Forces

Chapters 6 and 13

Friction

* So far we have been constantly ignoring friction.

* Obviously, friction is actually important otherwise we wouldn't have to say to ignore it every time.

* Friction is complicated and poorly understood. Basically, on the microscopic level there are all kinds of bumps and ridges on a material and when sliding something this bumps and ridges hit one another and it takes force to get them pass one another.

* We will focus on two types of friction kinetic friction and static friction.

Kinetic Friction

* One way to parameterize friction is to consider the friction force as proportional to the normal force.
* This makes some sense since the stronger you push two surfaces together the more they stick together.
* We can write this as a:

$$F_{fr} = \mu_k F_N$$

* where μ_k is a parameter that depends on the substance.

Static Friction

* The friction force of something moving is different than the friction force of something at rest, which is often greater.

* We can express this force in a similar way:

$$F_{fr} \leq \mu_s F_N$$

* Note this is a reactive force. It increase as you try to push an object until you reach the maximum value $\mu_s F_N$.

Friction Coefficients

TABLE 5–1 Coefficients of Friction[†]

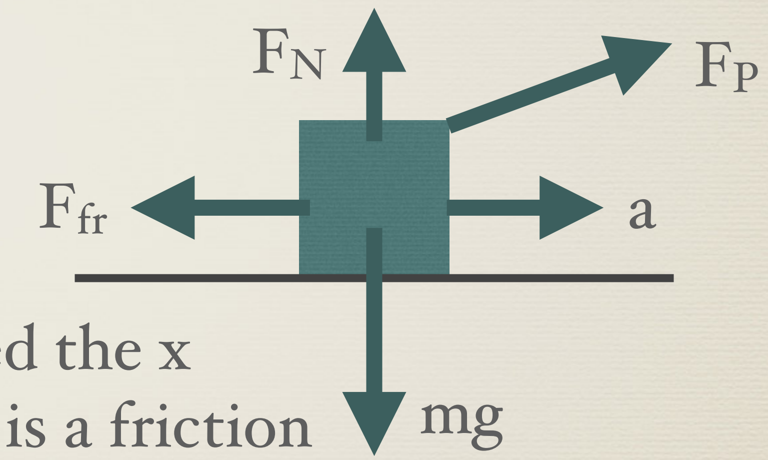
Surfaces	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

[†] Values are approximate and intended only as a guide.

Example 5-3

* A 10.0-kg box is pulled along a horizontal surface by a force of F_P=40.0N applied at a 30.0° above the horizontal. This is like our previous example but now include friction and we assume a coefficient of kinetic friction of 0.3. Calculate the acceleration.

known	**unknown**
m=10.0 kg	a=?
F_P=40.0N	
θ=30.0°	
μ_k= 0.3	



physics

To get the acceleration we need the x component of the force. But there is a friction force in the x direction so we need the normal force which is in the y direction.

$$F_{Px} = F_p \cos\theta = (40.0N)(\cos 30.0) = 34.6N \qquad F_{Py} = F_p \sin\theta = (40.0N)(\sin 30.0) = 20.0N$$

y:

$$F_N + F_{Py} = mg \qquad F_N = mg - F_{Py} = (10.0kg)(9.8m/s^2) - 20.0N = 78N$$

