MOMENTUM

Chapter 8

LINEAR MOMENTUM

- Since conservation of energy is such a useful concept, we might wonder if there are any other conserved quantities. It turns out there are.

- Linear momentum is a property that is generally conserved in collisions. It is the product of mass and velocity.

$$\vec{p} = m\vec{v}$$

- Linear momentum doesn't have it's own unit.

EXAMPLE 8.1

- **Calculating Momentum: A Football Player and a Football:** (a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

m = 110 kg

v = 8 m/s

$$p = mv = (110kg)(8m/s) = 880kgm/s$$

m_{fb} = 0.410 kg

v_{fb} = 25 m/s

$$p_{fb} = m_{fb}v_{fb} = (0.410kg)(25m/s) = 10.3kgm/s$$

MOMENTUM AND NEWTONS LAWS

- Now that we know about momentum we can see a new way or writing Newton's second law.

$$F_{net} = \frac{\Delta p}{\Delta t}$$

- if m is constant then we see that $\Delta p = m\,\Delta v$ and since $a = \Delta v/\Delta t$ we recover F=ma. The advantage of writing it in terns of momentum is that if the mass is not conserved this formula still works.

EXAMPLE 8.2

- **Calculating Force: Venus Williams' Racquet:** During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

v = 58m/s F = ?
m = 0.057kg
t = 5.0ms = 0.005 s

$$F_{net} = \frac{\Delta p}{\Delta t} = \frac{mv - 0}{t} = \frac{(0.057kg)(58m/s)}{0.005s} = 660N$$

IMPULSE

- The effect of a force is to change momentum. The longer the force is applied the more the momentum will change. Impulse is a measure of this. For a constant force

$$\Delta p = F_{net}\Delta t$$

- Note we won't use a special symbol for this and the units are the same as momentum.

Casino Royal (2006)

How much force does it take to stop his fall from the crane?

We can estimate the height of the crane above the roof from the size of the people at about 6.5 meters. Neglecting air resistance and using energy conservation we can find his velocity as he impacts the roof.

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh} = \sqrt{2(9.8m/s^2)(6.5m)} = 11.3m/s$$

When either man lands on the roof their y momentum will go from mv to 0. If we take the force that causes this momentum change to be constant we have

$$\Delta p = Ft \rightarrow F = \frac{\Delta p}{t}$$

Let's say Bond's mass is 80kg and he takes 0.1s to stop his fall. Then the force would be

$$F = \frac{(80kg)(11.3m/s)}{0.1s} = 9040N$$

Ouch! That's going to hurt. Let's say Foucan's mass is 65kg and he takes 0.5s to stop his fall

$$F = \frac{(65kg)(11.3m/s)}{0.5s} = 1469N$$

Much better. This is why extending the time to stop the fall is a critical part of arts like freerunning.

CONSERVATION OF MOMENTUM

- We have seen that force will change momentum, but what if there is no net force on a system. Then the momentum will be conserved.
- The most common example is a collision between two objects. Though any situation where there is no net outside force will conserve momentum.
- If a collision also conserves kinetic energy it is called an elastic collision. Otherwise it is called an inelastic collision.

EXAMPLE 8.4

- **Calculating Velocities Following an Elastic Collision:** Calculate the velocities of two objects following an elastic collision, given that m_1=0.500 kg, m_2=3.50 kg, v_1=4.00 m/s, and v_2=0.

$$p_1 = p_1' + p_2'$$

$$m_1 v_1 = m_1 v_1' + m_2 v_2'$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$v_2' = \frac{m_1}{m_2}(v_1 - v_1')$$

$$m_1(v_1^2 - v_1'^2) = m_2\left(\frac{m_1}{m_2}(v_1 - v_1')\right)^2$$

$$(m_1 + m_2) v_1'^2 + 2 m_1 v_1 v_1' + (m_1 - m_2) v_1 = 0$$

$$v_2' = \frac{0.5}{3.5}(4 - -3) = 1 m/s$$

$$v_1'^2 - v_1' - 12 = 0$$

$$v_1' = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad = 4 m/s \;\; or \;\; -3 m/s$$

EXAMPLE 8.5

- **Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie:** (a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible.

