Vectors – Graphical Approach

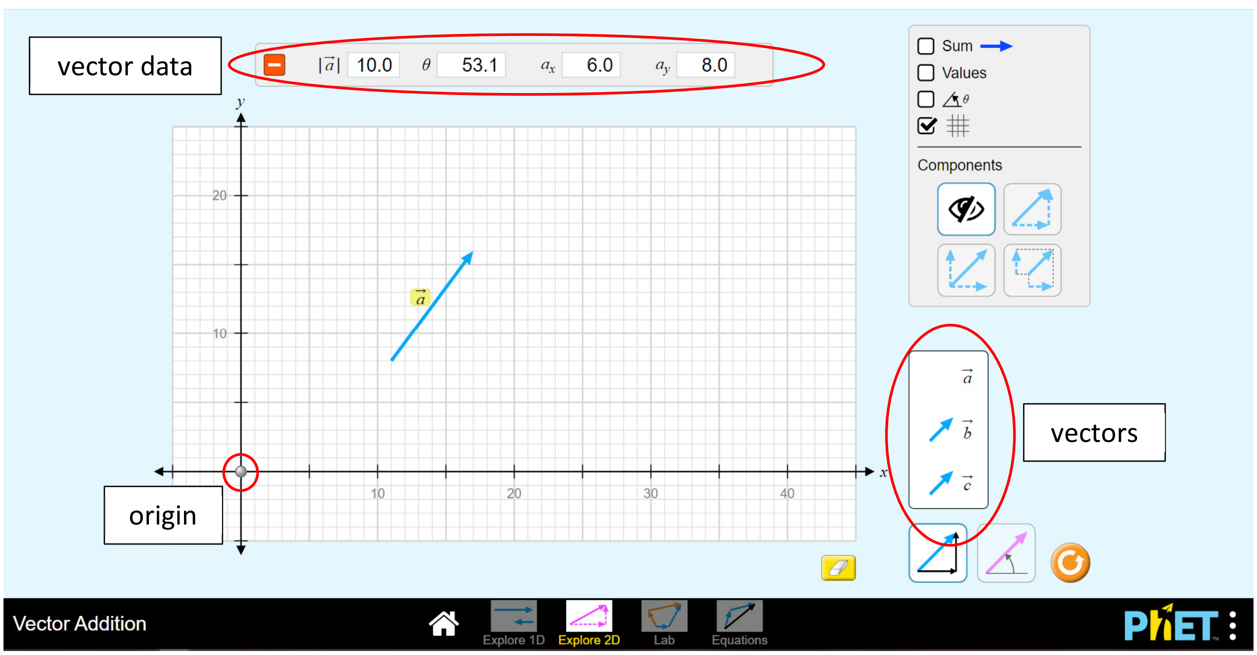
Simulation

Instead of using a physical apparatus, this lab uses an open-source online interactive simulation provided by PhET. The simulation can be accessed at:

https://phet.colorado.edu/en/simulation/vector-addition

The simulation can be downloaded and run later without an internet connection.

Start up the vector addition simulation and select "Explore 2D". You can select three vectors to display on the graph. Drag one of the vectors onto the graph. You can resize and rotate the vector by clicking and dragging the arrow tip. Click anywhere else on the vector, hold, and drag to move it around. The origin of the coordinate system can be moved by clicking and dragging the grey circle. Move the origin of the coordinate system so that it is in the center of the screen. Finally, there is a reset button in the lower right corner which will reset the experiment. Play around with the simulation and see what the various buttons do.



Briefly, the four numbers at the top, the vector data, are as follows. $|\vec{a}|$ is the length or magnitude of the vector, θ is the "standard" angle of the vector. The "standard" angle is the one the vector makes with the positive x-axis. Checkmark the "Values" option, and the angle option below it, in the upper right of the window, in order to display the magnitude and the standard angle. a_x and a_y are the components of the vector. To see these displayed, click the button to the right of or below the eye, in the "Components" options, in the middle right of the window.

Exercise 1: Place one vector on the screen and move it around i.e. change the position of the entire object, without altering the angle or the length. Does any of the vector data change when you move the vector as a whole? Take a screen shot.

We now discuss vectors in more detail.

- A **scalar** quantity is one which has a size, but not a direction.
    - Examples are temperature, mass, volume.
- A **vector** quantity is one which is characterized by both a magnitude (i.e. a size) and a direction.
    - Examples: Displacement, velocity, force
- The **magnitude of a vector** is a positive number, which conveys the size. The physical interpretation of the magnitude depends on what the vector is:
    - Displacement: The magnitude is the distance.
    - Velocity: The magnitude is the speed.
    - Force: The magnitude is the strength of the force.

We are used to working with numbers, as purely mathematical objects, without specifying what scalar quantities the numbers are measuring (i.e. we can say 3+2=5 or we can say 3kg +2kg=5kg).

Similarly, we can work with vectors, as purely mathematical objects, just to get used to the arithmetic operations we can do with vectors.

