CIRCULAR MOTION AND GRAVITY

Chapter 6

CIRCULAR MOTION

- We have been focusing on objects moving in a straight line or a parabola, but what about something that is rotating. In that case we can talk about the angle it has rotated instead of the distance it has traveled.

- We can define an angular velocity as the change in angle with time.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- In this case we don't measure the angle in degrees, but instead in radians.

CIRCULAR MOTION

- The distance a point has moved as this object rotates will depend on how far it its from the axis of rotation.

- The distance $\Delta s = R\Delta\theta$. Note if we go all around the circle we travel a distance $2\pi R$ which is why it makes sense to say the angle we have gone is 2π, so radians.

$$\Delta\theta = \frac{\Delta s}{r}$$
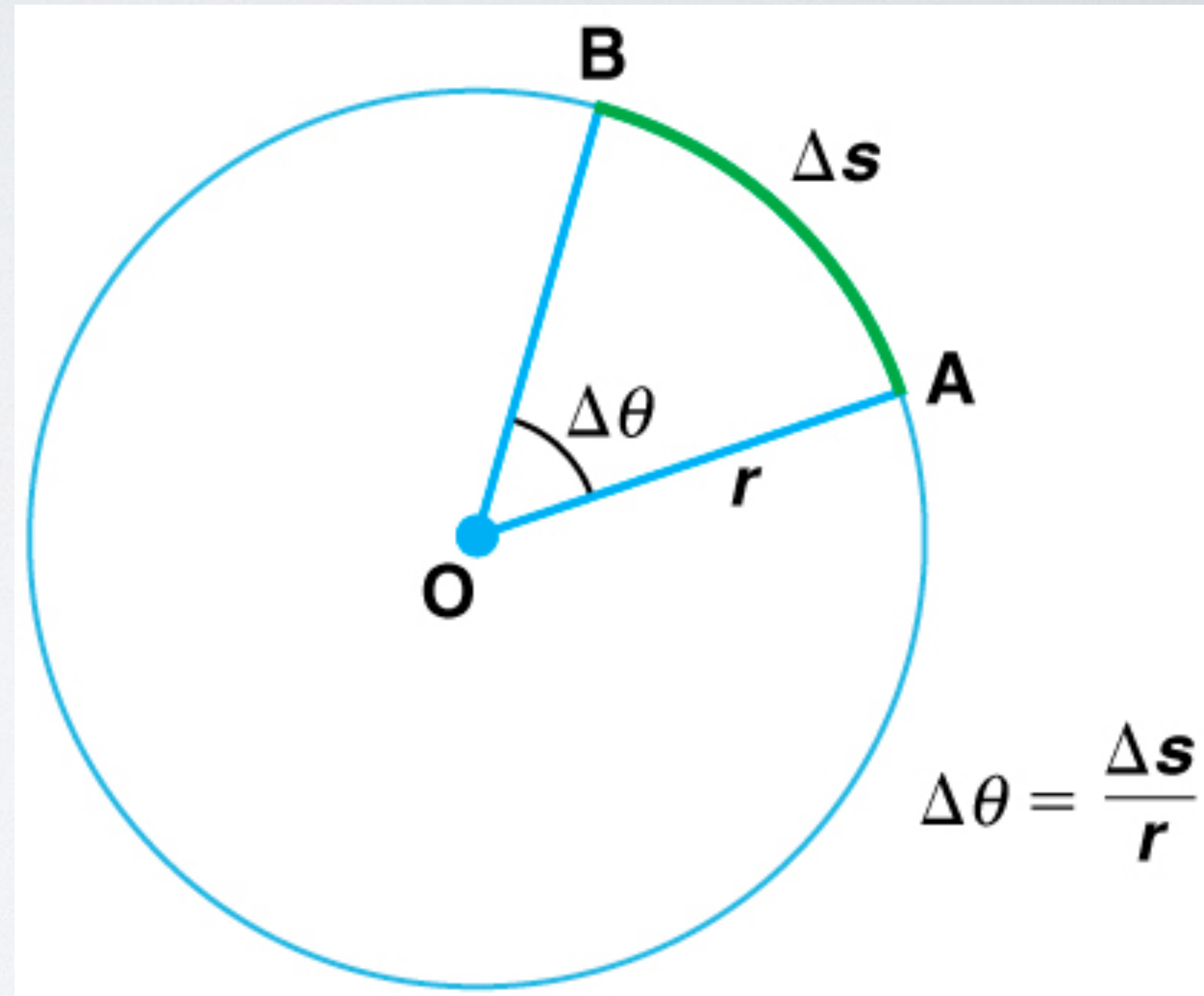
ANGULAR VELOCITY

- We can now connect the regular linear velocity and angular velocity. Since

$$v = \frac{\Delta s}{\Delta t} = \frac{R\Delta\theta}{\Delta t} = R\omega$$

- we see that v = ωR, or ω = v/R. Note that the conversion between angular and linear velocity will depend on a points distance from the axis of rotation.

