ROTATIONAL MOTION

Chapter 10

ANGULAR ACCELERATION

- We have discussed how we can measure the angular velocity of a rotating object as

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- Likewise we can define angular acceleration as the rate at which angular velocity changes
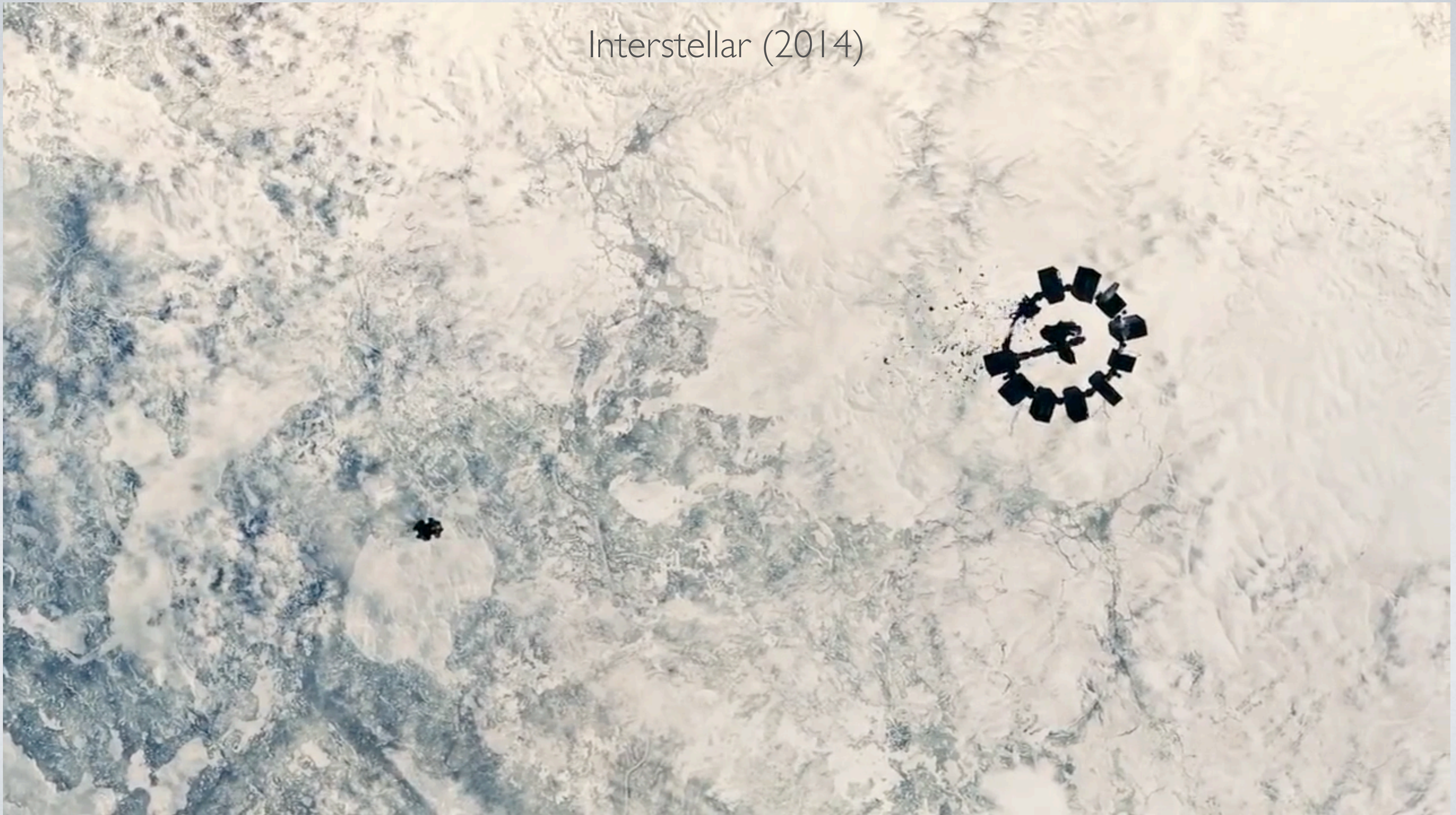
$$\alpha = \frac{\Delta\omega}{\Delta t}$$

- Both of these can be converted to their linear equivalents by multiplying by radius

$$v = r\omega \qquad\qquad a_t = r\alpha$$

-

Interstellar (2014)

What is the angular velocity of the space station?

$$\omega = 68 \text{ rpm} = 68 \times 2\,\pi\,/60\text{s} = 7.1 \text{ rad/s}$$

What is the angular acceleration of the spacecraft?