- Vectors have a graphical representation: an arrow.
    - The length of the arrow represents the magnitude of the vector.
    - The direction of the arrow represents the direction of the vector.

Note that the magnitude of a vector quantity can, in general, have units, e.g. meters or Newtons. So, the length of the arrow is a representation of the magnitude, but it's not identical to it, unless we are working with vectors as purely abstract mathematical objects.

The direction is dimensionless and is essentially identical for the arrow and the actual quantity. The only adjustment is associating the orientation/directions of the planes, e.g. the plane of a page can represent east, west, north south, or the plane of a page can represent up, down, left, right. We must also specify how exactly the direction is associated with a number. The **direction** can be given by an **angle**, with respect to another direction which is known and fixed.

- The **"standard" angle** is the one with the positive x-axis.
    - Equivalently, it's the angle with the horizontal line which intersects the base of the vector and extends to the right.
    - To see that these are the same, place your vector with its base at the origin. The displayed angle will be with the positive x-axis. Then move your vector elsewhere. The angle remains unchanged, but the horizontal line is no longer the x-axis.

Notation:

- A **vector** quantity is denoted by putting an arrow over the variable name
    - Examples: $\vec{A}, \vec{v}, \vec{F}$
- The **magnitude** of a vector is denoted by drawing the symbol without any decorations or by using the absolute value symbol
    - Example: The magnitude of $\vec{A}$ would be written as A or $|\vec{A}|$
- The **angle** in the simulation is denoted by θ. One can put the vector name as a subscript: θ_A.

Unit Vectors, Multiplication of Vector by Scalar, and Vector Addition

Click the crossed out eye button for the next few exercises so that the components are not displayed.

Exercise 2: Take two vectors which are the same i.e. have the same length and angle and place the base of one vector at the arrow tip of the other. Can you think of the combination as a new vector? What would be its magnitude and direction? What if you take a third vector with the same length and angle. What does the combination of three vectors look like?

Write down the length and angle of your original vector, the length and angle of the double vector, the length and angle of the triple vector. You might click the "sum" option, if you are not sure. Take a screen shot.

- In general, we can multiply a vector by a number. The angle doesn't change, whereas the magnitude is multiplied by the number i.e. the number stretches (or contracts) the vector.
  - If $c\,\vec{A} = \vec{B}$ then B=c A and $\theta_A=\theta_B$

Exercise 3: Display a rightward horizontal vector with a length of 1. Display an upward vertical vector with a length of 1. A vector with length 1 is called a **unit vector**. The horizontal unit vector has the special notation: $\hat{x}$ or $\hat{\imath}$. The vertical unit vector has the special notation: $\hat{y}$ or $\hat{\jmath}$. (Pick either the first notation for both or the second notation for both, but don't mix them.) State the angle associated with each unit vector. Take a screen shot. Now flip the direction of the vectors (i.e. to make them leftward and downward). These are -$\hat{x}$ and -$\hat{y}$ respectively. State the angles of these unit vectors. Notice that the angles in exercise 3 follow the general rule

- If $-\vec{A} = \vec{B}$ then B=A and $\theta_A=\theta_B$+180 degrees

Exercise 4: Display two vectors: $\vec{a}$ =$a_x\hat{x}$ and $\vec{b}$ =$b_y\hat{y}$ where a_x and b_y are two numbers of your choice, which can be positive or negative. State the values of a_x and b_y and take a screen shot.

(Another notation is that $\hat{A} = \vec{A}/A$ i.e. a vector divided by its magnitude is a unit vector in the direction of the original vector. The unit vector is written as the same letter, but with a ^ instead of the arrow.)

Exercise 5: What would it mean to add vectors $\vec{a}$ =$a_x\hat{x}$ and $\vec{b}$ =$b_y\hat{y}$? As in exercise 2, put the base of one of the vectors at the tip of the arrow of the other one. (Order doesn't matter.) Click the "Sum" option and arrange the three vectors into a triangle. Take a screen shot. Check that if you reverse the order of the vectors, the sum doesn't change.

Notice that if you click on the sum vector, $\vec{s}$, $s_x=a_x$ and $s_y=b_y$.

- Exercise 5 demonstrates how **we can write any vector as a sum of a purely vertical vector and a purely horizontal vector:** $\boldsymbol{\vec{s} = \vec{a} + \vec{b} = s_x\hat{x} + s_y\hat{y}}$
- You can check (and it can be proven) that **the magnitude of any vector is related to the components as** $\boldsymbol{s = \sqrt{s_x{}^2 + s_y{}^2}}$