CENTRIPETAL ACCELERATION

- If an object is moving in a circle it must be accelerating even if its speed remains constant. This is a result of Newtons first law which says objects will naturally move in a straight line. If they don't there must be a net force acting on them.

- Velocity if a vector, so even if the magnitude is not changing the direction does change when moving in a circle. The acceleration is given by

$$a_c = \frac{v^2}{R}$$

EXAMPLE 6.1

- **How Fast Does a Car Tire Spin?** Calculate the angular velocity of a 0.300 m radius car tire when the car travels at 15.0m/s (about 54km/h).

R = 0.300 m

v = 15.0 m/s

$\omega = v/R = 15.0 \text{ m/s} / 0.300 \text{m} = 50 \text{ rad/s}$

EXAMPLE 6.2

- **How Does the Centripetal Acceleration of a Car Around a Curve Compare with That Due to Gravity?** What is the magnitude of the centripetal acceleration of a car following a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h)? Compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed.

R = 500m
v = 25.0m/s

$a_c = v^2/R = (25.0m/s)^2/500m = 1.25 \ m/s^2$

compare to gravity -
$a_c /g = 1.25/9.81 = 0.128$

CENTRIPETAL FORCE

- If an object is moving in a circle then it is experiencing centripetal acceleration, a_c, which means there must be a net force on the object.

- The magnitude of this force must be $F=ma_c$, but this doesn't tell us what is causing the force. The acceleration is the result of a force not the cause.

UNIVERSAL GRAVITY

- Newton realized that the gravity that we fell here on Earth, must be the same force that keeps the planets going around the Sun. But the strength of the force couldn't be the same as on Earth.

- He guessed that the force should depend on the masses of two objects attracting one another and the distance between them squared. So he got

$$F = G\frac{mM}{r^2}$$

- where G is a universal constant, G = 6.67 10^{-11} N m^2/kg^2.

GRAVITY ON EARTH

- So how is it that on Earth gravity can be expressed as F = mg.

- Well on Earth the 2nd mass is the mass of the Earth and the distance is always the distance to Earth's center. Those values are basically the same everywhere on Earth. So we get

$$F = G\frac{mM}{r^2} = m\left(\frac{GM_E}{R_E^2}\right) = mg$$

- it just happens to be the case that $GM_E/R_E^2 = 9.81$ m/s^2.

If we ignore air resistance he falls with constant acceleration.
From the clip we can measure the time as about 6s.

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

If this was Earth then we could use a_y = g = 9.8 m/s². However, the scene takes place on Pandora so the acceleration is not 9.8m/s². From the internet we can find one estimate of Pandora's mass and size as 45% of Earth's mass and 75% of Earth's size. So
M=(0.45)(5.98 × 1024 kg) = 2.7 × 1024 kg
R=(0.75)(6.37 × 106m) = 4.8 × 106 m

$$F_g = G\frac{Mm}{R^2} \qquad => g = \frac{F}{m} = G\frac{M}{R^2} \qquad\qquad g_P = G\frac{2.7 \times 10^{24}kg}{(4.8 \times 10^6 m)^2} = 7.8m/s^2$$

$$0 = y_0 - \frac{1}{2}(7.8m/s^2)(6s)^2 \quad => y_0 = 140m$$

His velocity on hitting the water would be $\qquad v = v_0 + at = (-7.8m/s^2)(6s) = 47m/s$

That is very fast, 170 km/hr. In comparison the highest dive into water done on Earth was Oliver Favre in 1987 from a height of 54m. While the atmosphere on Pandora is thicker than Earth, this still seems like a very high drop.

EXAMPLE 6.6

- **Earth's Gravitational Force Is the Centripetal Force Making the Moon Move in a Curved Path** (a) Find the acceleration due to Earth's gravity at the distance of the Moon. (b) Calculate the centripetal acceleration needed to keep the Moon in its orbit (assuming a circular orbit about a fixed Earth), and compare it with the value of the acceleration due to Earth's gravity that you have just found.