It looks like it takes about 20s for them to
reach the same angular velocity, so

$$\alpha = \Delta\omega\,/\,\Delta t = 7.1 \text{ rad/s} / 20\text{s} = 0.36 \text{ rad/s}^2$$

EXAMPLE 10.1

- **Calculating the Angular Acceleration and Deceleration of a Bike Wheel** Suppose a teenager puts her bicycle on its back and starts the rear wheel spinning from rest to a final angular velocity of 250 rpm in 5.00 s. (a) Calculate the angular acceleration in rad/s² (b) If she now slams on the brakes, causing an angular acceleration of –87.3 rad/s² , how long does it take the wheel to stop?

a) ω = 250rpm = 250 (2π/rotation)(1 minute / 60s) = 26.2 rad/s

t = 5.00s

α = ?

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{26.2 rad/s}{5s} = 5.24 rad/s^2$$

b) α = -87.3 rad/s²

t = ?

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{-\omega}{t} \qquad t = \frac{-\omega}{\alpha} = \frac{-26.2 rad/s}{-87.3 rad/s^2} = 0.300s$$

EXAMPLE 10.2

- **Calculating the Angular Acceleration of a Motorcycle Wheel:**
  A powerful motorcycle can accelerate from 0 to 30.0 m/s (about 108 km/h) in 4.20s. What is the angular acceleration of its 0.320-m-radius wheels?

v = 30.0 m/s
t = 4.20s
r = 0.32m
α = ?

$$a = \frac{\Delta v}{\Delta t} = \frac{v}{t} = \frac{30.0m/s}{4.20s} = 7.14m/s^2$$

$$\alpha = \frac{a}{r} = \frac{7.14m/s^2}{0.32m} = 22.3rad/s^2$$

CONSTANT ANGULAR ACCELERATION

- Our definitions of angular velocity and angular acceleration are exactly like are definitions of linear acceleration and velocity,

- Therefore the equations of constant angular acceleration look just like the ones for constant linear acceleration with the variables changed.

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

EXAMPLE 10.3

- **Calculating the Acceleration of a Fishing Reel:** A deep-sea fisherman hooks a big fish that swims away from the boat pulling the fishing line from his fishing reel. The whole system is initially at rest and the fishing line unwinds from the reel at a radius of 4.50 cm from its axis of rotation. The reel is given an angular acceleration of 110rad/s² for 2.00s. (a) What is the final angular velocity of the reel? (b) At what speed is fishing line leaving the reel after 2.00s elapses? (c) How many revolutions does the reel make? (d) How many meters of fishing line come off the reel in this time?

r = 4.5 cm = 0.045m

α = 110 rad/s²

t = 2.00s

ω = ?

v = ?

θ = ?

L = ?

$$\omega = \alpha t = (110 rad/s^2)(2s) = 220 rad/s$$

$$v = r\omega = 0.045m(220 rad/s) = 9.90 m/s$$

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}110 rad/s^2(2s)^2 = 220 rad$$

$$revolutions = \frac{220 rad}{2\pi} = 35.0 revs$$

$$L = r\theta = (0.045m)(220 rad) = 9.90m$$

NEWTON'S 2ND LAW

- We have learned about angular acceleration and torque, which is an angular force, how do the two relate?

- Let's consider a force acting on a point mass a distance r from a pivot point rotating it. Then

