Projectile motion

Lab Activity by John Aldon Estes, SUNY College at Old Westbury

In this lab you will study the motion of a projectile in free-fall. You will demonstrate that the projectile follows projectile motion with constant acceleration in the vertical direction and constant velocity in the horizontal direction. You will then explore the kinematics of projectile motion by studying the maximum height for various firing angles and speeds.

Learning outcomes:

1. Students will understand the theory of projectile motion and be able to experimentally demonstrate that objects in free-fall follow projectile motion.
2. Understand how the maximum height of a projectile can be determined.
3. Understand how the path of a projectile would be altered when placed on a celestial body different from Earth.

Checklist for lab report:

- Projectile Motion
    - 3 questions
    - 2 exercises
    - 2 graphs
- Maximum Height of a Projectile
    - 2 exercises
    - 2 questions

Part 1: Projectile Motion

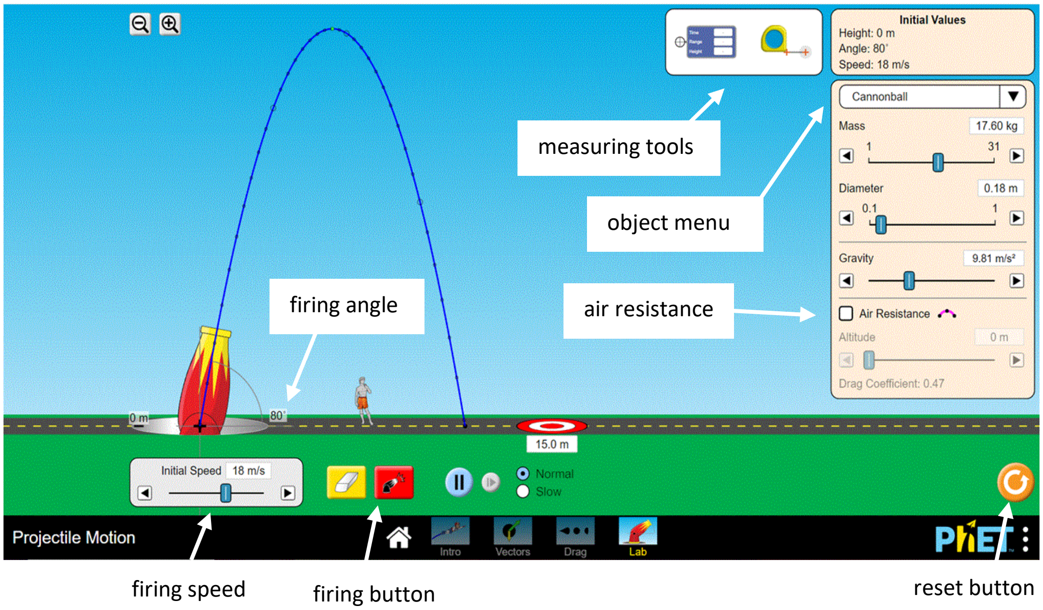
Simulation

Instead of using a physical apparatus, this lab uses an open-source online interactive simulation provided by PhET. The simulation can be accessed at:

://httpsphet.colorado.edu/en/simulation/projectile-motion

The simulation can be downloaded and run later without an internet connection.

Start up the projectile motion simulation and select Lab. On the lower left you can select the firing speed and by clicking and dragging the cannon, you can adjust the firing angle. Next to the speed selector is an eraser button and a firing button. On the right you have settings which control the properties of the projectile, the gravitational acceleration, and the air resistance. (Note that the altitude slider controls the density of the air. Leave this setting at zero meters.) There is also a pull down menu which will select different objects to fire. On the upper right is a set of measurement tools, the first gives the position and time of points along the trajectory, while the second can be used to measure distances. Finally, there is a reset button in the lower right corner which will reset the experiment.



Initial Values
Height: 0 m
Angle: 80°
Speed: 18 m/s

Cannonball
Mass 17.60 kg
Diameter 0.18 m
Gravity 9.81 m/s²
☐ Air Resistance
Altitude 0 m
Drag Coefficient: 0.47

Initial Speed 18 m/s

○ Normal
○ Slow

Turn off air resistance and play around with the simulation by trying to hit the target. If you run into trouble, you can always reset the simulation with the reset button.

