FLUID DYNAMICS

Chapter 12

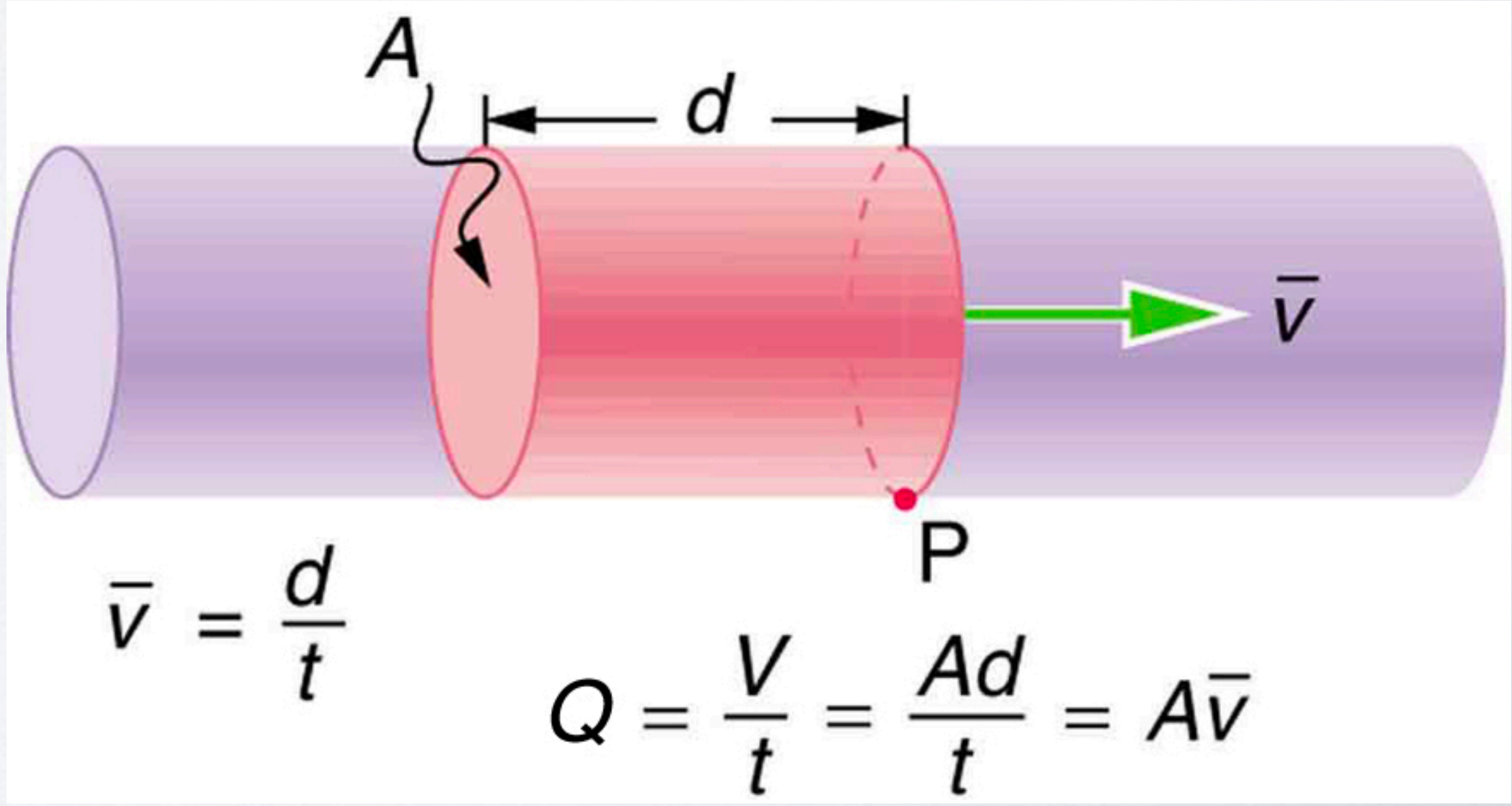
FLOW RATE

- So far we have just discussed fluid statics, but of course a fluid can also me flowing.
- An important quantity for a moving fluid is its flow rate, which is the volume of fluid that moves past some point in a given time.

$$Q = \frac{V}{t}$$

- If the fluid is flowing though something like a pipe then the flow rate can be rewritten in terms of the fluids speed.

$$Q = \frac{V}{t} = \frac{Ad}{t} = A\bar{v}$$



EXAMPLE 12.1

- **Calculating Volume from Flow Rate: The Heart Pumps a Lot of Blood in a Lifetime:** How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min?