$$F = ma = mr\alpha$$

$$\tau = rF = mr^2\alpha$$

- so the relation between torque and angular acceleration is not just mass but something called the moment of inertia, which gets the symbol I. Then we have
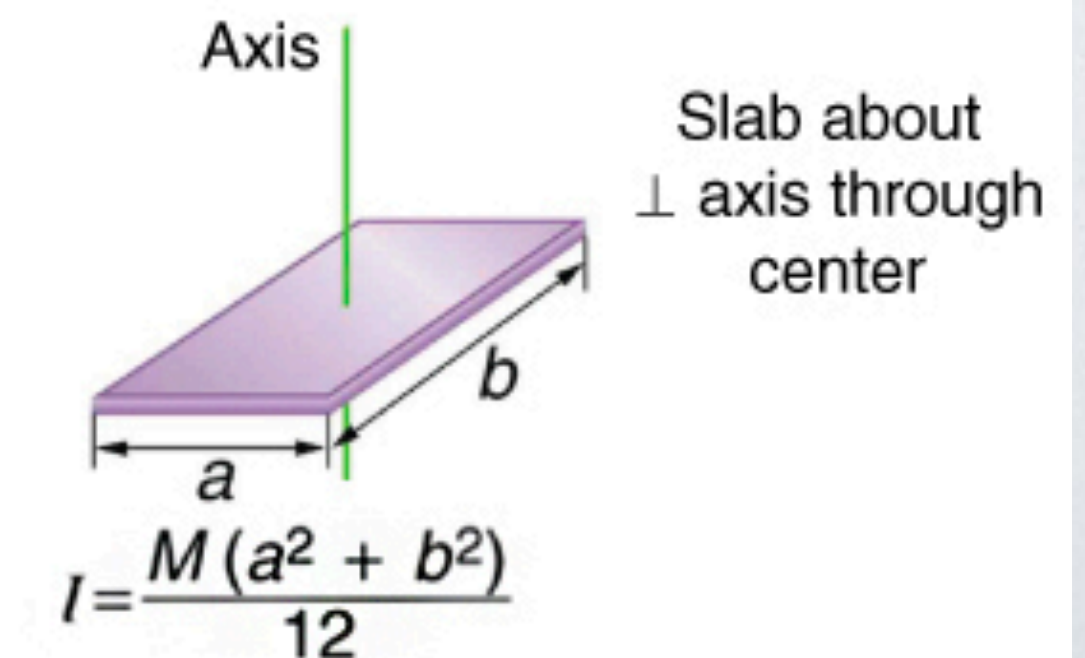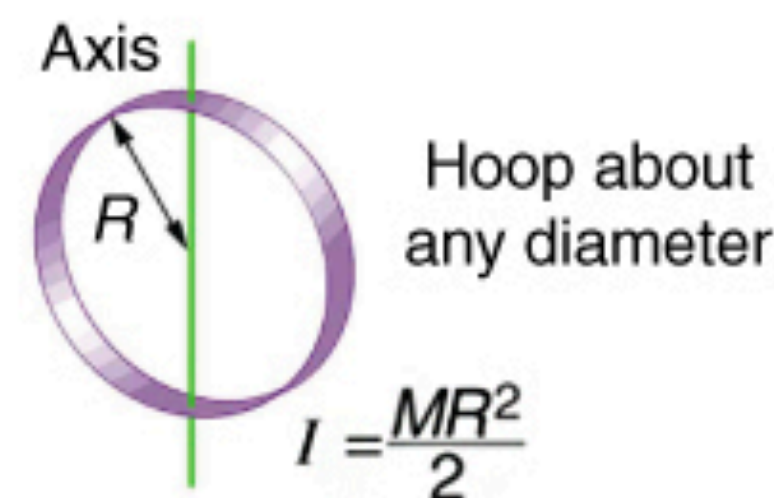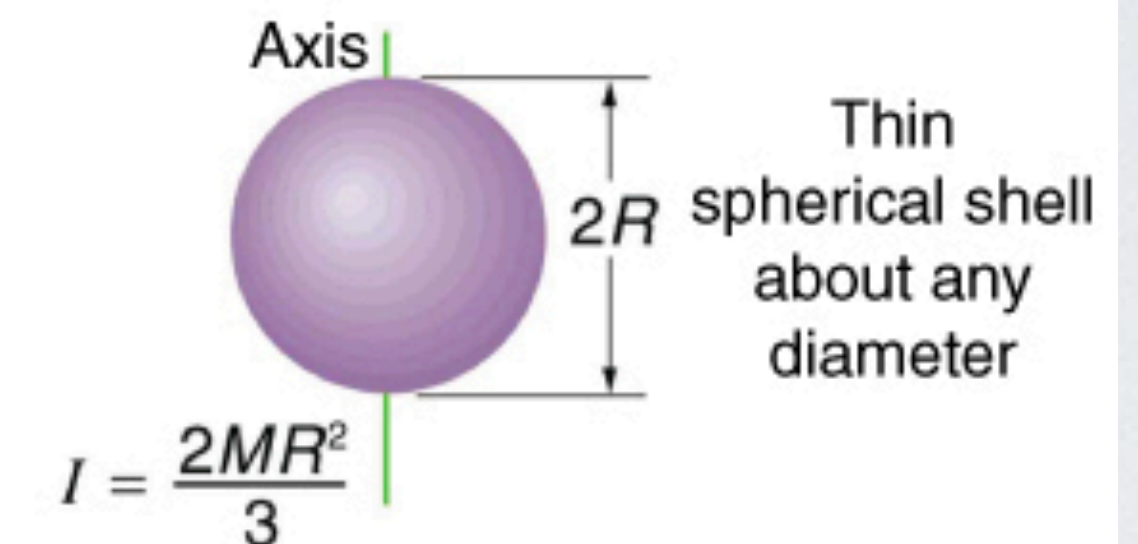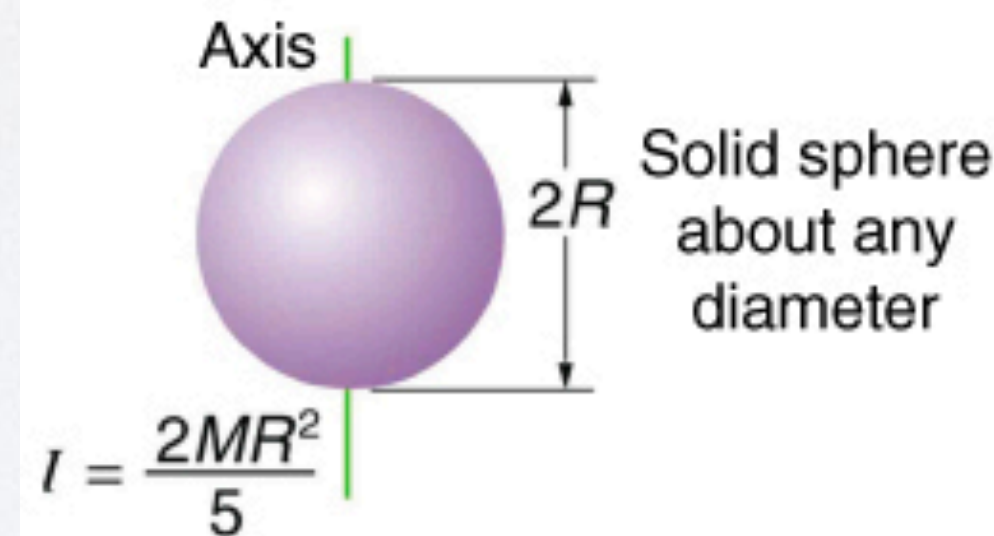
$$\tau = I\alpha$$