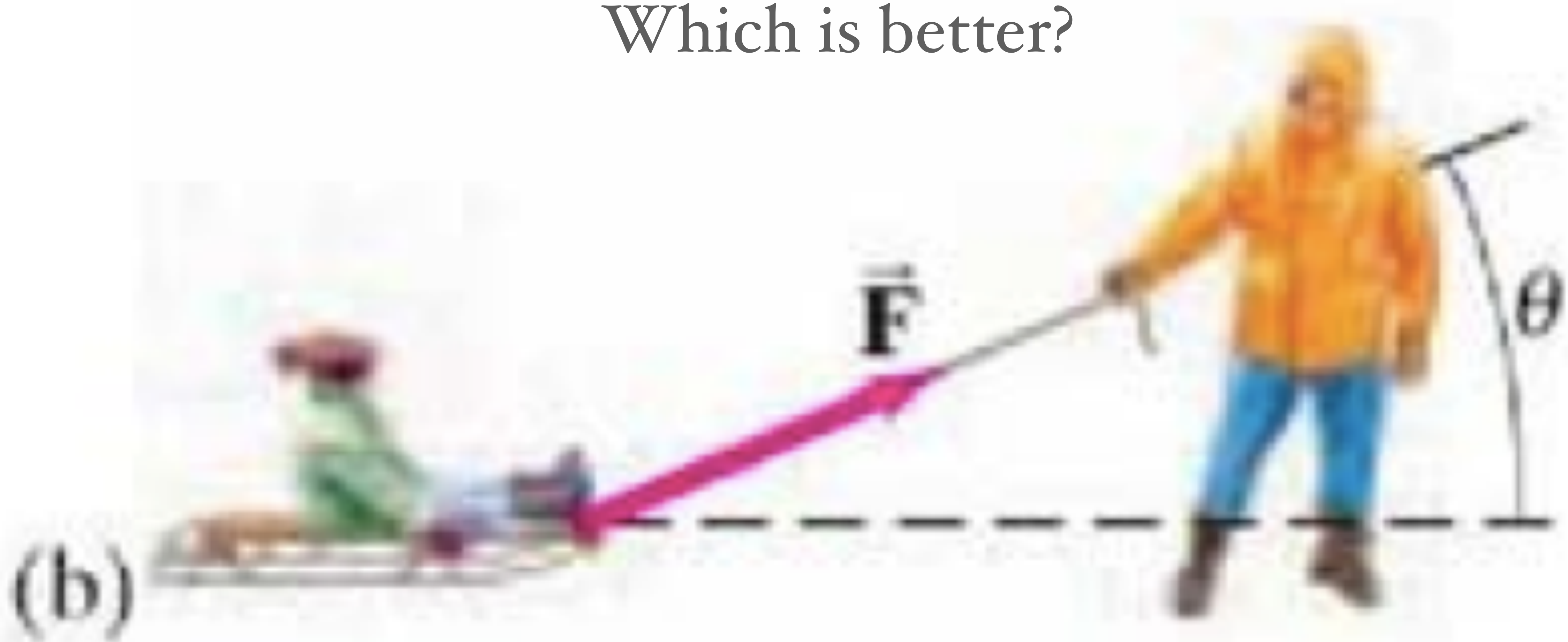
x:

$$F_{fr} = \mu_k F_N = (0.3)(78N) = 23.4N$$

$$F_x = F_{Px} - F_{fr} = 34.6N - 23.4N = 11.2N$$

$$a_x = \frac{F_x}{m} = \frac{11.2N}{10.0kg} = \boxed{1.12m/s^2}$$

Which is better?



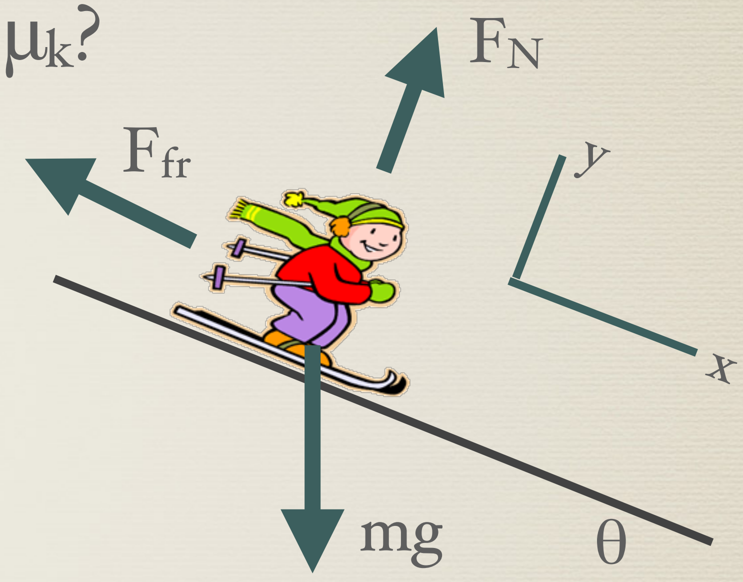
Example 5-6

* A skier is descending a 30° slope, at constant speed. What can you say about the coefficient of kinetic friction μ_k?

