Momentum and Interactions

Simulation

This lab uses an open-source online interactive simulation provided by PhET.

https://phet.colorado.edu/sims/html/collision-lab/latest/collision-lab_all.html

The simulation models collisions of objects in a completely general way. In this context, "collision" just means that there is a finite time interaction between objects. In other words, objects are initially far apart and do not have any effect on one another, then they have an interaction, i.e. they exert forces on each other, and finally, they either move apart with no further influence on each other, or they bond and move together.

To make the simulation a bit less abstract, you might think of the balls of the simulation as billiard balls, or as gas molecules.

Theoretical Background

Momentum:

Suppose we want to analyze a collision of two objects (or even more generally, an interaction between two objects). Let's call them A and B. The mass of A is $\mathrm{M_A}$ and the mass of B is $\mathrm{M_B}$. The velocity of A before the collision is $\vec{\mathrm{v}}_{\mathrm{Ai}}$ and the velocity of B before the collision is $\vec{\mathrm{v}}_{\mathrm{Bi}}$ ("i" for initial). What is the effect of the collision/interaction on A and B?

We have the relation between force and acceleration: $\vec{\mathrm{F}} = \mathrm{M}\vec{\mathrm{a}}$. We rewrite it as follows, in terms of the initial velocity, since that is something we know. "t" is time. The index "f" refers to "final" i.e. post collision.

$$\vec{\mathrm{F}} = \mathrm{M}\frac{\vec{\mathrm{v}}_\mathrm{f} - \vec{\mathrm{v}}_\mathrm{i}}{\mathrm{t}_\mathrm{f} - \mathrm{t}_\mathrm{i}}$$

For simplicity, we have assumed above that the time interval is very small so we can approximate the acceleration as constant so that it is just a ratio of the velocity change to the time interval. We don't know what forces A and B exert on each other, but we do know that the force exerted by A on B must be equal in magnitude and opposite in direction to the force exerted by B on A. We also know that the duration of the force action must, of course, be the same.

$$\vec{\mathrm{F}}_{\mathrm{BonA}}(\mathrm{t_f} - \mathrm{t_i}) = \mathrm{M_A}(\vec{\mathrm{v}}_{\mathrm{Af}} - \vec{\mathrm{v}}_{\mathrm{Ai}}) \; = -\vec{\mathrm{F}}_{\mathrm{AonB}}(\mathrm{t_f} - \mathrm{t_i}) = -\,\mathrm{M_B}(\vec{\mathrm{v}}_{\mathrm{Bf}} - \vec{\mathrm{v}}_{\mathrm{Bi}})$$

The above equation tell us that the **change in (mass)*(velocity)** from the interaction is equal and opposite for the two objects, A and B. Or equivalently,

$$\mathrm{M_A}\vec{\mathrm{v}}_{\mathrm{Af}} + \mathrm{M_B}\vec{\mathrm{v}}_{\mathrm{Bf}} \; = \; \mathrm{M_A}\vec{\mathrm{v}}_{\mathrm{Ai}} + \mathrm{M_B}\vec{\mathrm{v}}_{\mathrm{Bi}}$$

Since this (mass)*(velocity) product is ubiquitous, it is given a name: <u>**momentum**</u>. We denote it as $\vec{\mathrm{p}}$.

$$\vec{\mathrm{p}} = \mathrm{M}\,\vec{\mathrm{v}}$$

The above equation can then be rewritten very simply as final momentum equals initial momentum.

$$\vec{\mathrm{p}}_\mathrm{f} \; = \; \vec{\mathrm{p}}_\mathrm{i}$$

where $\vec{\mathrm{p}}$ is the **total** momentum of object A and B together, $\vec{\mathrm{p}}_\mathrm{A} + \vec{\mathrm{p}}_\mathrm{B}$.

The principle that the total momentum (i.e. the momentum sum) of interacting objects remains constant is called <u>**conservation of momentum**</u>.

It is noteworthy that, we have a vector equation restricting the final state of the objects, without knowing anything about the details of their interaction, i.e. the details of the force exerted by A and B on each other. If we know the velocity of one object post collision, we can find the other one.

There is one important caveat. The discussion above, only takes into account the interaction of A and B. If there are other forces concurrently acting on A and/or B, the momentum of A and B will have changes from those forces. In the simulation, there are no forces, other than the ones between the colliding balls.

<u>Elasticity:</u>

A very informative way to parametrize different collisions, is by how well kinetic energy is conserved in the process. This parameter is the "elasticity". The kinetic energy which is "lost" is converted into a different form. For example, if you clap your hands together, they feel warm, because kinetic energy is converted into heat in this case. If two molecules bond during a collision, kinetic energy is converted into chemical or electrical potential energy.

100% elasticity means that the kinetic energy is perfectly conserved, i.e. the **sum** of kinetic energies of A and B stays the same before and after interaction.

$$\mathrm{K_{Ai} + K_{Bi} = K_{Af} + K_{Bf}}$$

where $\mathrm{K} = (1/2)\,\mathrm{Mv}^2$. The collision of billiard balls is a good macroscopic example of elastic collision.