MOMENT OF INERTIA

- The moment of inertia for more than one point would just be there sum

$$I = \sum_i m_i r_i^2$$

- for extended objects moments of inertia have been calculated for standard shapes.

The moment of inertia for any shape can be calculated using integration. The results for standard shapes can be found here or online.

Axis

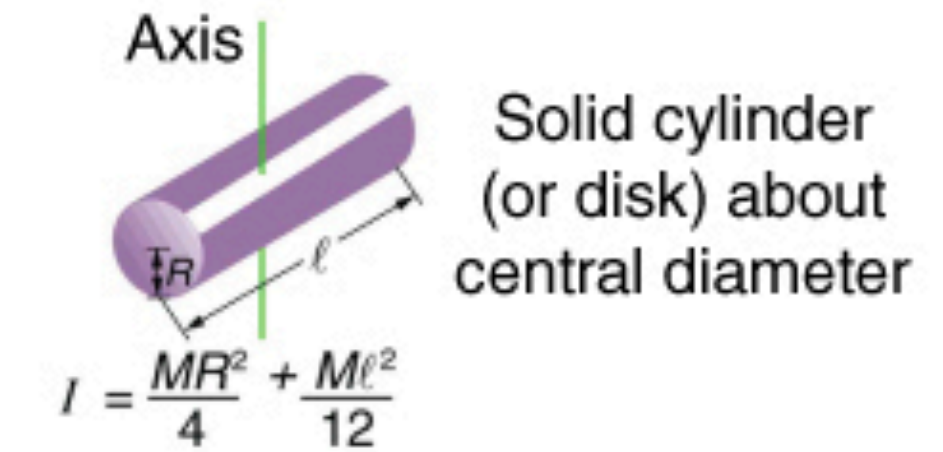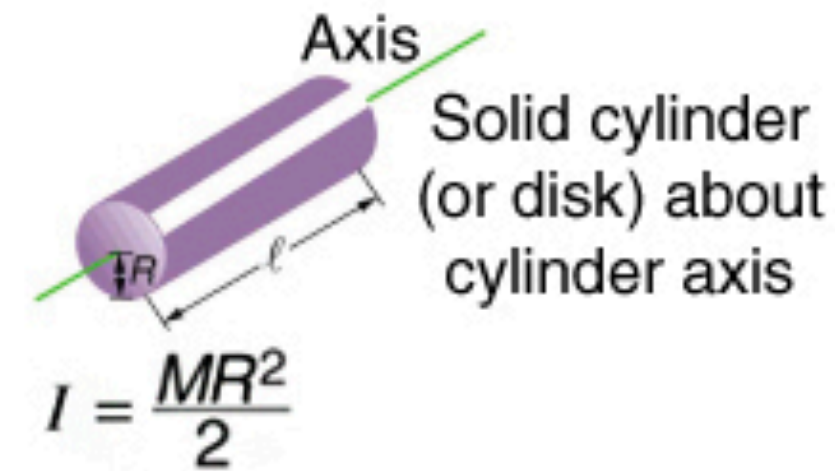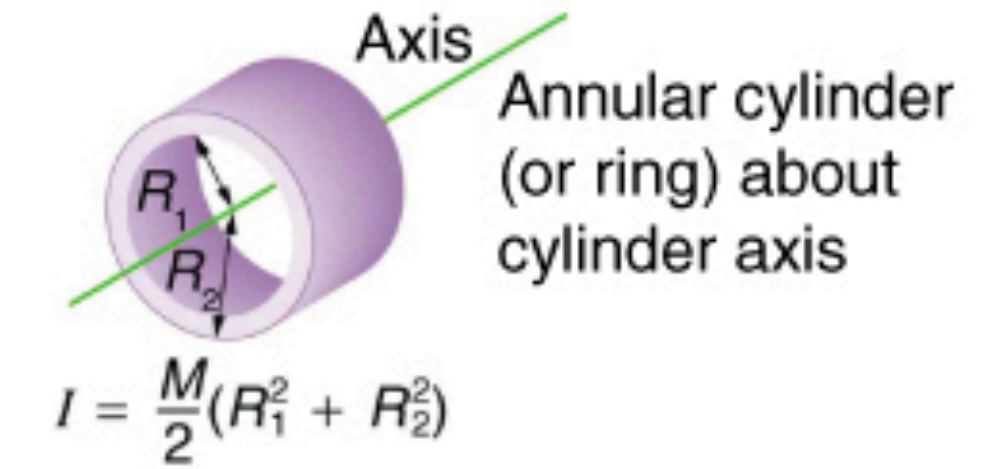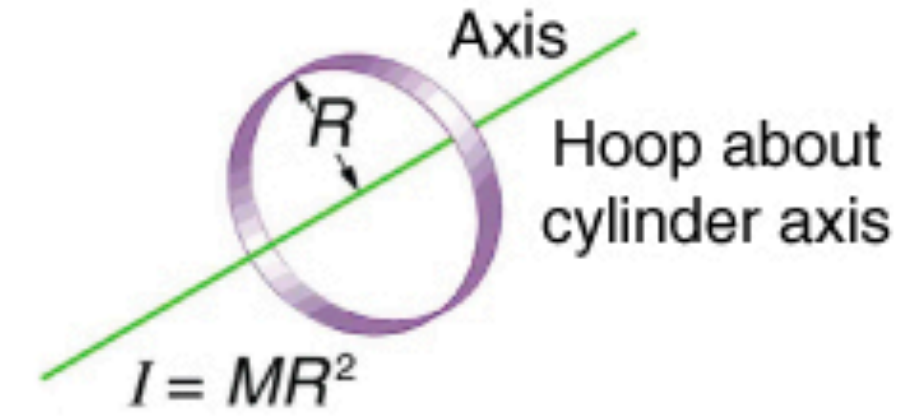$I = MR^2$