known

a = 0

θ=30.0°



y:

$$F_N - mg\cos\theta = 0 \quad => F_N = mg\cos\theta$$

x:

$$mg\sin\theta - F_{fr} = 0 \qquad F_{fr} = \mu_k F_N = \mu_k mg\cos\theta \qquad mg\sin\theta - \mu_k mg\cos\theta = 0$$

$$\mu_k = \tan\theta = \tan(30) = \boxed{0.58}$$

Circular Motion

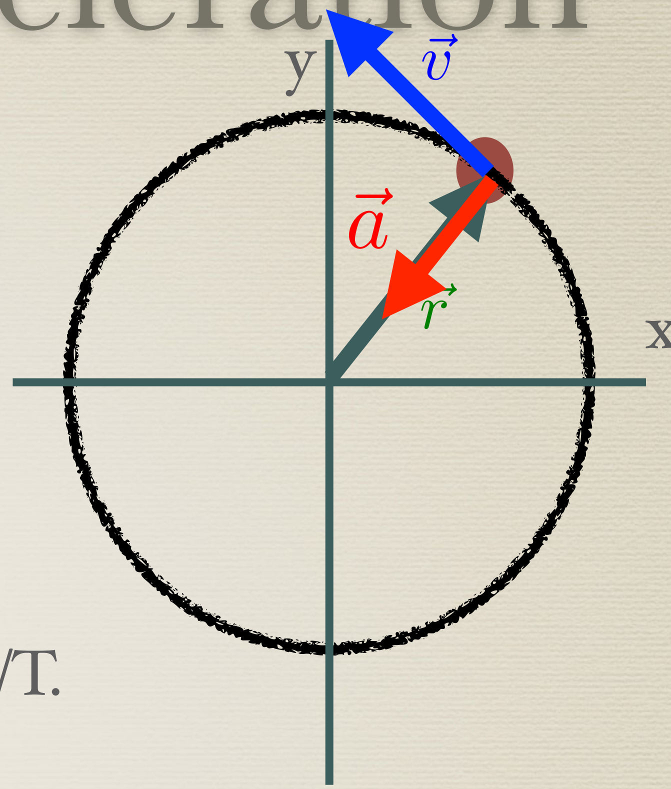
* Uniform circular motion is when an object travels in a circle at a constant speed.

* Such motion requires acceleration as the objects velocity is constantly changing. Not the magnitude but the direction.

* This acceleration is called centripetal acceleration and is given by

$$\vec{a}_R = -\frac{v^2}{r}\hat{r}$$

Deriving Centripetal Acceleration



The motion of an object in a circle can be give by

$$\vec{r} = r\cos\left(\theta(t)\right)\hat{i} + r\sin\left(\theta(t)\right)\hat{j}$$

where θ(t) is the angular position as a function of time.
For uniform velocity this will just be linear in time θ(t) = ωt

The object goes from 0 to 2π in a period T. So θ(T)= 2π and ω=2π/T.
The object travels a distance 2πr in one period so the velocity is

$$v = \frac{2\pi r}{T} \quad \text{=>} \quad T = \frac{2\pi r}{v} \quad \text{=>} \quad \omega = \frac{v}{r}$$

$$\vec{r} = r\cos\frac{vt}{r}\hat{i} + r\sin\frac{vt}{r}\hat{j} \quad \text{=>} \quad \vec{v} = \frac{d\vec{r}}{dt} = -\frac{v}{r}r\sin\frac{vt}{r}\hat{i} + \frac{v}{r}r\cos\frac{vt}{r}\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\frac{v^2}{r}\cos\frac{vt}{r}\hat{i} - \frac{v^2}{r}\sin\frac{vt}{r}\hat{j}$$

$$|a| = \frac{v^2}{r}$$

Example 5-9

* The Moon's nearly circular orbit about the Earth has a radius of about 384,000km and a period of T=27.3 days. Determine the acceleration of the Moon towards the Earth.

known

r = 384,000km = 3.84×10^8 m

T = 27.3d = (27.3d)(24h/1d)(60m/1h)(60s/1m) = 2.36×10^6 s

unknown

a_R = ?

v = ?

physics

$$a_R = \frac{v^2}{r}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(3.84 \times 10^8 m)}{2.36 \times 10^6 s} = 1022 m/s$$

$$a_R = \frac{(1022\, m/s)^2}{3.84 \times 10^8 m} = \boxed{2.72 \times 10^{-3} m/s^2}$$

Centripetal Force

* The net force that causes centripetal acceleration can be called the centripetal force.

* Note this is different than other forces we have discusses. Centripetal force is the result of other forces, not a source of a force.