$m_2 = 70.0$ kg
$m_1 = 0.15$ kg
$v_1 = 35.0$ m/s

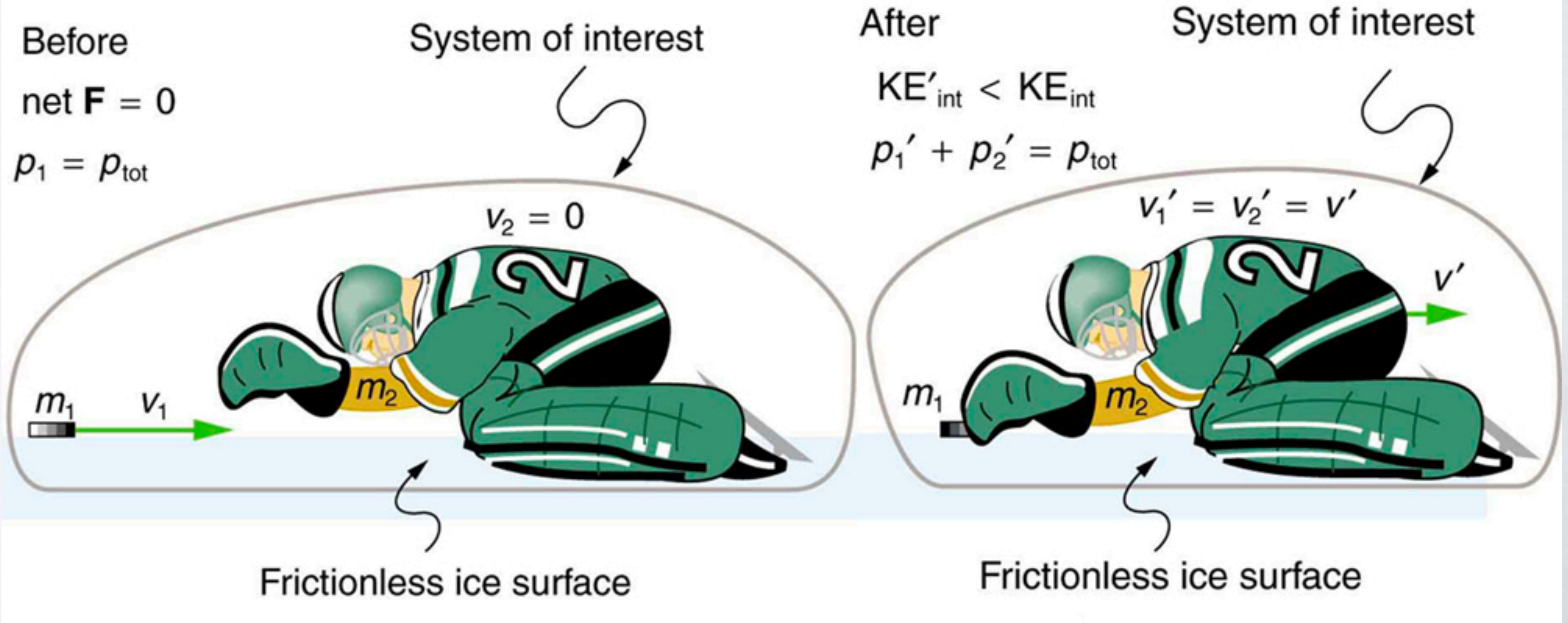
$$m_1 v_1 = (m_1 + m_2) v'$$

$$v' = \frac{m_1 v_1}{m_1 + m_2} = \frac{(0.15kg)(35.0m/s)}{70.0kg + 0.15kg} = 7.48 \times 10^{-2} m/s$$

$$KE_{int} = \frac{1}{2} m_1 v_1^2 = \frac{1}{2}(0.15kg)(35m/s)^2 = 91.9J$$

$$KE' = \frac{1}{2}(m_1 + m_2) v'^2$$
$$= \frac{1}{2}(70.15kg)(0.0748m/s)^2 = 0.196J$$

$$KE_{loss} = KE_{int} - KE' = 91.9J - 0.196J = 91.7J$$



MOMENTUM IN 2D

- Momentum is a vector, which means that in 2 dimensions each component of momentum is conserved. So just like any 2 dimensional problem we get two separate one dimensional momentum conservation equations.

- For a collision

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}$$

$$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}$$

EXAMPLE 8.7

- **Determining the Final Velocity of an Unseen Object from the Scattering of Another Object:** Suppose the following experiment is performed. A 0.250-kg object, (m_1), is slid on a frictionless surface into a dark room, where it strikes an initially stationary object with mass of 0.400 kg (m_2). The 0.250-kg object emerges from the room at an angle of 45.0° with its incoming direction. The speed of the 0.250-kg object is originally 2.00 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity (v'_2) and (θ_2) of the 0.400-kg object after the collision.