Hoop about cylinder axis

Axis

$I = \dfrac{MR^2}{2}$

Solid cylinder (or disk) about cylinder axis

Axis

$I = \dfrac{M\ell^2}{12}$

Thin rod about axis through center $\perp$ to length

Axis

$I = \dfrac{2MR^2}{5}$

Solid sphere about any diameter

Axis

$I = \dfrac{MR^2}{2}$

Hoop about any diameter

Axis

$I = \dfrac{M}{2}(R_1^2 + R_2^2)$

Annular cylinder (or ring) about cylinder axis

Axis

$I = \dfrac{MR^2}{4} + \dfrac{M\ell^2}{12}$

Solid cylinder (or disk) about central diameter

Axis

$I = \dfrac{M\ell^2}{3}$

Thin rod about axis through one end $\perp$ to length

Axis

$I = \dfrac{2MR^2}{3}$

Thin spherical shell about any diameter

Axis

$I = \dfrac{M(a^2 + b^2)}{12}$

Slab about $\perp$ axis through center

EXAMPLE 10.7

- **Calculating the Effect of Mass Distribution on a Merry-Go-Round:** Consider the father pushing a playground merry-go-round in the Figure. He exerts a force of 250 N at the edge of the 50.0-kg merry-go-round, which has a 1.50 m radius. Calculate the angular acceleration produced (a) when no one is on the merry-go-round and (b) when an 18.0-kg child sits 1.25 m away from the center. Consider the merry-go-round itself to be a uniform disk with negligible retarding friction.