0% elasticity or "completely inelastic" means that the maximum amount of energy is lost in the collision. If the objects are treated as point particles (equivalently, we assume objects collide head on), 0% elasticity implies that the objects bond and move as one, post collision, i.e.

$$\vec{\mathrm{v}}_{\mathrm{Af}} = \vec{\mathrm{v}}_{\mathrm{Bf}}$$

For the in between cases, we have

$$(\eta/100)(\mathrm{K_{Ai} + K_{Bi}}) = \mathrm{K_{Af} + K_{Bf}}$$

where η is the percent elasticity.

Activity

Start the simulation and select "introduction". Check off "more data", as well as "kinetic energy" and "values", and whatever else you like.

Exercise 1: <u>Completely inelastic collision</u> (A and B stick together after collision)

Set the "Elasticity" to "0%" ("Inelastic").

Try the following cases in the simulations and fill out the table.

(1) In the first two cases, set $\mathrm{v_{Bi}} = 0$. For this "stationary target" case, your simulation data should show that the ratio of final to initial total kinetic energy is given by the mass ratio in the last column.

(2) In the last two cases make both velocities not zero. Choose the third case such that A and B are moving in opposite directions and collide head on. Choose the fourth case so A and B are moving in the same direction. (Note the kinetic energy ratio in these cases depends on the initial velocities.)

In all cases, show agreement between the final velocity in the simulation and the formula in the 6th column. This formula is just momentum conservation with the assumption that the final speeds of A and B are equal.

Table 1: Completely inelastic collisions

M_A (kg)	M_B (kg)	v_{Ai} (m/s)	v_{Bi} (m/s)	v_f (m/s)	$v_f = \frac{M_A v_{Ai} + M_B v_{Bi}}{M_A + M_B}$	$\frac{K_f}{K_{Ai} + K_{Bi}}$	$\frac{M_A}{M_A + M_B}$
			0				
			0				
							NA
							NA

Exercise 2: Elastic collision (Total kinetic energy before and after collision stays the same.)

Part A - Stationary target

Reset the simulation. Set the "Elasticity" to "100%" ("Elastic"). Try the following cases in the simulations and fill out the table.

Set $v_{Bi} = 0$. In the first case, choose $M_A > M_B$, choose $M_B > M_A$ for the second case, and $M_A = M_B$ for the third case. Your simulation data should show agreement between the final velocity v_{Af} in the simulation and the formula in the 6th column, as well as the final velocity v_{Bf} in the simulation and the formula in the last column. These formulas come from combining momentum conservation and kinetic energy conservation.

Table 2: Elastic collisions with Stationary target

M_A (kg)	M_B (kg)	v_{Ai} (m/s)	v_{Bi} (m/s)	v_{Af} (m/s)	$v_{Af} = \frac{M_A - M_B}{M_A + M_B} v_{Ai}$	v_{Bf} (m/s)	$v_{Bf} = \frac{2\,M_A}{M_A + M_B} v_{Ai}$
			0				
			0				
			0				

Question 1.

a) If the larger mass is moving and the smaller mass is the stationary target in an elastic collision, which way will the larger mass be moving after the collision (same direction or opposite direction as its initial motion)?

b) If the smaller mass is moving and the larger mass is the stationary target in an elastic collision, which way will the smaller mass be moving after the collision (same direction or opposite direction as its initial motion?)

c) In both part A and B, is the larger or smaller mass moving faster after the collision?

d) If the masses of the stationary target and the moving mass are equal in an elastic collision, what happens to the initially moving mass post collision?

e) Can any of the above answers change with a different choice of initial speed? (Hint: look at the formulas.)

Part B

Keep the "Elasticity" at "100%" ("Elastic"). Try the following cases in the simulations and fill out the table.

In the first case, take $M_A = M_B$. Simplify the formula for the final velocities for this case. Hint: You should have no mass dependance.

For the second and third case, choose any parameters. For both cases, v_{AF} in the simulation and the formula in the 6th column, as well as the final velocity v_{Bf} in the simulation and the formula in the last column, should agree.

Table 3: Elastic collisions

M_A (kg)	M_B (kg)	v_{Ai} (m/s)	v_{Bi} (m/s)	v_{Af} (m/s)	$v_{AF} = \frac{M_A - M_B}{M_A + M_B} v_{Ai} + \frac{2\,M_B\,v_{Bi}}{M_A + M_B}$	v_{Bf} (m/s)	$v_{BF} = \frac{(M_B - M_A)}{M_A + M_B} v_{Bi} + \frac{2\,M_A\,v_{Ai}}{M_A + M_B}$
M_B	M_A						

Question 2.

What is the algebraic formula for the final velocities of equal mass objects in an elastic collision, in terms of the initial velocities?