$t = 75\ \text{yr} = 2.365\times10^{9}\ \text{s}$

$Q = 5.00\ \text{L/min} = 8.33\times10^{-5}\ \text{m}^3/\text{s}$

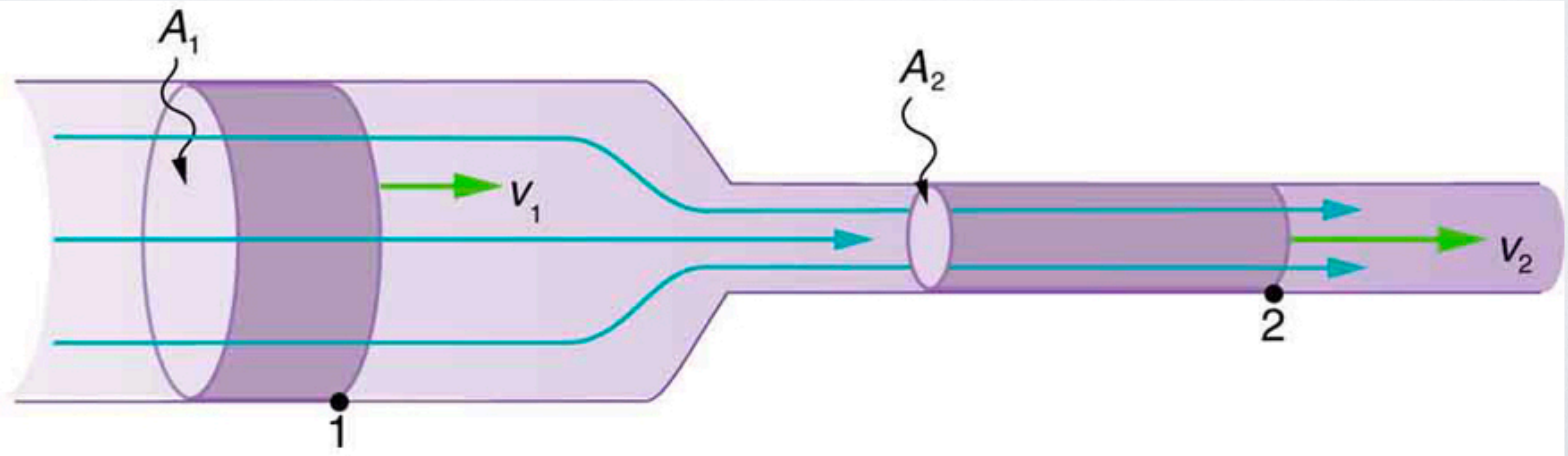
$V = ?$

$$Q = \frac{V}{t} \quad \rightarrow V = Qt$$

$$V = (8.33 \times 10^{-5} m^3/s)(2.365 \times 10^9 s) = 2.0 \times 10^5 m^3$$

- If your fluid is liquid, generally its density can not change much. This means that the volume flow rate is also the mass flow rate. Since in these cases mass is never created or destroyed we get that when the cross sectional area changes the velocity must change to keep the flow rate unchanged.

$$A_1\bar{v}_1 = A_2\bar{v}_2$$



EXAMPLE 12.2

- **Calculating Fluid Speed: Speed Increases When a Tube Narrows:** A nozzle with a radius of 0.250 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

$r_1 = 0.9 \text{ cm} = 9.00\times10^{-3} \text{ m}$

$r_2 = 0.25 \text{ cm} = 2.50\times10^{-3} \text{ m}$

$Q_1 = 0.500 \text{ L/s} = 5\times10^{-4} \text{ m}^3/\text{s}$

$v_1 = ?$

$v_2 = ?$

$$Q = A\bar{v} \qquad \rightarrow \bar{v} = \frac{Q}{A}$$

$$\bar{v}_1 = \frac{Q_1}{\pi r_1^2} = \frac{5\times10^{-4}m^3/s}{\pi(9.0\times10^{-3}m)^2} = 1.96m/s$$

$$A_1\bar{v}_1 = A_2\bar{v}_2$$

$$\bar{v}_2 = \frac{A_1}{A_2}\bar{v}_1 = \frac{r_1^2}{r_2^2}\bar{v}_2 = \frac{0.9^2}{0.25^2}1.96m/s = 25.5m/s$$

BERNOULLI'S EQUATION

- In the previous example the water traveling through the nozzle was moving much faster than the water in the hose. This means the water gained kinetic energy, but where did that energy come from?