F = 250N
m = 50.0 kg
R = 1.5m
α_a = ?
m = 18kg
r = 1.25m
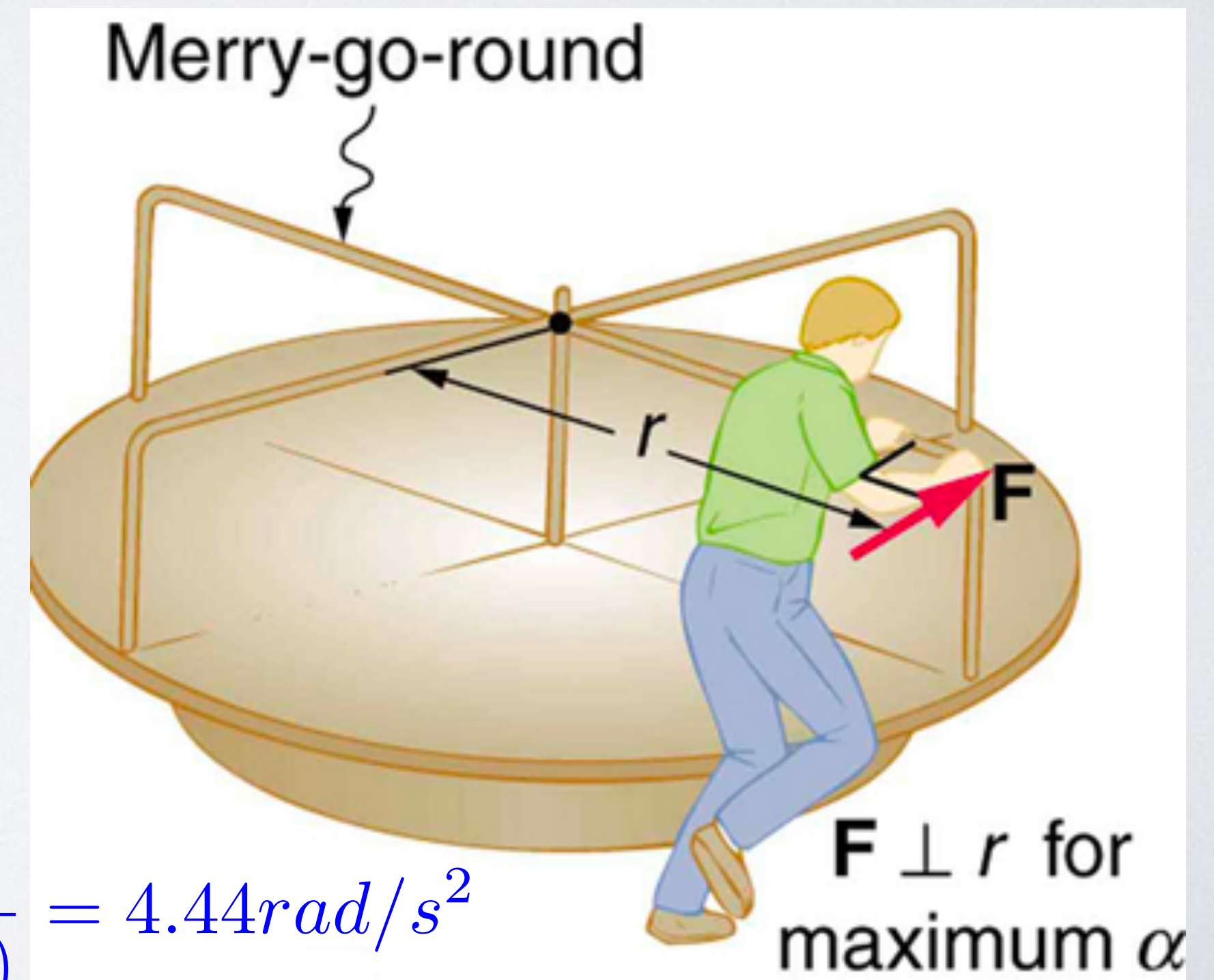α_b = ?

$$\tau = I\alpha \qquad \rightarrow \alpha = \tau/I$$

$$\tau = RF\sin\theta \qquad I_{disk} = \frac{1}{2}MR^2$$

$$\alpha_a = \frac{RF}{\frac{1}{2}MR^2} = \frac{2F}{MR}$$
$$= \frac{2(250N)}{(50kg)(1.5m)} = 6.67rad/s^2$$

$$\alpha_b = \frac{RF}{\left(\frac{1}{2}MR^2 + mr^2\right)} = \frac{(1.5m)(250N)}{\left(\frac{1}{2}(50kg)(1.5m)^2 + (18kg)(1.25m)^2\right)} = 4.44rad/s^2$$

Merry-go-round

r

F

$F \perp r$ for maximum α

ROTATIONAL ENERGY

- A rotating object also has rotational kinetic energy, given by

$$KE_r = \frac{1}{2}I\omega^2$$

- An object can have both rotational and linear kinetic energy at the same time.

Let's say people are roughly a cylinder with

R= 0.5m   m = 80kg   h= 100m

$$v = R\omega \qquad I = \frac{1}{2}mR^2$$

Let's assume no friction, then energy is conserved.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}mR^2)(v/R)^2 = \frac{3}{4}mv^2$$

$$=> v^2 = \frac{4}{3}gh \quad => v = \sqrt{\frac{4}{3}gh} \ = \sqrt{\frac{4}{3}(9.8m/s^2)(100m)} = 36\,m/s$$

Clearly this doesn't happen.
There are nonconservative forces so that at the end they have zero velocity.

EXAMPLE 10.9

- **Calculating Helicopter Energies:** A typical small rescue helicopter, has four blades, each is 4.00 m long and has a mass of 50.0 kg. The blades can be approximated as thin rods that rotate about one end of an axis perpendicular to their length. The helicopter has a total loaded mass of 1000 kg. (a) Calculate the rotational kinetic energy in the blades when they rotate at 300 rpm. (b) Calculate the translational kinetic energy of the helicopter when it flies at 20.0 m/s, and compare it with the rotational energy in the blades. (c) To what height could the helicopter be raised if all of the rotational kinetic energy could be used to lift it?

L = 4.00m

m = 50.0kg

M_h = 1000kg

$$KE_r = \frac{1}{2}I\omega^2 \qquad I = 4\frac{1}{3}mL^2$$

$$\frac{KE_t}{KE_r} = \frac{2}{5.26} = 0.380$$

$$KE_r = \frac{2}{3}mL^2\omega^2 = \frac{2}{3}(50kg)(4m)^2(31.4rad/s)^2 = 5.26 \times 10^5 J$$

ω = 300 rpm =300 (2π)(1min/60s) = 31.4 rad/s

KE_b= ?