$M_E = 5.98 \times 10^{24}$ kg $R_{EM} = 3.84 \times 10^8$ m

$$a = G\frac{M}{R^2} = 6.67 \times 10^{-11} Nm^2/kg^2 \frac{5.98 \times 10^{24} kg}{(3.84 \times 10^8 m)^2} = 2.70 \times 10^{-3} m/s^2$$

$T_M = 27.3$ days $= 86,400$ s

$$\omega = \frac{2\pi}{T_M} = \frac{2\pi}{86400s} = 2.66 \times 10^{-6} rad/s$$
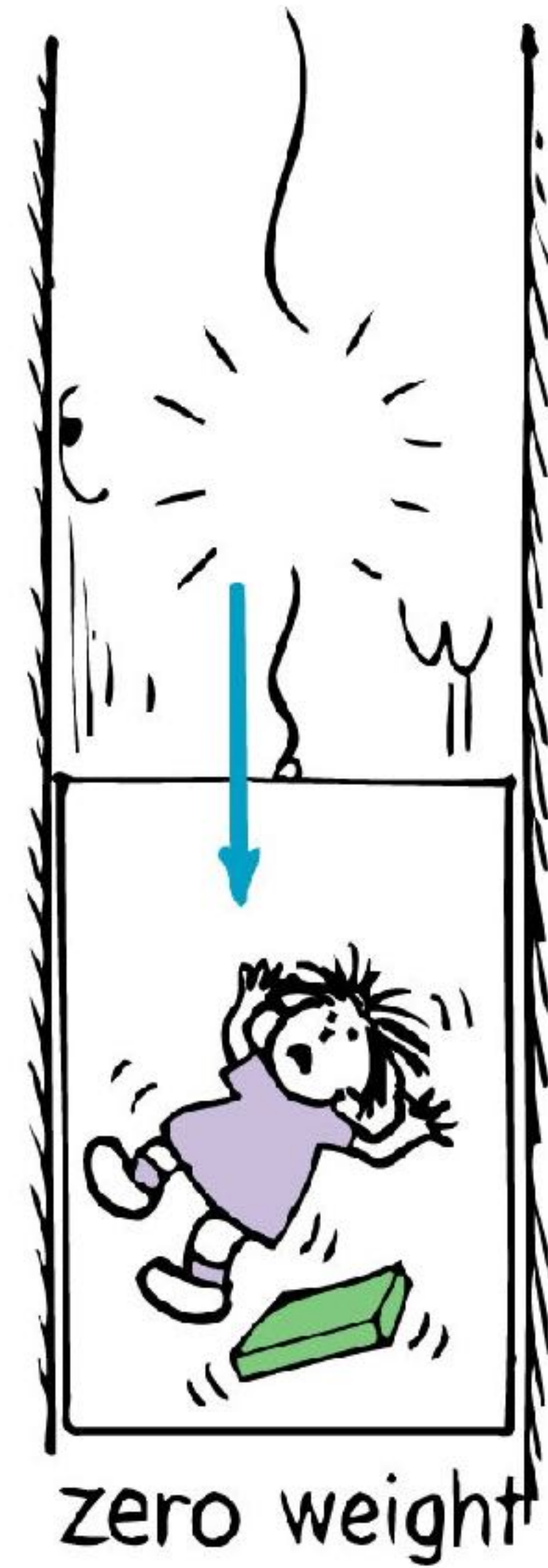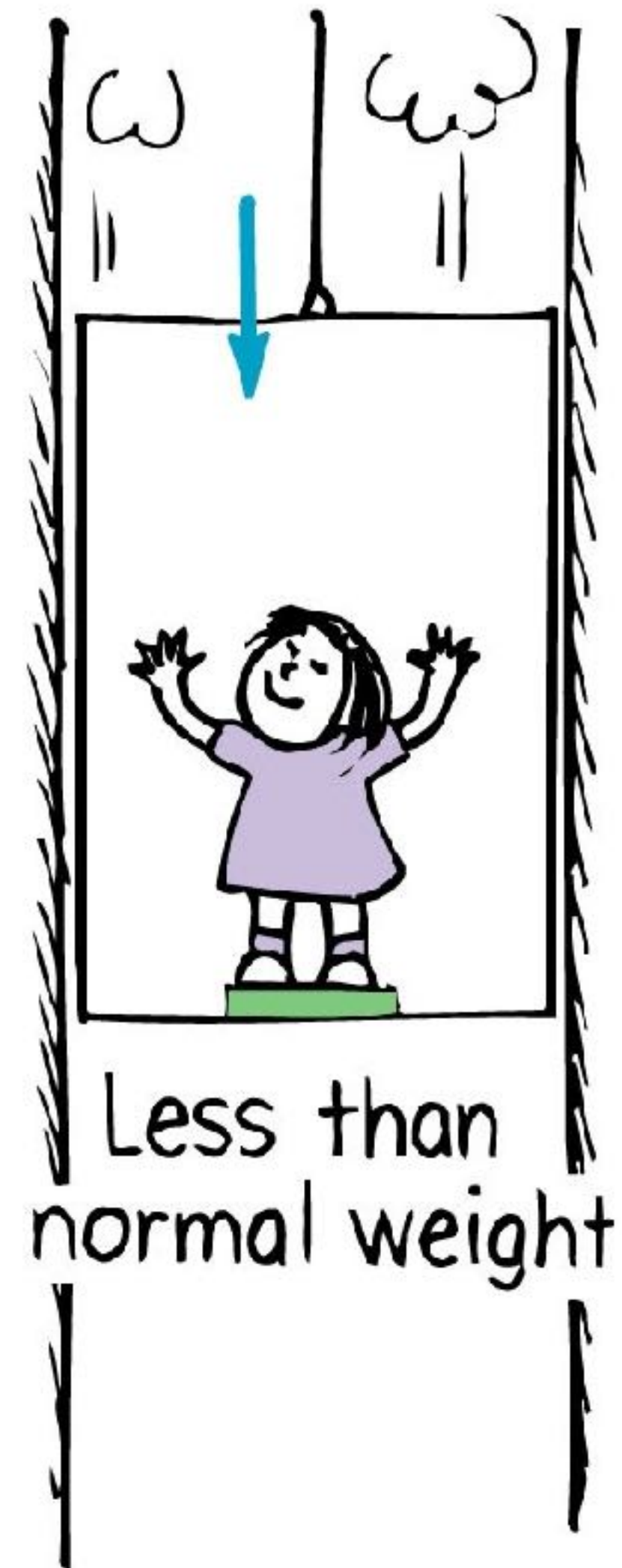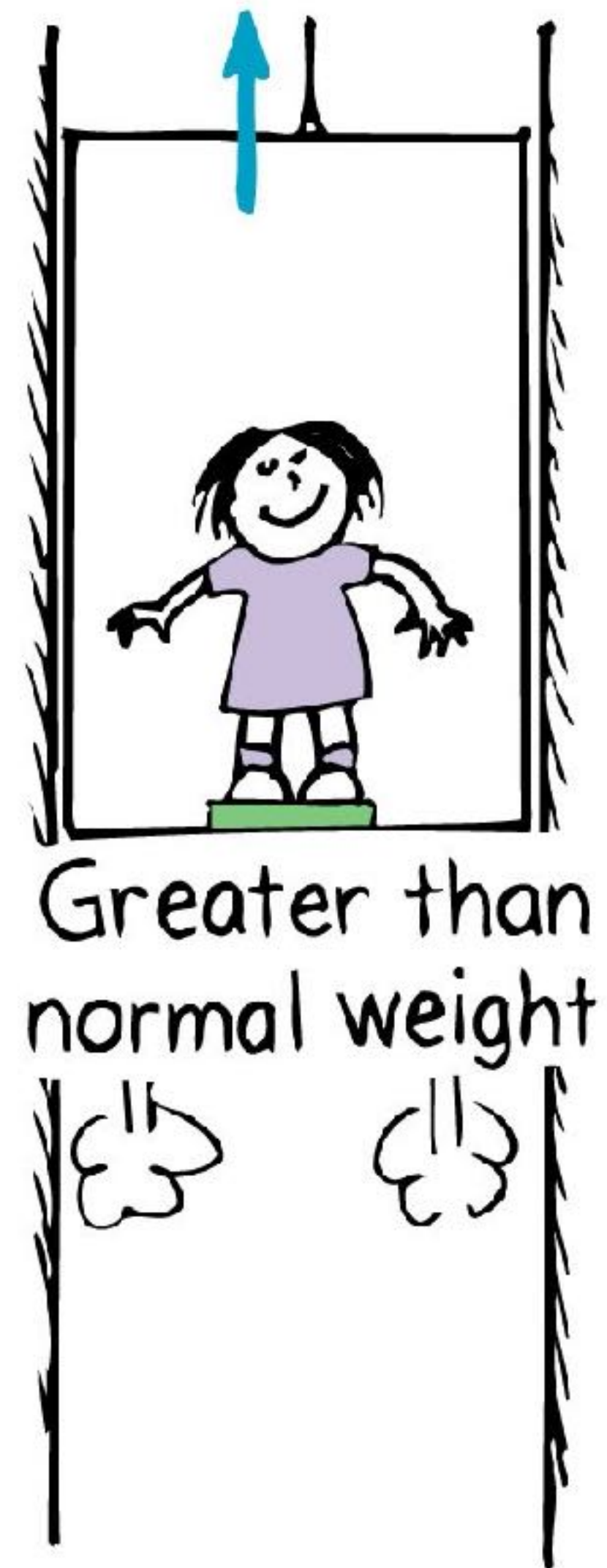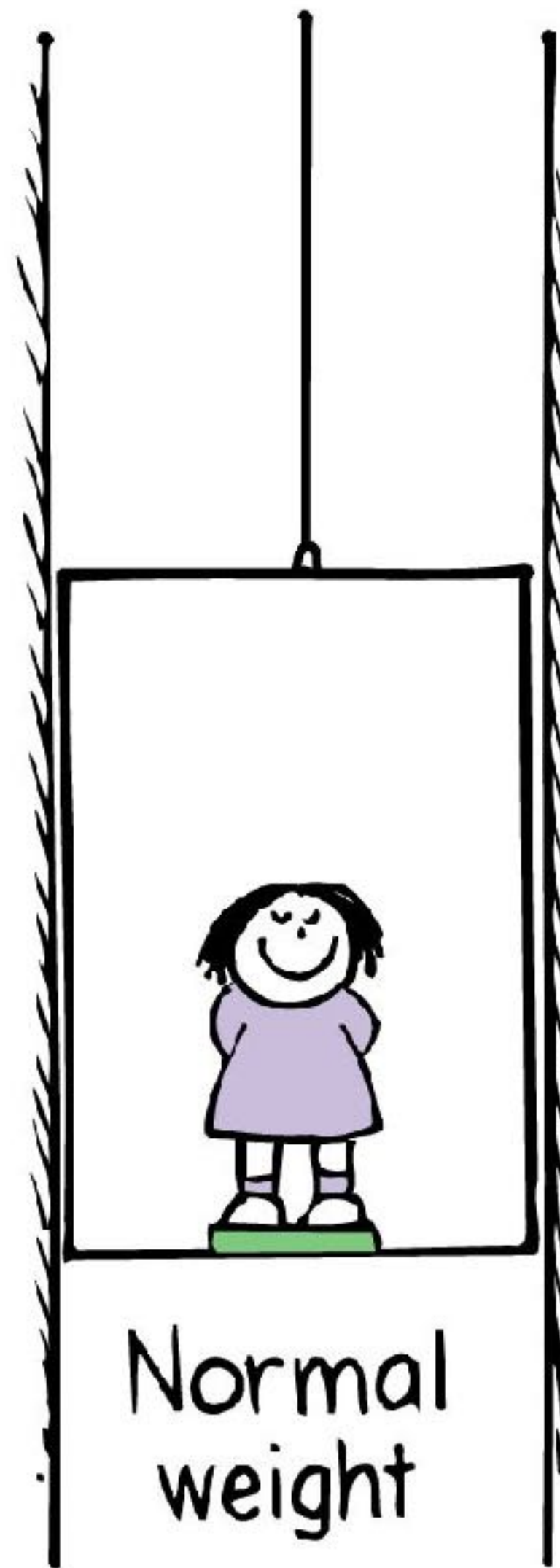
$$a_c = \omega^2 R = (2.66 \times 10^{-6} rad/s)^2 (3.84 \times 10^8 m) = 2.72 \times 10^{-3} m/s^2$$

WEIGHTLESSNESS

- Weightlessness is the absence of a normal force. When you are weightless you still feel gravity, just nothing opposes it.

- In space gravity still exists, in fact gravity is felt infinitely far away from an object. The reason things float in a satellite is because everything is accelerating together, so there is no relative acceleration. Everything is still being pulled towards the Earth.

- You can experience weightlessness on Earth. Just go on a ride that drops you. Without a normal force, you will accelerate at g and be weightless.

Normal weight

Greater than normal weight

Less than normal weight

zero weight

HOME WORK

- Chap 6 - 4, 7, 17, 20