- Bernoulli wrote down an equation to account for the different types of energy in a fluid, which are conserved.

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

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- We can see that the 2nd term is kinetic energy per volume.
- And the 3rd term is gravitational potential energy per volume.
- The first term we can recognize is like work if we remember that P = F/A then multiply both by d we get P = Fd/Ad = W/V.
- Thus energy conservation tells us that if a fluid starts moving faster it's pressure must decrease.

EXAMPLE 12.4

- **Calculating Pressure: Pressure Drops as a Fluid Speeds Up:** In Example 12.2, we found that the speed of water in a hose increased from 1.96 m/s to 25.5 m/s going from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is 1.01×10^5 N/m^2 (atmospheric, as it must be) and assuming level, frictionless flow.

$v_1 = 1.96$ m/s
$v_2 = 25.5$ m/s

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

level means h is the same

$P_2 = 1.01 \times 10^5$ N/m^2

$\rho = 1000$kg/m^3

$P_1 = ?$

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 \quad = 1.01 \times 10^5 N/m^2 + \frac{1}{2}(1000kg/m^3)((25.5m/s)^2 - (1.96m/s)^2)$$

$$= 4.24 \times 10^5 N/m^2$$

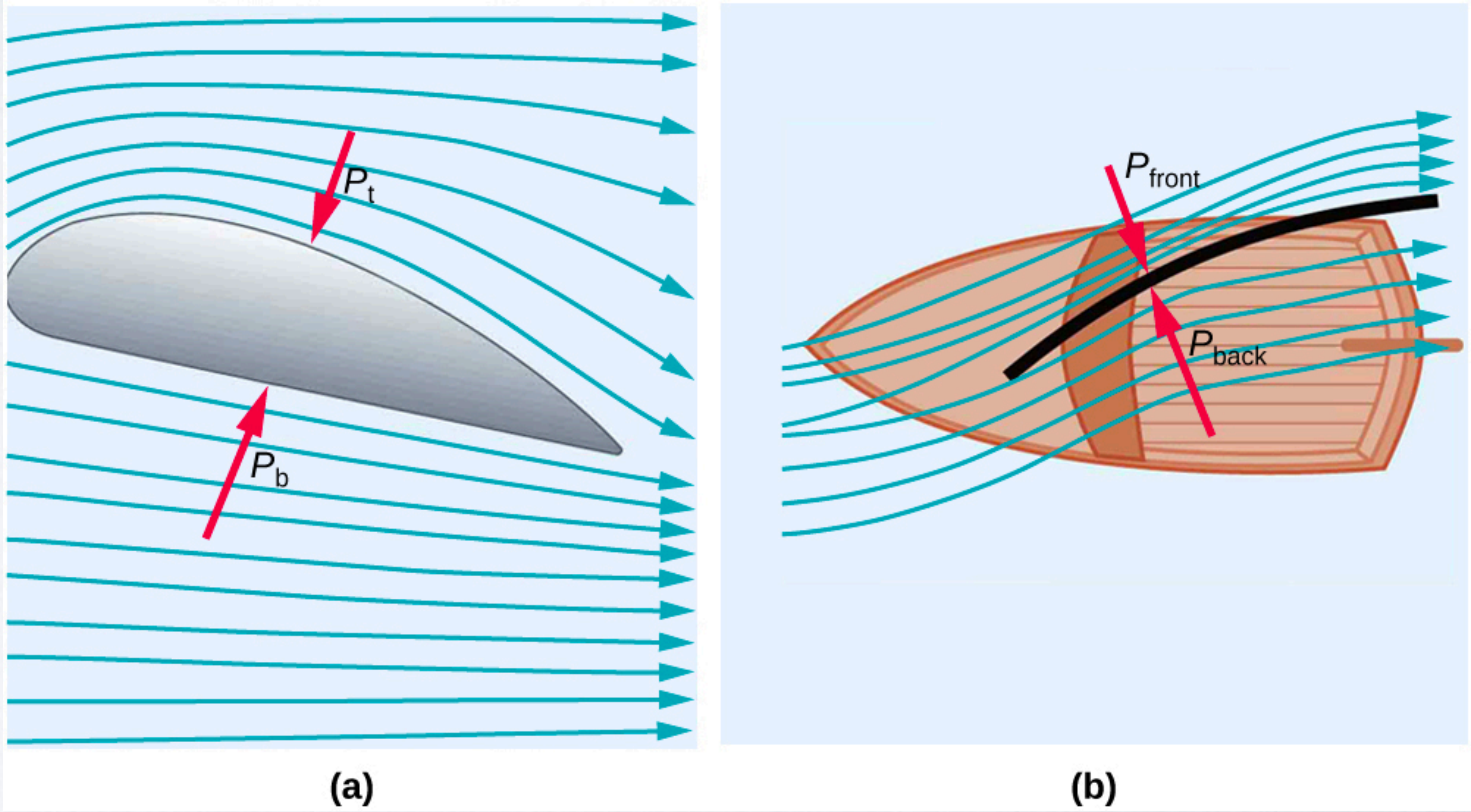
a bit more than 4 atm

WINGS AND SAILS

- Bernoulli's equation shows us how sailboats and airplanes work.

- Wings and sails are curved such that air moves faster on one side than the other.

- Therefore the pressure is lower on one side and that pressure difference exerts a force that can lift a plane or pull a sailboat.



AN EXAMPLE