$m_1 = 0.25$ kg
$m_2 = 0.4$ kg
$\theta_1 = 45$
$v_1 = 2.00$ m/s
$v'_1 = 1.50$ m/s
$v'_{12} = ?$
$\theta_2 = ?$

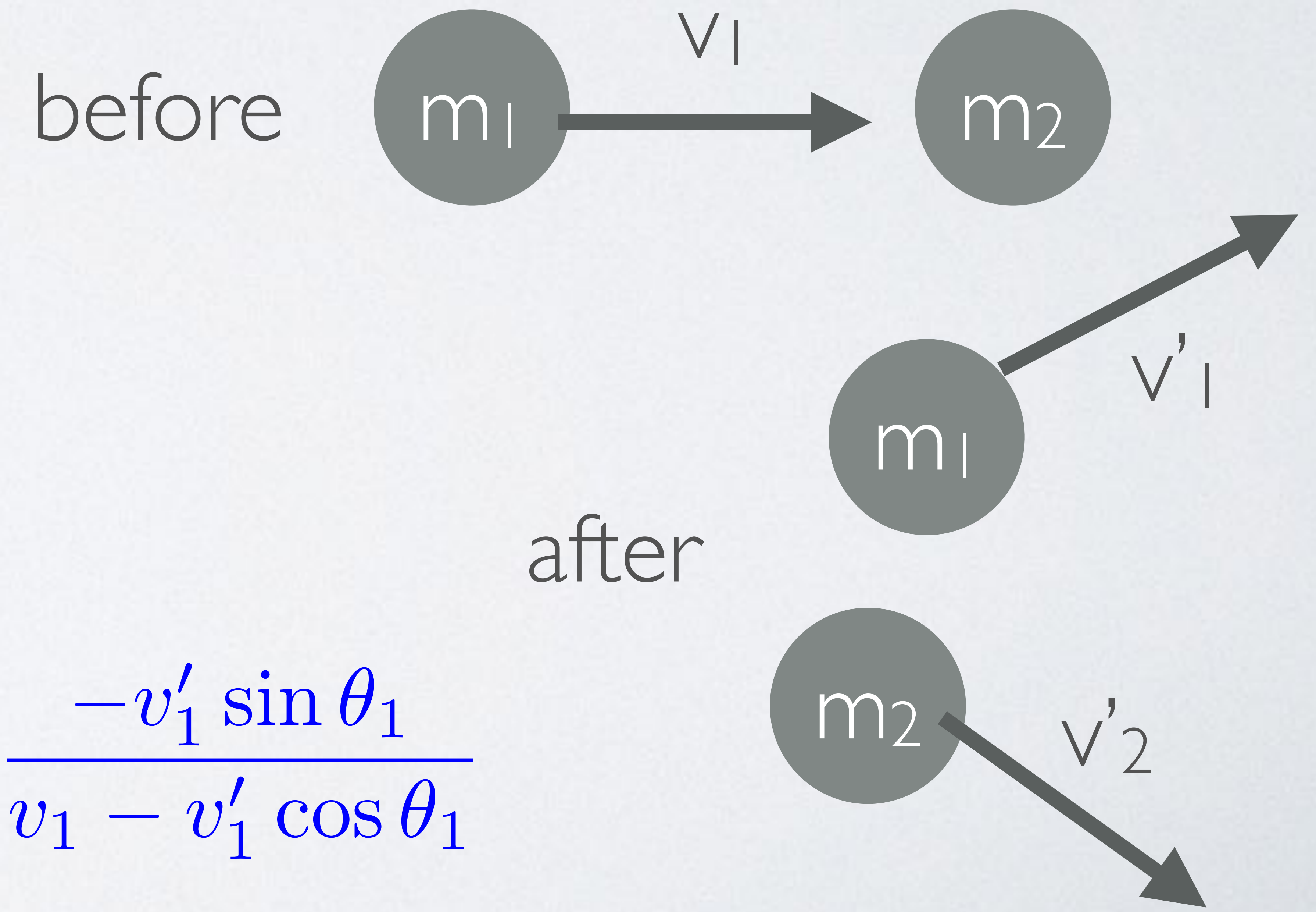
$$p_{1x} = p'_{1x} + p'_{2x}$$
$$0 = p'_{1y} + p'_{2y}$$

$$m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$$
$$0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2$$

