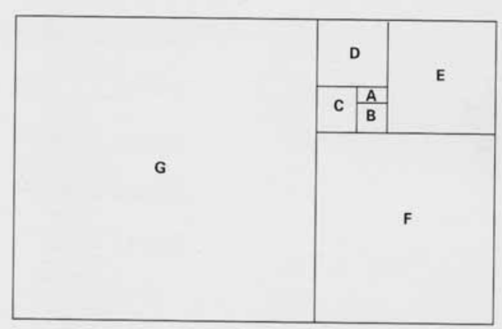


Diameter AB = BC + CD
Diameter BC = CD + DE
Diameter CD = DE + EF
etc.





Golden Rectangle	+	Square	=	Golden Rectangle
A	+	B	=	AB
AB	+	C	=	ABC
ABC	+	D	=	ABCD
ABCD	+	E	=	ABCDE
ABCDE	+	F	=	ABCDEF
ABCDEF	+	G	=	ABCDEFG



Golden Section and the Fibonacci Sequence

The special proportioning properties of the golden section have a close relationship to a sequence of numbers called the Fibonacci sequence, named for Leonardo of Pisa who introduced it to Europe about eight hundred years ago along with the decimal system. This sequence of numbers, 1, 1, 2, 3, 5, 8, 13, 21, 34... is calculated by adding the two previous numbers to produce the third. For example, 1+1=2, 1+2=3, 2+3=5 etc. The proportioning pattern of this system is very close to the proportioning system of the golden section. The early numbers in the sequence begin to approach the golden section, and any number beyond the fifteenth number in the sequence that is divided by the following number approximates 0.618, and any number divided by the previous number approximates 1.618.

Fibonacci Number Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, etc.

1+1=2,
1+2=3,
2+3=5,
3+5=8,
5+8=13,
8+13=21,
13+21=34,
21+34=55
34+55=89

$^{2}/_{1}$ = 2.0000

$^{3}/_{2}$ = 1.5000

$^{5}/_{3}$ = 1.66666

$^{8}/_{5}$ = 1.60000

$^{13}/_{8}$ = 1.62500

$^{21}/_{13}$ = 1.61538

$^{34}/_{21}$ = 1.61904

$^{55}/_{34}$ = 1.61764

$^{89}/_{55}$ = 1.61818

$^{144}/_{89}$ = 1.61797

$^{233}/_{144}$ = 1.61805

$^{377}/_{233}$ = 1.61802

$^{610}/_{377}$ = 1.61803 Golden Section

Golden Section Dynamic Rectangles

All rectangles can be divided into two categories: static rectangles with ratios of rational fractions such as 1/2, 2/3, 3/3, 3/4, etc., and dynamic rectangles with ratios of irrational fractions such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ϕ (golden section), etc. Static rectangles do not produce a series of visually pleasing ratios of surfaces when subdivided. The subdivisions are anticipated and regular without much variation. However, dynamic rectangles produce an endless amount of visually pleasing harmonic subdivisions and surface ratios when subdivided, because their ratios consist of irrational numbers.

The process of subdividing a dynamic rectangle into a series of harmonic subdivisions is very simple. Diagonals are struck from opposite corners and then a network of parallel and perpendicular lines are constructed to the sides and diagonals.

Golden Section Dynamic Rectangles
These diagrams from *The Geometry of Art and Life,* illustrate a range of harmonic subdivisions of golden section rectangles. The small red line rectangles (left) show the golden section rectangle construction. The gray and red rectangles (middle) show the red golden section rectangle construction with the harmonic subdivisions in gray line. The black line rectangles (right) show only the harmonic subdivisions.