Ballistic Pendulum

In the ballistic pendulum experiment, a ball is shot using a spring loaded launcher. The ball is caught by a pendulum, and then, the ball plus pendulum swing up to some maximum height. The downward return swing is prevented by a "catcher" so that the maximum height can be easily observed. We can calculate the ball speed at launch from the height change of the ball plus pendulum and their masses. We can also measure the speed of the launched ball using photogates and the two measurements can be compared.

Watch the video of the experiment here:

https://www.dropbox.com/s/oi52y4yzy3u9krq/Ballistic%20Pendulum.mp4?dl=0

The data for the experiment can be found in the following folder. Look at the excel file.

https://cuny907-my.sharepoint.com/:f:/g/personal/german_kolmakov93_login_cuny_edu/Erokln6x8j9Eq7ss4ZPQQSEBk5hcM3E71f2WF4dpOdbNtg?e=NRqhl5

The device used in the experiment is *75425 CENCO Ballistic Pendulum*. Some photographs and descriptions of this device can be found on the last page.

Theoretical Background and Details of Experiment

Measurement of ball speed using photogates:

To measure the speed of the ball using photogates, we measure the distance between the photogates, d, and divide by the amount of time, t, that it took the ball to travel between the photogates. The timer is programmed to start when the ball enters the first photogate, i.e. when it blocks the light beam of the first photogate. The timer is programmed to stop when the ball enters the second photogate, i.e. blocks its light beam. We denote the speed measurement obtained from the photogate time, v'.

$$v' = d/t$$

The pendulum is not used in this measurement. It is raised, so as to not be in the way.

Measurement of ball speed using ballistic pendulum:

We discussed in the last lab activity that all collisions preserve momentum. The collision of the ball and pendulum here is completely inelastic, which is just a way to say that the ball and pendulum are stuck together and travel as a single object at a single speed. As we discussed in the previous class, equating the momentum of the ball pre-collision to the momentum of the ball plus pendulum right after the collision yields

$$v\, m = V\,(M + m)$$

where m is the mass of the ball, M is the mass of the pendulum, v is the speed of the ball prior to collision, and V is the speed of the ball plus pendulum right after the collision.

The above equation takes into account the full interaction of the ball and the pendulum, but we need to ask ourselves whether there is anything else going on concurrently with the collision that is not taken into account by that conservation of momentum equation. Indeed, there is an obvious difference between the current set up and the inelastic collisions we analyzed in the simulation in the previous class. The pendulum is attached to a rod and the rod is attached at the top to the stand. Our equation above does not take into account the interaction with the rod.

The approximation that we are making here is that the rod is “massless”. If we approximate the rod as having 0 mass then its momentum is 0 and does not alter the above equation. The equation also does not take into account the gravity force acting on the ball prior to the collision. And we might worry about whether the force of the attachment on the rod has an effect. For now, we assume that all these approximations are justified and proceed.

After the collision, the pendulum plus ball move under the action of the gravity force. Under these circumstances, the mechanical energy, i.e. the sum of the kinetic energy and the gravitational potential energy of the pendulum plus ball is conserved (i.e. remains constant throughout the motion). Since the rod is approximated as massless, it does not contribute to the energy conservation equation. The force of the attachment on the rod does not do any work since the attachment point is stationary. The conservation of mechanical energy equation is as follows and can be solved for V.

$$\frac{1}{2}(M + m)V^2 + (M + m)g\, y_1 = (M + m)g\, y_2$$

$$V = \sqrt{2\, g\, h}$$

where $h = y_2 - y_1$, y_1 and y_2 refer to the initial and final height of the pendulum plus ball respectively, and g is the gravitational acceleration, $9.8\, m/s^2$. At the maximum height, the speed of the pendulum plus ball is 0. One caveat is that the effect of the “catcher mechanism” is not taken into account.

Another question you should have is which height are we measuring? Do we measure the bottom of the pendulum? Or the top? If your intuition is to measure the height of the center, you are correct. Since the shape of the pendulum and the ball is quite symmetric, you might guess that the geometric center (which is roughly the same point for both the ball and the pendulum) is a kind of average location for the mass. This concept can be defined rigorously and is referred to as “center of mass.”