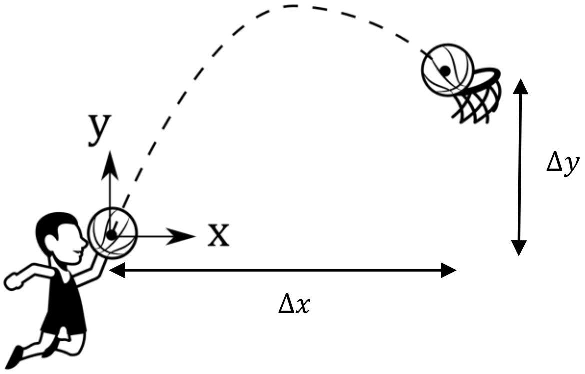
Question 1: Without air resistance does the path of the projectile depend on what object is fired? Does the changing the firing angle change where the object hits? Which firing angle gives the largest firing distance?

<u>Free-fall:</u> A body is in free-fall whenever the only force acting on it is gravity. In most cases, there are additional forces acting on the object and the object is in free-fall when the additional forces are negligible compared to gravity. Any object in free fall will accelerate towards with the ground with an acceleration of $g = 9.81\ \mathrm{m/s^2}$.

On other celestial bodies, such as a planet or moon, the same facts regarding free-fall hold, but the gravitational force is different. As a result, the acceleration of an object in free-fall takes different values.

$$g_{\text{Earth}} = 9.81\ \frac{\text{m}}{\text{s}^2}$$
$$g_{\text{Moon}} = 1.63\ \frac{\text{m}}{\text{s}^2}$$
$$g_{\text{Mars}} = 3.72\ \frac{\text{m}}{\text{s}^2}$$
$$g_{\text{Venus}} = 8.87\ \frac{\text{m}}{\text{s}^2}$$
$$g_{\text{Neptune}} = 11.15\ \frac{\text{m}}{\text{s}^2}$$

<u>Projectile motion:</u> An object launched into the air at an angle will typically experience free-fall and its path will follow a parabolic like trajectory. This kind of motion is called projectile motion.



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The equations governing projectile motion are the equations for constant acceleration with constant velocity in the x-direction ($a_x = 0$) and constant acceleration in the y-direction ($a_y = -g$):

$$\Delta x = v_x t \qquad \Delta y = -\frac{1}{2} g\, t^2 + v_{0,y}\, t$$
$$v_y = -g\, t + v_{0,y}$$

Exercise 1: Our first goal will be to demonstrate that projectiles follow this kind of motion. To do so, reset the simulation and select an initial speed and firing angle. Make sure the air resistance is off and fire the projectile. Check that the entire path of the projectile is visible on your screen. You can use the plus and minus buttons on the upper left side of the screen to zoom in and out. If you can’t get the entire path of the projectile on the screen, erase the current path, lower the initial speed and try again. Record the initial speed and firing angle in table 1 below.

Using the measurement tool, record the data for five data points while the projectile is going up, five data points while the projectile is going down and the data point at the maximum height of the projectile in table 1 below. In total this gives eleven data points. Note that the range corresponds to the x-displacement of the projectile and the height corresponds to both the y-displacement.

Table 1: projectile motion

Initial speed (m/s)	Firing angle (degrees)	Time (s)	Range (m)	Height (m)

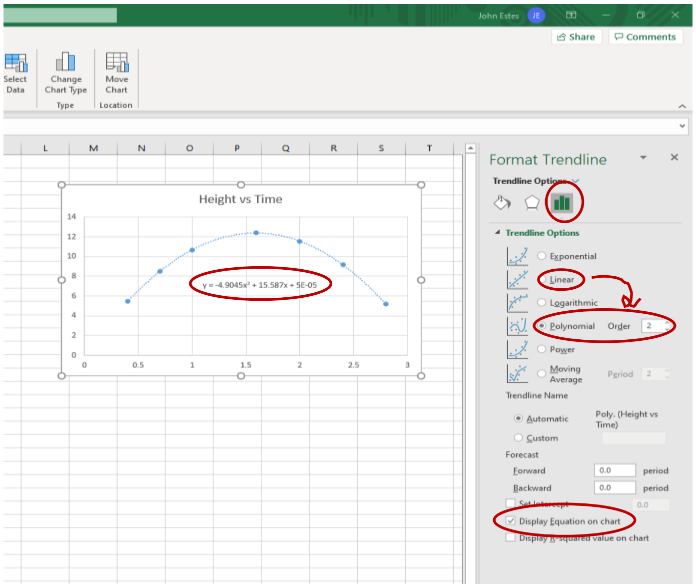
Make a plot of Range vs Time and Height vs Time.