* If the net forces on an object cause it to move in a circle then those net forces cause a centripetal acceleration.

$$F_c = ma_R = \frac{mv^2}{r}$$

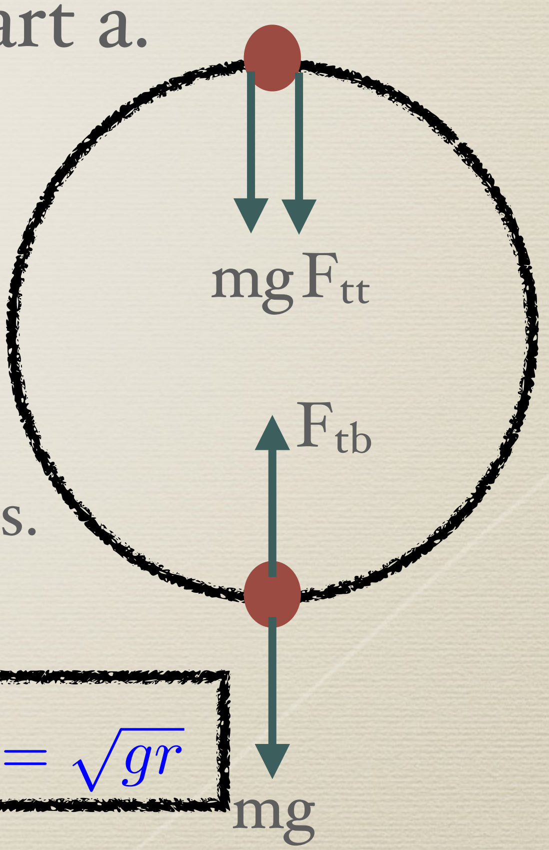
Example 5-12

* A 0.150 kg ball on the end of a 1.10 m long cord (negligible mass) is swung in a *vertical* circle. a) Determine the minimum speed the ball must have at the top of its arc so that he ball continues moving in a circle. b) Calculate the tension in the cord at the bottom of the arc assuming the ball is moving at twice the speed in part a.

known	**unknown**
m = 0.150 kg	v_t = ?
r = 1.10 m	F_{tb} = ?

not uniform circular motion

However, we can use centripetal equation at the top and bottom points.



top:

$$F_{tt} + mg = m\frac{v^2}{r}$$

smallest F_{tt} can be is zero

$$mg = m\frac{v^2}{r} \qquad \boxed{v = \sqrt{gr}}$$

bottom:

$$F_{tb} - mg = m\frac{v^2}{r} = m\frac{4gr}{r} = 4mg \qquad \boxed{F_{tb} = 5mg}$$

Highway Curves

* Circular motion doesn't require that you make a full circle. The acceleration to keep you on just a fraction of a circle is the same.

* An example of this is a car turning on a highway. If we approximate the turn as along a circular path, then the force required is the force required to create the right centripetal acceleration.

* Often curves are banked (tilted) so that gravity provides some of this force.

Example 5-14

* A 1000-kg car rounds a curve on a flat road of radius 50m at a speed of 15 m/s (54km/h). Will the car follow the curve or will it skid? Assume: a) the pavement is dry and the coefficient of static friction is μ_s=0.60; b) the pavement is icy and μ_s=0.25.

known

m = 1000 kg
r= 50 m
v = 15 m/s
μ_s=0.60

unknown

F_{fr} = ?
ma_R = ?

$$F_{fr} \geq ma_r = m\frac{v^2}{r} \qquad F_{fr} \leq \mu_s F_N = \mu_s mg \qquad => \mu_s mg \geq m\frac{v^2}{r}$$

$$=> \mu_s \geq \frac{v^2}{gr} = \frac{(15m/s)^2}{(9.80m/s^2)(50m)} = 0.46$$

if μ_s=0.60 > 0.46 then the car will follow the curve

but if μ_s=0.25 < 0.46 then the car will skid

Drag Force

* Often the force on an object is proportional to its velocity.

* We can call this a drag force. This is also a friction force, but we are trying to give different names to a bunch of related things; air resistance, friction, drag force.

* We will use drag force to mean a force proportional to velocity.

$$F_D = -bv$$

Terminal Velocity

* In the atmosphere, most objects will reach a terminal velocity because of the drag force.

* Take for example a freely falling object, but consider a drag force on it.

$$\sum F = mg - bv$$

always some v_T where bv=mg

$$v_T = \frac{mg}{b}$$

Universal Gravity

* Newton also came up with a rule for gravity.
* The force between two objects is proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$\vec{F}_{12} = G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$$

* Notice this automatically satisfies Newton's 3rd law.