- How fast must the air above an airplane wing be moving to keep an 8000kg plane aloft if the total wing area is 30m² and the plane is flying at 90m/s at the density of air is 1.2 kg/m³.

ΔPA

mg

m = 8000kg

A = 30m²

v_1 = 90.0m/s

ρ = 1.2 kg/m³

v_2 = ?

$$\Delta PA = mg$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\Delta P = P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) => v_2^2 = \frac{2\Delta P}{\rho} + v_1^2 = \frac{2mg}{A\rho} + v_1^2$$

$$v_2^2 = \frac{2(8000kg)(9.8m/s^2)}{(30m^2)(1.2kg/m^3)} + (90m/s)^2 \quad = 4,355m^2/s^2 + 8,100m^2/s^2 = 12,455m^2/s^2$$

$$v_2 = \sqrt{12,455m^2/s^2} = 111m/s$$

Note the air above the wing is only going 21m/s faster than the air below.

EXAMPLE 12.5

- **Calculating Pressure: A Fire Hose Nozzle:** Fire hoses used in major structure fires have inside diameters of 6.40 cm. Suppose such a hose carries a flow of 40.0 L/s starting at a gauge pressure of $1.62\times10^6 N/m^2$. The hose goes 10.0 m up a ladder to a nozzle having an inside diameter of 3.00 cm. Assuming negligible resistance, what is the pressure in the nozzle?

d_1 = 6.4 cm = 0.064 m

Q = 40 L/s = 0.04 m^3/s

P_1 = 1.62×10^6 N/m^2

h_2 = 10 m

d_2 = 3.0 cm = 0.030 m

P_2 = ?

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho g h_2$$

$$Q = Av$$

$$v_1 = \frac{Q}{A_1} = \frac{Q}{\pi(d_1/2)^2} = \frac{0.04m^3/s}{\pi(0.032m)^2} = 12.4m/s$$

$$v_2 = \frac{Q}{\pi(d_2/2)^2} = \frac{0.04m^3/s}{\pi(0.015m)^2} = 56.6m/s$$

$$P_2 = 1.62\times10^6 N/m^2 + \frac{1}{2}(1000kg/m^3)((12.4m/s)^2 - (56.6m/s)^2) - (1000kg/m^3)(9.8m/s^2)(10m)$$
$$= 1800N/m^2$$

almost zero

HOME WORK

- Chap 12 - 5, 13, 21, 23