KE_t= ?

h = ?

$$KE_t = \frac{1}{2}M_h v^2 = \frac{1}{2}(1000kg)(20.0m/s)^2 = 2.00 \times 10^5 J$$

$$M_h gh = KE_r$$

$$h = \frac{KE_r}{M_h g} = \frac{5.26 \times 10^5 J}{1000kg(9.8m/s^2)} = 53.7m$$

ANGULAR MOMENTUM

- Finally we have angular momentum, the rotational equivalent of linear momentum. Angular momentum, get's the letter L.
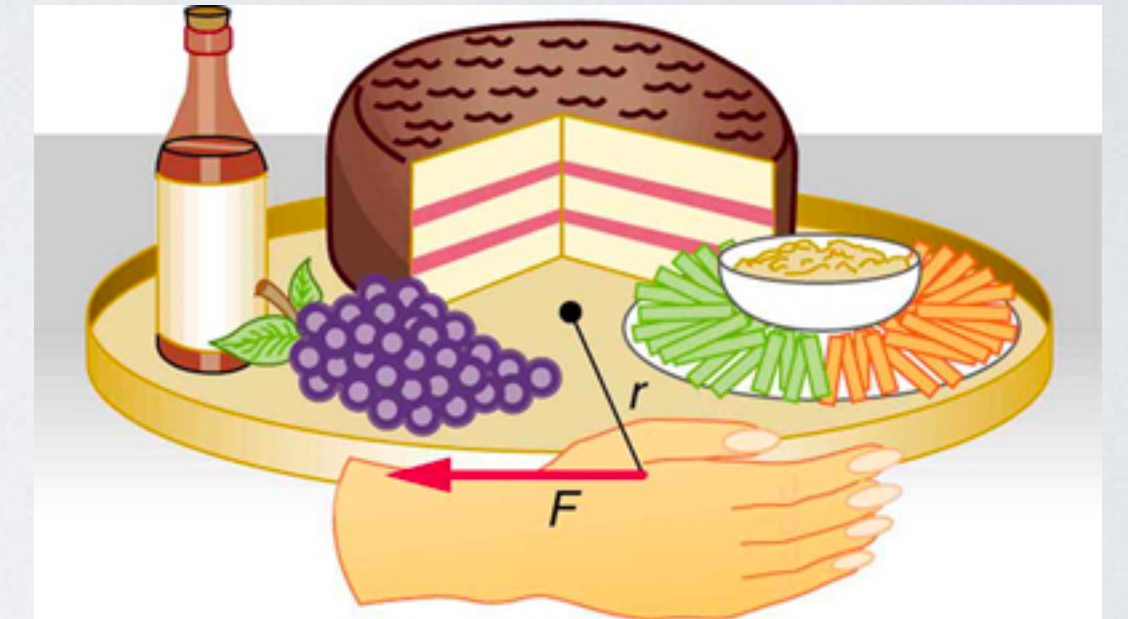
$$L = I\omega$$

- Angular momentum is conserved if there are no outside torques and it changes if there is a net torque.

$$\tau = \frac{\Delta L}{\Delta t}$$

EXAMPLE 10.12

- **Calculating the Torque Putting Angular Momentum Into a Rotating Food Tray**: The Figure shows a rotating food tray, often called a lazy Susan, being turned by a person in quest of sustenance. Suppose the person exerts a 2.50 N force perpendicular to the lazy Susan's 0.260-m radius for 0.150 s. (a) What is the final angular momentum of the lazy Susan if it starts from rest, assuming friction is negligible? (b) What is the final angular velocity of the lazy Susan, given that its mass is 4.00 kg and assuming its moment of inertia is that of a disk?

F = 2.5N
r = 0.26m
t = 0.15s
L = ?
m = 4.00kg
ω = ?

$$\tau_{net} = \frac{\Delta L}{\Delta t}$$

$$L = \tau_{net}t$$

$$L = I\omega$$

$$\omega = \frac{L}{I}$$

$$I = \frac{1}{2}mr^2$$

$$\omega = \frac{2L}{mr^2} = \frac{2(9.75 \times 10^{-2})}{(4kg)(0.26m)^2} = 0.721 rad/s$$

$$L = rFt = (0.26m)(2.5N)(0.15s) = 9.75 \times 10^{-2} kgm^2/s$$

EXAMPLE 10.14

- **Calculating the Angular Momentum of a Spinning Skater:** Suppose an ice skater, such as the one in the Figure, is spinning at 0.800 rev/s with her arms extended. She has a moment of inertia of $2.34kg \cdot m^2$ with her arms extended and of $0.363kg \cdot m^2$ with her arms close to her body. (These moments of inertia are based on reasonable assumptions about a 60.0-kg skater.) (a) What is her angular velocity in revolutions per second after she pulls in her arms? (b) What is her rotational kinetic energy before and after she does this?