$$-m_1 v'_1 \sin\theta_1 = m_2 v'_2 \sin\theta_2$$

$$m_1 v_1 - m_1 v'_1 \cos\theta_1 = m_2 v'_2 \cos\theta_2$$

$$\tan\theta_2 = \frac{-v'_1 \sin\theta_1}{v_1 - v'_1 \cos\theta_1}$$



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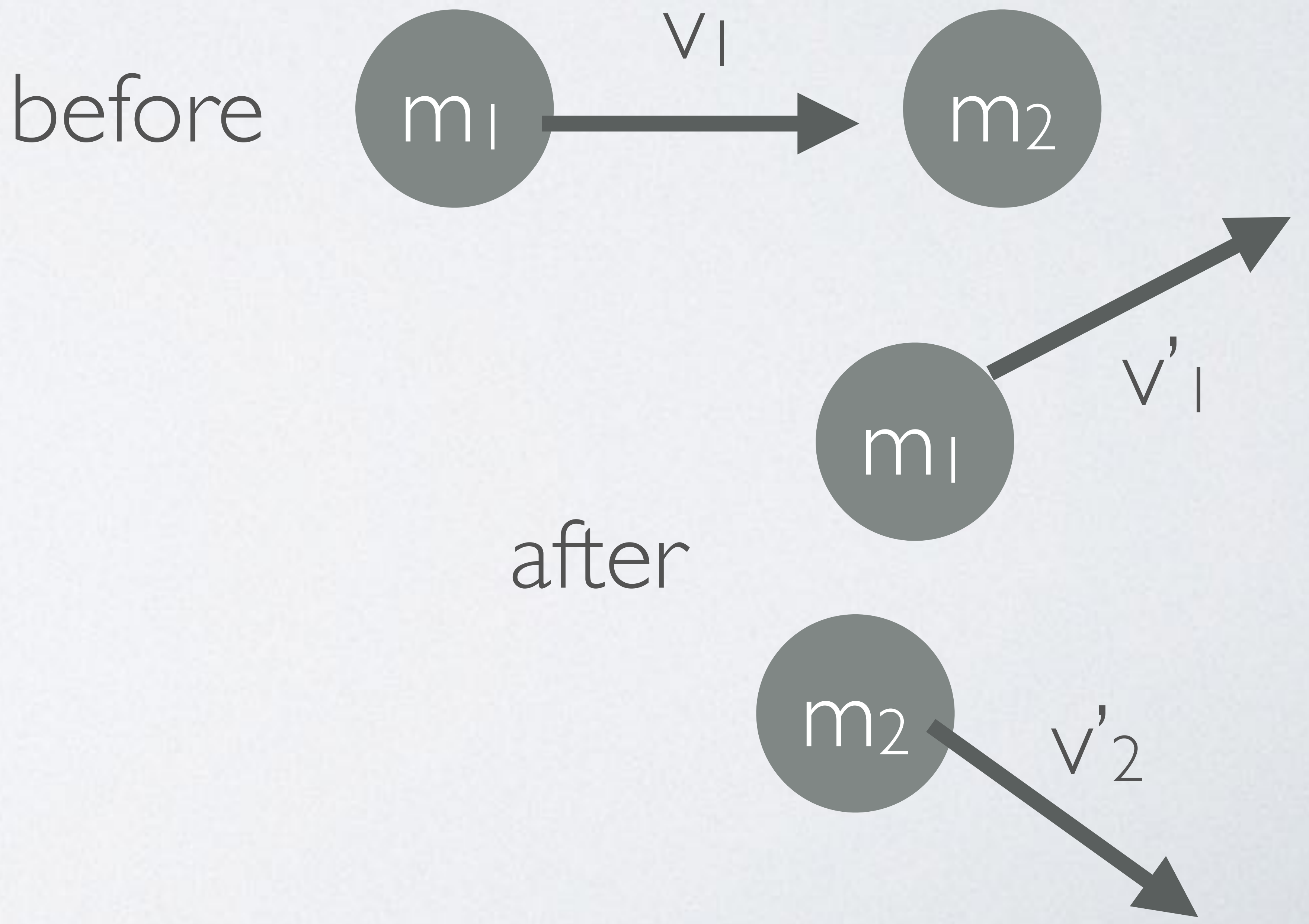
$m_2 = 0.4$ kg
$\theta_1 = 45$
$v_1 = 2.00$ m/s
$v'_1 = 1.50$ m/s
$v'_{12} = ?$
$\theta_2 = ?$

$$\tan\theta_2 = \frac{-v_1'\sin\theta_1}{v_1 - v_1'\cos\theta_1}$$

$\theta_2 = 312$ or -48

$$v_2' = -\frac{m_1}{m_2}v_1'\frac{\sin\theta_1}{\sin\theta_2} \quad = 0.886 m/s$$



HOME WORK

- Chap 8 - 2, 12, 26, 32, 36, 45