Gravity Near the Earth's Surface

* Near the Earth's surface we have learned two ways of expressing the force of gravity on an object.

$$mg = G\frac{mM_E}{R_E^2}$$

* Where M_E and R_E are the mass and radius of the Earth. Notice that the mass of the object can be divided from both sides, so we see why Galileo's observation must be true.

$$g = G\frac{M_E}{R_E^2} \;\; = (6.67 \times 10^{-11} N \odot m^2/kg^2)\frac{5.98 \times 10^{24} kg}{(6.38 \times 10^6 m)^2} = 9.80 m/s^2$$

* Make sure you don't confuse g and G, they are totally different things, different values, different units and different meanings.

Avatar

How far does he fall?

If we ignore air resistance he falls with constant acceleration.
From the clip we can measure the time as about 6s.

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

If this was Earth then we could use $a_y = g = 9.8\ m/s^2$. However, the scene takes place on Pandora so the acceleration is not $9.8 m/s^2$. From the internet we can find one estimate of Pandora's mass and size as 45% of Earth's mass and 75% of Earth's size. So

$M=(0.45)(5.98 \times 10^{24}\ kg) = 2.7 \times 10^{24}\ kg$

$R=(0.75)(6.37 \times 10^6 m) = 4.8 \times 10^6\ m$

$$F_g = G\frac{Mm}{R^2} \quad => g = \frac{F}{m} = G\frac{M}{R^2} \qquad g_P = G\frac{2.7 \times 10^{24} kg}{(4.8 \times 10^6 m)^2} = 7.8 m/s^2$$

$$0 = y_0 - \frac{1}{2}(7.8m/s^2)(6s)^2 => y_0 = 140m$$

His velocity on hitting the water would be $v = v_0 + at = (-7.8m/s^2)(6s) = 47m/s$

That is very fast, 170 km/hr. In comparison the highest dive into water done on Earth was Oliver Favre in 1987 from a height of 54m. While the atmosphere on Pandora is thicker than Earth, this still seems like a very high drop.

Example 6-4

* Gravity on Everest. Estimate the effective value of g on the top of Mt. Everest, 8850m above sea level. That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude?

known

h = 8850m

R_E = 6389km = 6.389×10^6 m

M_E = 5.98×10^{24} kg

unknown

g = ?