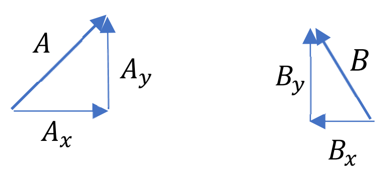
- You can check (and it can be proven) that **the angle is given by** $\boldsymbol{\theta_s = \tan^{-1}\left({}^{s_y}/_{s_x}\right)}$ **if** $s_x>0$**.**
    - **The angle is given by** $\boldsymbol{\theta_s = \tan^{-1}\left({}^{s_y}/_{s_x}\right) + 180°}$ **if** $s_x<0$**.**
    - Make sure to include the sign of s_x, s_y in the formula. Note that the result of the $\tan^{-1}$ function is negative if the ratio is negative. Recall a negative angle is measured clockwise.
    - If $s_x=0$, you cannot use the above formula, but you know the vector is purely vertical so its angle is either 90° ($s_y>0$) or 270° ($s_y<0$)

<u>Exercise 6:</u> Fill out the table. Use the values from exercise 5 in the first 4 lines. You can use the simulation for these or compute by hand. The last four lines require a calculation to 3 decimal places and cannot be done within the simulation. Show your work for the last two lines.

Magnitude	Angle	x-component	y-component
		a_x	b_y
		$-a_x$	$-b_y$
		a_x	$-b_y$
		$-a_x$	b_y
		-2.952	0.000
		0.000	-3.848
		2.452	-5.000
		-6.939	4.000

What if we know the angle and magnitude of a vector? How can we figure out the components?

- <u>Trigonometry:</u> A vector can always be interpreted as a right triangle with
    - hypotenuse = magnitude
    - horizonal side = x-component
    - vertical side = y-component



Note that when using the trigonometric approach, you still must put in the signs of the components by hand.

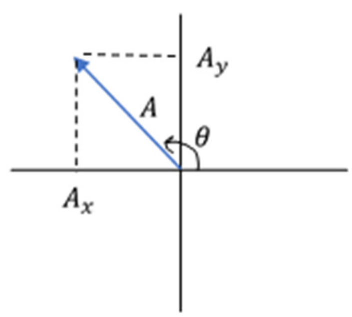
In the left example both A_x and A_y would be positive, while in the right example B_x would be negative and B_y would be positive.

While you can always set up a right triangle and use trigonometry to determine the components or the magnitude and angle, it is convenient to have a general procedure to follow.

- **Given the magnitude** $\boldsymbol{A}$ **and angle** $\boldsymbol{\theta}$ **to the positive x-axis, the components are given by**

  $\boldsymbol{A_x = A\cos(\theta)\ ;\ A_y = A\sin(\theta)}$

  Note that the signs will work out correctly, only if you use the angle to the positive x-axis.



- Graphical vector addition: The procedure for adding vectors that we used in exercise 2 and 5, can be used with any vector and any number of vectors. Just take each successive vector and put its base at the tip of the arrow of the previous vector. The sum vector goes from the base of the first vector to the tip of the arrow of the last one. The order of the vectors does not change the result.

- Algebraic Vector addition: We also know that any vector $\vec{A} = A_x\hat{x} + A_y\hat{y}$ and $\vec{B} = B_x\hat{x} + B_y\hat{y}$ so

$$\vec{S} = \vec{A} + \vec{B} = A_x\hat{x} + A_y\hat{y} + B_x\hat{x} + B_y\hat{y} = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} = S_x\hat{x} + S_y\hat{y}$$

Where $S_x = A_x + B_x$, $S_y = A_y + B_y$

Exercise 7: Reset the PhET simulation and move the origin so that it is in the center of your screen. Place vector $\vec{a}$ in the first quadrant, vector $\vec{b}$ in the second quadrant and vector $\vec{c}$ in the third quadrant. Arrange the vectors so that they each start at the origin and point outwards. You can pick any length and direction, but make sure the vectors each have a different length and that you put them in the appropriate quadrant. Record the magnitude, angle, and components of each vector in the table below.

Click the "Sum" box to display the vector $\vec{s}$, which is the sum of the three vectors. Write down its information in the table below. Now move the 3 vectors and arrange them so as to graphically demonstrate that $\vec{s} = \vec{a} + \vec{b} + \vec{c}$. In other words, use the graphical vector addition procedure described above. (Move the origin if you need to.) Take a screen shot. Fill out the table below.

Vector	Magnitude	Angle	x-component	y-component
$\vec{a}$				
$\vec{b}$				
$\vec{c}$				
$\vec{s}$				

Exercise 8: Reset the PhET simulation. Choose three vectors so that so that the sum is zero. (Think about what it would mean graphically.) Click Sum to check. Adjust the vectors until $\vec{s} = 0$. Record your values for the three vectors and the sum of the first two in the table below.

Vector	Magnitude	Angle	x-component	y-component
$\vec{a}$				
$\vec{b}$				
$\vec{c}$				
$\vec{a} + \vec{b}$				

Check the components shown by the simulation using the formulas given above. Show your work. Show that the components add up to 0.

Now think of the three vectors as forces pulling an object at the center. You can think of each vector as a rope with someone pulling it. Move the vectors (without changing them) and place the base of all three vectors at the origin. (You might have to move the origin.) Take a screen shot.

-Lab activity adapted from lab by John Estes and lab by German Kolmakov