

Heat and Gasses

Atomic Theory

- * We now know that all matter is made up of atoms. The Greeks originally proposed this, but it took until the 18th and 19th centuries to show that this was true.
- * The mass of a hydrogen atom is 1.67×10^{-27} kg and the size is about 10^{-10} m. So anything we can see has many many many atoms. One kilogram of hydrogen consists of 6.0×10^{26} atoms.
- * Temperature is caused by the motion of atoms. Temperature is actually kinetic energy. But since we can't count the kinetic energy of each atom in a substance we discuss the temperature of all the atoms.

Temperature Scale

- * There are three different units used for temperature; Fahrenheit, Celsius and Kelvin.
- * **Fahrenheit** is the scale only used in America.
- * **Celsius** is the scale used everywhere else in the world. For this scale zero is when water freezes and 100 is when water boils.
- * **Kelvin** is the scale used in physics. One degree in Kelvin is the same as one degree in Celsius, but zero Kelvin is -273.15 Celsius also known as absolute zero.

Thermal Expansion

- * Most substances expand when heated. The amount of expansion can be described by the following formula

$$\Delta L = \alpha L_0 \Delta T$$

- * where L_0 is the original length of the object, ΔT is the change in temperature, ΔL is the increase in length of the object and α is a constant called the coefficient of expansion that depends on the substance.

Example 17-3

- * The steel bed of a suspension bridge is 200m long at 20°C. If the extremes of temperature to which it might be exposed are -30°C to 40°C, how much will it contract and expand?

known

$$L_0 = 200\text{m}$$

unknown

$$\Delta L = ?$$

physics

$$\Delta L = \alpha L_0 \Delta T$$

we look up the coefficient of expansion for steel and find $\alpha = 12 \times 10^{-6}/^\circ\text{C}$

$$\Delta L = (12 \times 10^{-6}/^\circ\text{C})(200\text{m})(40^\circ\text{C} - 20^\circ\text{C}) = 4.8 \times 10^{-2}\text{m}$$

$$\Delta L = (12 \times 10^{-6}/^\circ\text{C})(200\text{m})(-30^\circ\text{C} - 20^\circ\text{C}) = -12.0 \times 10^{-2}\text{m}$$

Ideal Gas Law

- * In the case of gasses things are a little more complicated. Gasses expand, but they can also change their pressure. The formula for gasses is thus a little more complicated.

$$PV = nRT$$

- * Where n is the amount of moles of gas and R is a constant, $R=8.314 \text{ J}/(\text{mol} \cdot \text{K})$, and the temperature must be in Kelvin.
- * The ideal gas law can also be expressed in terms of the total number of molecules N with a different constant called Boltzman's constant $k=1.38 \times 10^{-23} \text{ J}/\text{K}$.

$$PV = NkT$$

Absolute Zero

- * From the ideal gas law you see that at constant pressure as you decrease the temperature of a gas the volume decreases.
- * Since volume can't be less than zero, this suggests there is a minimum temperature that a gas can reach. This is called absolute zero and is the basis for the Kelvin scale.
- * Based on the atomic theory we can understand that since temperature is a measure of the kinetic energy of atoms that at absolute zero the velocities of all atoms go to zero.

Example 17-11

- * A helium party balloon assumed to be a perfect sphere, has a radius of 18.0cm. At room temperature (20°C) its internal pressure is 1.05atm. Find the number of moles of helium and the mass of helium in the balloon.