$\omega = 0.8$ rev/s

$I = 2.34$ kgm^2
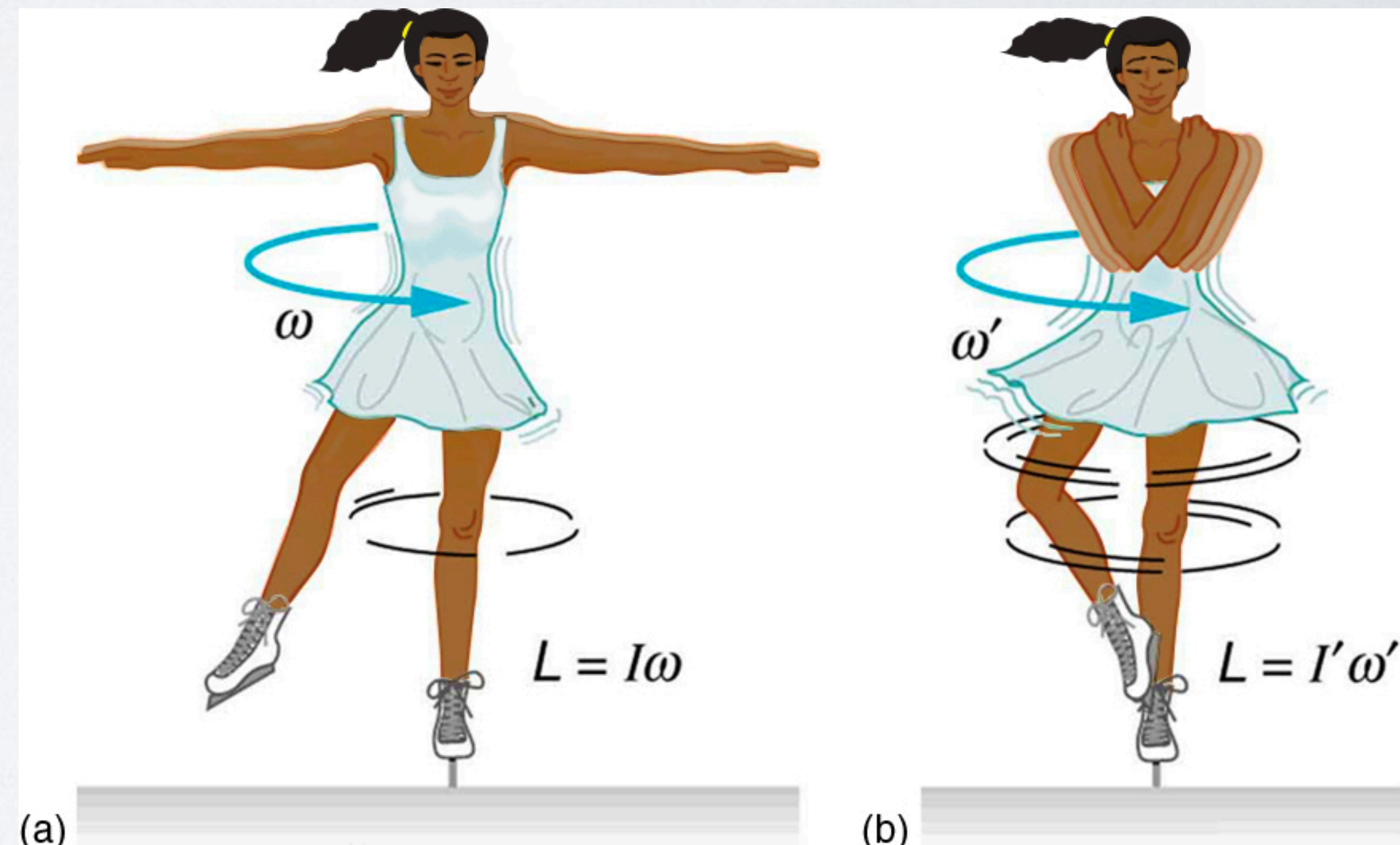
$I' = 0.363$ kgm^2

$\omega' = ?$ rev/s

KE = ?

KE' = ?

$$L = L' \qquad I\omega = I'\omega'$$

$$\omega' = \frac{I}{I'}\omega$$

$$= \frac{2.34}{0.363}0.8 = 5.16 rev/s$$

$$KE = \frac{1}{2}I\omega = \frac{1}{2}(2.34)(0.8 \times 2\pi) = 29.6J$$

$$KE' = \frac{1}{2}I'\omega' = \frac{1}{2}(0.363)(5.16 \times 2\pi) = 191J$$

ω

$L = I\omega$

(a)

ω'

$L = I'\omega'$

(b)

HOME WORK

- Chap 10 - 6, 16, 26, 28, 39, 45