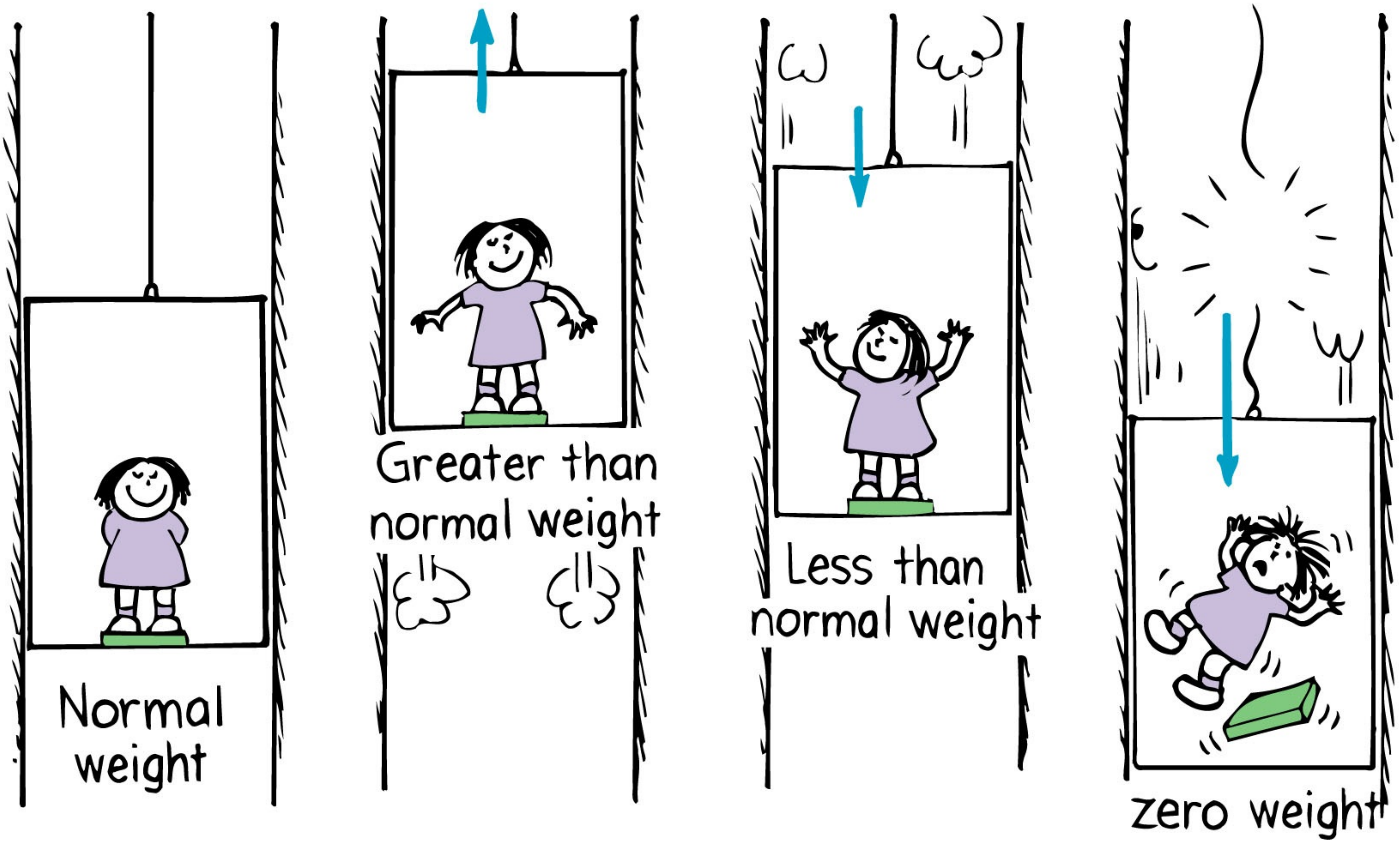
$$g = \frac{F}{m} = G\frac{M}{R^2} = G\frac{M_E}{(R_E + h)^2} \quad = 6.67 \times 10^{-11} Nm^2/kg^2 \frac{5.98 \times 10^{24} kg}{(6.389 \times 10^6 m + 8850m)^2}$$

$$= 9.74N/kg = 9.74m/s^2$$

$$((9.74 - 9.81)/9.81) \times 100 = -0.6\%$$

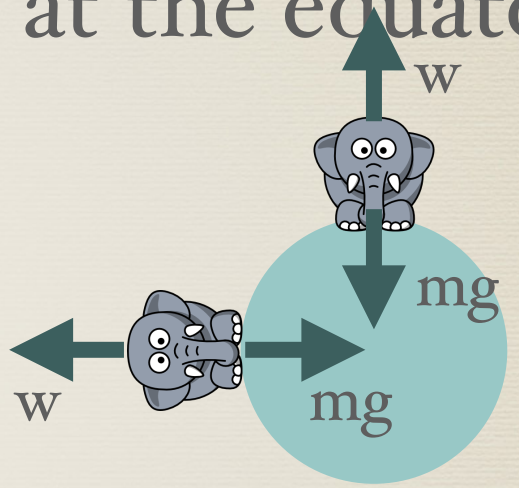
Weightlessness

* Weightlessness is the absence of a normal force. When you are weightless you still feel gravity, just nothing opposes it.
* In space gravity still exists, in fact gravity is felt infinitely far away from an object. The reason things float in a satellite is because everything is accelerating together, so there is no relative acceleration. Everything is still being pulled towards the Earth.
* You can experience weightlessness on Earth. Just go on a ride that drops you. Without a normal force, you will accelerate at g and be weightless.


Normal weight
Greater than normal weight
Less than normal weight
zero weight

Example 6-5

* Assuming the Earth is a perfect sphere, determine how the Earth's rotation affects the value of g at the equator compared to its value at the poles.



pole:

$$mg - w = 0$$

equator:

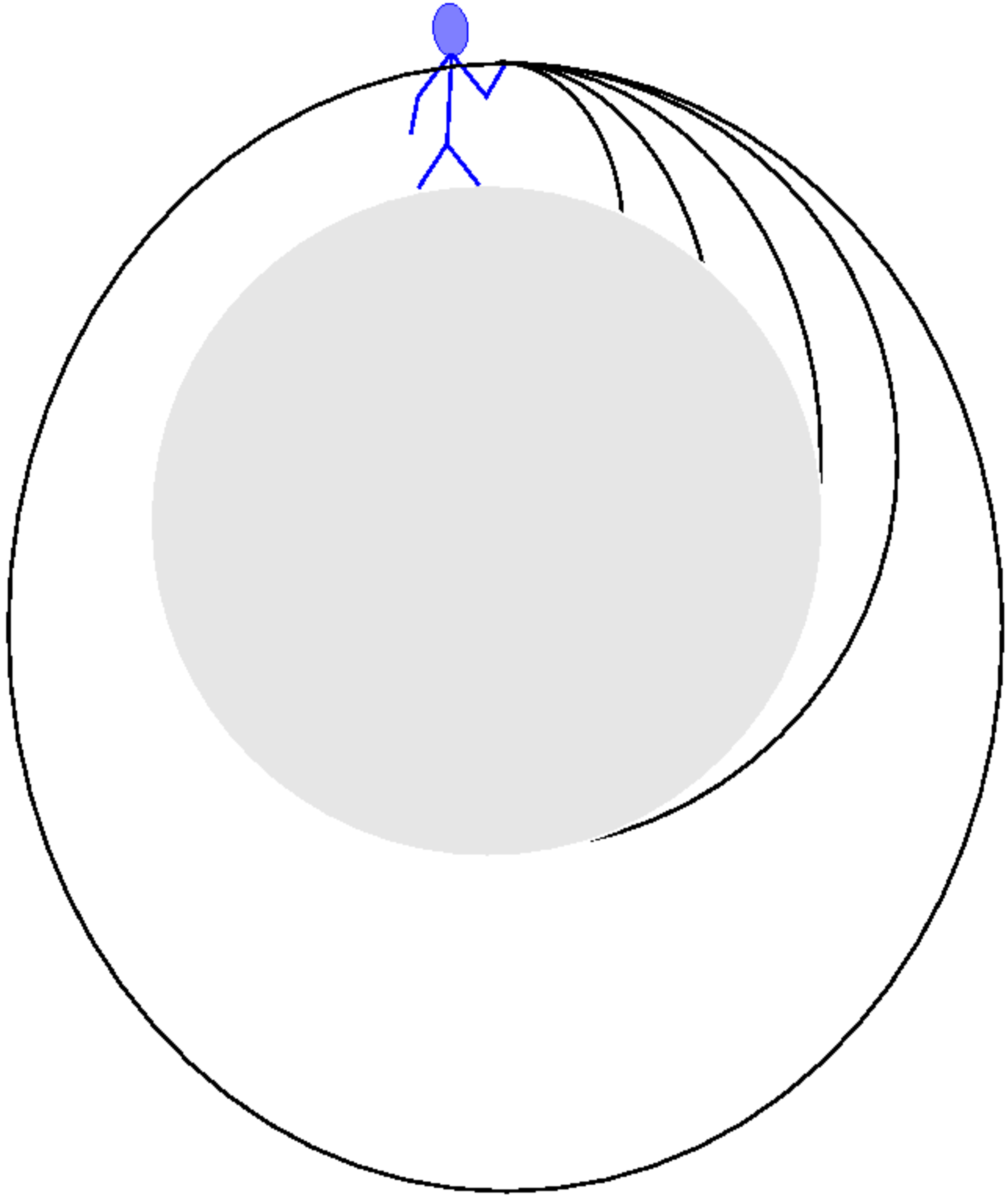
$$mg - w = m\frac{v_E^2}{R_E} \qquad w = mg'$$

$$g' = g - \frac{v_E^2}{R_E} \qquad v_E = \frac{2\pi R_E}{1d} = \frac{2\pi 6.38 \times 10^6 m}{(24h)(60m)(60s)} = 4.64 \times 10^2 m/s$$

$$\Delta g = g - g' = \frac{v_E^2}{R_E} = \frac{(4.64 \times 10^2 m/s)^2}{(6.38 \times 10^6 m)} = 0.0337 m/s^2$$

about 0.3% less

If you throw something with more velocity it will go farther before hitting the ground. If you could throw it fast enough it would miss the Earth as it fell. This is what a satellite does, it keeps falling towards the Earth but its horizontal velocity means it keeps missing the Earth as it falls.



Homework

* Chapter 6: 13,30,37,38,48,55,61,66,72