known

$$r = 18.0\text{cm} = 0.18\text{m}$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.18\text{m})^3 = 0.0244\text{m}^3$$

$$P = 1.05\text{atm} = (1.05)(1.013 \times 10^5) = 1.064 \times 10^5 \text{ N/m}^2$$

$$T = 20^\circ\text{C} = 20 + 273 = 293\text{K}$$

unknown

$$n = ?$$

physics

$$PV = nRT$$

$$n = \frac{PV}{RT} = \frac{(1.064 \times 10^5 \text{ N/m}^2)(0.0244\text{m}^3)}{(8.314\text{J}/(\text{mol} \cdot \text{K})(293\text{K})} = 1.066 \text{ mol}$$

$$m = (1.066\text{mol})(4.00\text{g/mol}) = 4.26\text{g}$$

Understanding the Ideal Gas Law

- * The ideal part of the Ideal Gas Law is that the molecules only interact by collisions and that those collisions are elastic.
- * Under these assumptions we can write down what is going on in a gas just using the physics we have learned.
- * Let's imagine a gas in a box. Every time a molecule hits a wall it will transfer some momentum to the wall. This change in momentum, or force, causes the pressure on the wall.

Understanding the Ideal Gas Law

When a molecule hits the wall the component of the velocity in the direction of the wall will transfer momentum. Let's call this the x direction. Since the collision is elastic if this is v_x before the collision it must be $-v_x$ after.

$$\Delta p = mv_x - m(-v_x) = 2mv_x$$

This molecule will hit the wall, bounce across the box, hit the other side and then come back to hit the wall again. The time it will take to do this is if the length of the box is L is

$$\Delta t = \frac{2L}{v_x} \quad \text{so} \quad F = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{\frac{2L}{v_x}} = \frac{mv_x^2}{L}$$

This is for just one molecule. If there are N molecules then each one will hit the wall with a different speed. But we can talk about the average square speed of a molecule, then

$$F = \frac{mN\bar{v}_x^2}{L} \quad \text{since there is nothing special about the x-direction we know}$$

$$\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 \quad \text{and} \quad \bar{v}^2 = \bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2 \quad \text{so} \quad \bar{v}_x^2 = \frac{1}{3}\bar{v}^2 \quad \text{and} \quad F = \frac{mN\bar{v}^2}{3L}$$

$$P = \frac{F}{A} = \frac{2}{3} \frac{N}{LA} \frac{1}{2} m\bar{v}^2$$

Understanding the Ideal Gas Law

$$P = \frac{F}{A} = \frac{2}{3} \frac{N}{LA} \frac{1}{2} m \bar{v}^2$$

$$PV = N \frac{2}{3} \left(\frac{1}{2} m \bar{v}^2 \right)$$

Notice that $\frac{1}{2} m \bar{v}^2$ is the kinetic energy K , so if

$$PV = NkT \quad \text{then} \quad kT = \frac{2}{3} \left(\frac{1}{2} m \bar{v}^2 \right) \quad \text{and} \quad K = \frac{3}{2} kT$$

So not only do we derive the ideal gas law using mechanics, but we find that temperature must be related to the total kinetic energy of the gas.

In a gas the average speed molecules are moving is zero, since velocity is a vector and then are moving in every direction. More interesting is the root mean square (or rms) speed of the molecules, which is directly related to the temperature of the gas.

$$v_{rms} = \sqrt{\bar{v}^2} = \sqrt{\frac{3kT}{m}}$$

Example 18-2

- * What is the rms speed of air molecules (O_2 and N_2) at room temperature ($20^\circ C$)?

known

$$T = 20^\circ C = 20 + 273 = 293K$$

unknown

$$v_{rms} = ?$$

physics

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

$$m(O_2) = (32amu)(1.66 \times 10^{-27} kg/amu) = 5.3 \times 10^{-26} kg$$

$$m(N_2) = (28amu)(1.66 \times 10^{-27} kg/amu) = 4.6 \times 10^{-26} kg$$

$$v_{rms}(O_2) = \sqrt{\frac{3(1.38 \times 10^{-23} J/K)(293K)}{5.3 \times 10^{-26}}} = 480m/s$$

$$v_{rms}(N_2) = \sqrt{\frac{3(1.38 \times 10^{-23} J/K)(293K)}{4.6 \times 10^{-26}}} = 510m/s$$

Homework

- * Vol 2, Chapter 1 - 44, 53, 62, 69
- * Vol 2, Chapter 2 - 20, 27, 46