Question 2: Do your plots confirm that the object follows projectile motion? Explain your reasoning. Note that the Range corresponds to the horizontal displacement of the projectile Δx and the Height corresponds to the vertical displacement of the projectile Δy.

Graph 1: Make a linear fit to the Range vs Time graph and obtain the x-component of the velocity.

Graph 2: Make a quadratic fit to the Height vs Time graph and obtain the y-component of the initial velocity and the acceleration. To do a quadratic fit, first add a linear trendline. When you go to "Display Equation on chart" in the trendline options, change the selection from "Linear" to "Polynomial" and make sure the order is "2". To read off the information, compare the equation given on your graph to the equation below, remember that "x" on your graph is "t" in the equation.

$$\Delta y = -\frac{1}{2} g\, t^2 + v_{0,y}\, t$$



y = -4.9045x² + 15.587x + 5E-05

● Polynomial Order 2

● Automatic Poly. (Height vs Time)

Forward 0.0 period

Backward 0.0 period

☑ Display Equation on chart

Exercise 2: Compute the initial speed and firing angle of the projectile using the information you obtained from the graph. Note that from the previous exercises, you have the x-component and y-component of the initial velocity.

Question 3: Do your values for the initial speed, firing angle and acceleration agree with the simulation settings?

Part 2: Maximum Height of a Projectile

At the maximum height, the vertical velocity (y-component) of the projectile vanishes ($v_y = 0$). This is because the maximum height is the turning point at which the y-component of velocity goes from being up (positive) to down (negative). Using this fact, we can solve the kinematic equations for the maximum height in terms of the launch angle, θ, and initial speed, v_0:

$$\Delta y_{max} = \frac{{v_0}^2 \sin^2(\theta)}{2\, g}$$

Exercise 3: Use the above equation to compute the maximum height for the projectile for each case listed in table 2. Record your results in table 2 under $\Delta y_{max,th}$ (theoretical maximum vertical displacement). You can use Excel to do the computations for you as in “Lab 1 – Measurements”.

After you have computed the theoretical values, use the PhET simulation to determine the simulated values for the maximum height. Record your results under $\Delta y_{max,sim}$ (simulated maximum vertical displacement). You can use the measurement tools to read off the data at specific points along the trajectory. Make sure the air resistance is turned off by resetting the simulation.

Table 2: maximum height of a projectile

v_0 (m/s)	θ (degrees)	$\Delta y_{max,th}$ (m)	$\Delta y_{max,sim}$ (m)
13	90		
13	65		
13	45		
13	25		

Exercise 4: On Venus the acceleration due to gravity is about 10% lower than Earth’s, $g_{Venus} = 8.87\ \mathrm{m/s^2}$. In table 3 below, guess how the maximum height will be changed. Record your guess under $\Delta y_{max,guess}$. Afterwards, use the PhET simulation to simulate the experiment and record the simulated values for the maximum height under $\Delta y_{max,sim}$.

Table 3: maximum height of a projectile on Venus

v_0 (m/s)	θ (degrees)	$\Delta y_{max,guess}$ (m)	$\Delta y_{max,sim}$ (m)
13	90		
13	65		
13	45		
13	25		

Question 4: We saw that the maximum height of a projectile was altered on Venus compared to Earth. How would the maximum height be alerted on Neptune compared to Earth (larger or smaller)? Explain your reasoning.

Question 5: How would the firing range of the projectile be changed on Venus and Neptune compared to Earth? Use the simulation to check your answers.