Using the expression for V and the first equation, you can solve for v in terms of masses and h. (Try to find this formula and check your answer in the excel file.)

Let us briefly return to the question of whether the approximations we have made are valid. The mass of the rod in our set up is about 1/5 the mass of the pendulum, which isn’t negligibly small. However, the way this mass interacts is not in the same way as the others. In the analysis above, we have treated everything as a point particle. This is a reasonable approximation for the pendulum and the ball, but for the rod, it is a worse approximation than the massless one. That is why we cannot simply add the rod mass to the mass of the pendulum.

A full analysis of this experiment, which includes the mass of the rod, requires taking into account rotational motion, i.e. using conservation of *angular* momentum and keeping track of the *rotational* kinetic energy. Also, the center of mass of the rod is in the center of the rod, which makes the corresponding height change much smaller. Using the rotational calculation, it can be shown that the massless rod approximation gives a ball speed, v, which is about 5% smaller than the true value.

Activity

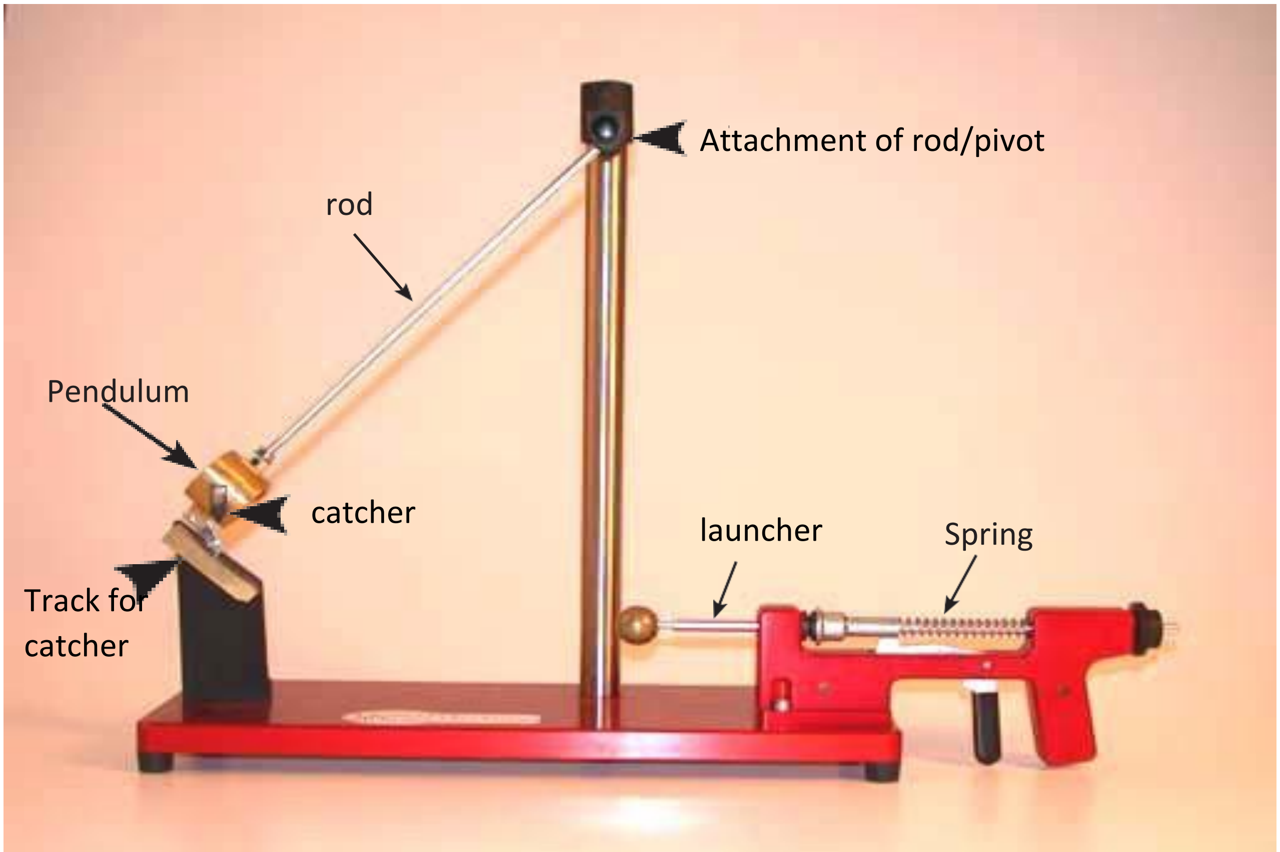
Fill out the data tables in the excel file, calculating the speed of the ball in two ways and comparing the results. The excel file is here:

https://cuny907-my.sharepoint.com/:f:/g/personal/german_kolmakov93_login_cuny_edu/Erokln6x8j9Eq7ss4ZPQQSEBk5hcM3E71f2WF4dpOdbNtg?e=NRqhl5

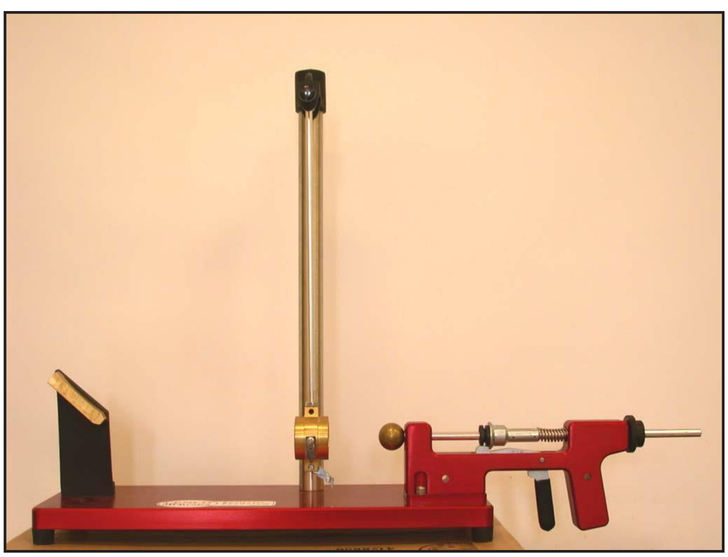
Answer the questions below. Photographs of the device are on the next page.

1. Consider the effect of the catcher mechanism on the final (maximum) height. The catcher mechanism is not taken into account in our calculations. Based on this, do you think the calculated velocity of the pendulum plus ball immediately after collision, higher or lower than the true velocity of the pendulum plus ball immediately after collision? Is the calculated initial velocity of the ball, higher or lower than the true velocity of the ball? Based on this, do we expect the speed of the ball calculated from the photogate time to be larger or smaller than the speed obtained from the ballistic pendulum calculation? Does this agree with the experimental data? If the effect of the massless rod approximation is to lower the ballistic pendulum result by about 5% compared to the true value, estimate the effect of the catcher mechanism (assuming this is the other main source of error).

2. Consider the effect of the gravity force on the speed of the ball i.e. the change in the speed of the ball between the launch and the collision. The distance the ball travels from the launch point to the center of the pendulum where it is embedded during the collision is under 1cm. Use the relation, $v_2^2 - v_1^2 = 2^*g^*x$, with x=1cm and v_1 equal to any of the initial speed values that you obtain. Solve for v_2 and remark on the size of the effect. (Don’t forget to convert x to meters.)



The above photograph shows the pendulum in the high position, prevented from falling by the “catcher mechanism”. The launcher (marked h on the photograph) is in the extended state. The ball has been placed back onto the end of the launcher and this demonstrates the position of the ball immediately prior to leaving the launcher when it’s shot. Compare with the photograph below to see the position of the pendulum prior to collision.



This photograph shows the spring loaded launcher compressed and ready to shoot the ball. But note that the ball is brought right up to the pendulum in the process of the launch so that the ball travels a distance on the order of 1cm or even less from the moment it leaves the launcher until the collision.