KINEMATICS 2D

Chapter 3

VECTORS

- Many of the quantities we have been discussing are vectors. If a quantity is a vector or not is very important in understanding what the quantity means and how to use it.
- A vector is something that has both a magnitude and a direction.
- Something which only has magnitude but no direction is called a scalar.

VECTORS

- What quantities that we've discussed so far are vectors?

distance velocity acceleration

- Which are scalars?

time

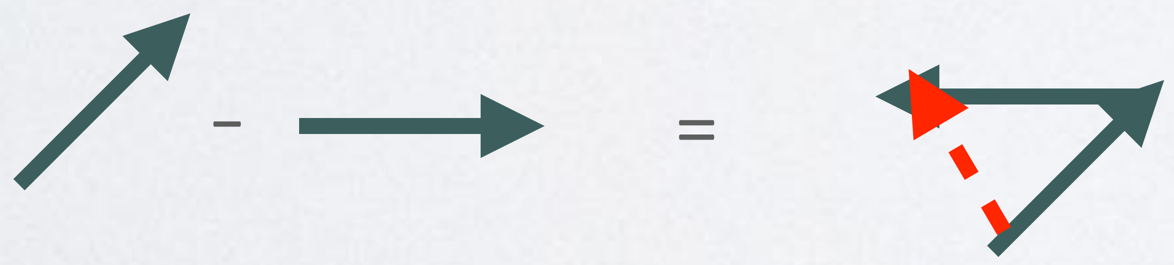
ADDING VECTORS

- The main feature of vectors that makes them more difficult then regular numbers is that you can’t add them the same way.
- You know how to add 5+5, but what is 5km north + 5km east?
- We will learn 2 ways to add vectors, the graphical method and using components.

THE GRAPHICAL METHOD

- One way to visualize adding vectors is to draw them as arrows.
- To add two vectors you place the tip of one on the tail of the other.

notice it doesn't matter what
order you add the vectors

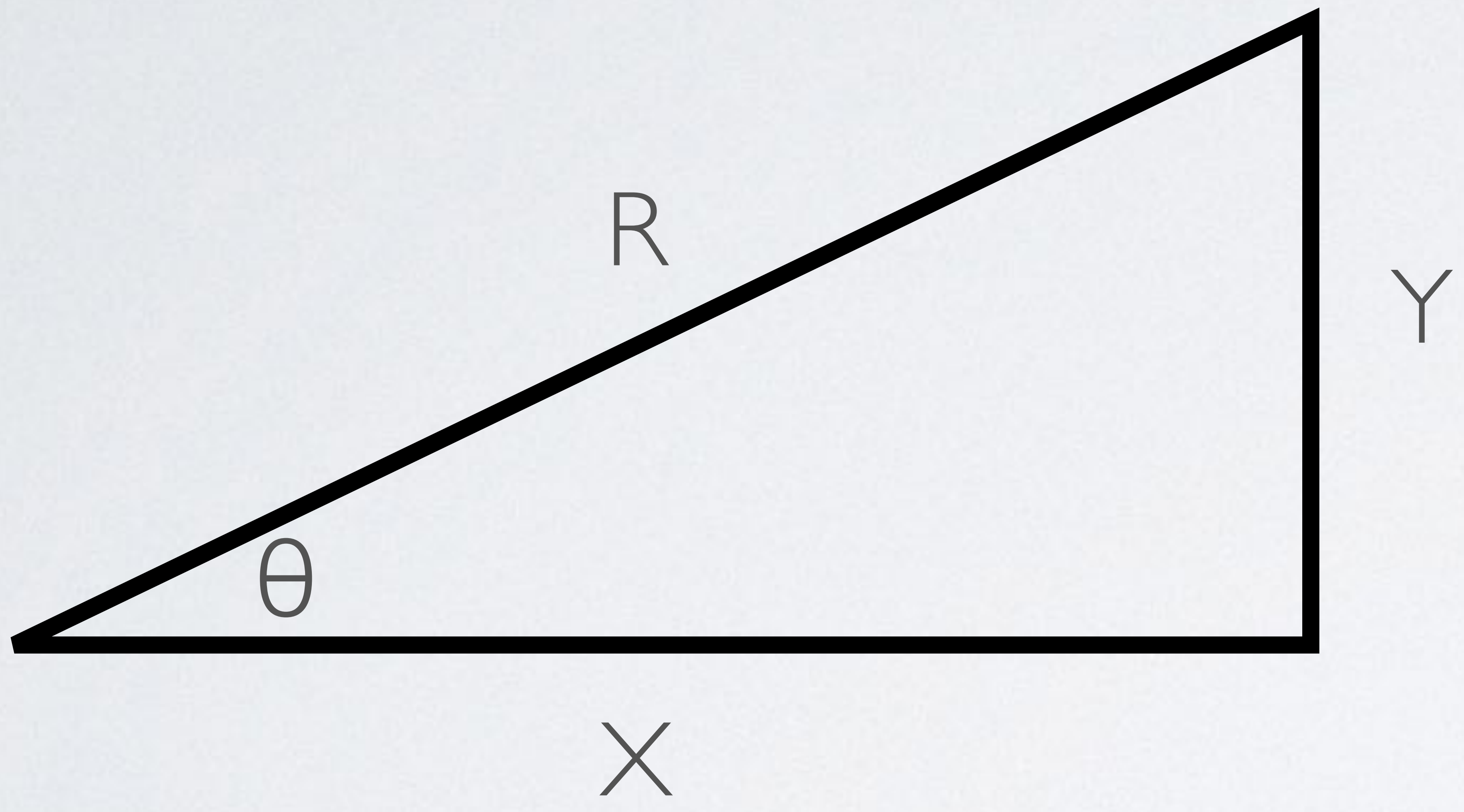


You aren't going to add vectors this way, but this should help you visualize what you expect your answer to be and avoid mistakes

ADDING BY COMPONENTS

- The real way to deal with vectors is to deal with their components.
- Once you realize that only the components matter, everything with vectors just becomes dealing with the components.
- Before we start with components let's review triangles.

TRIANGLES



$\sin\theta = Y/R$

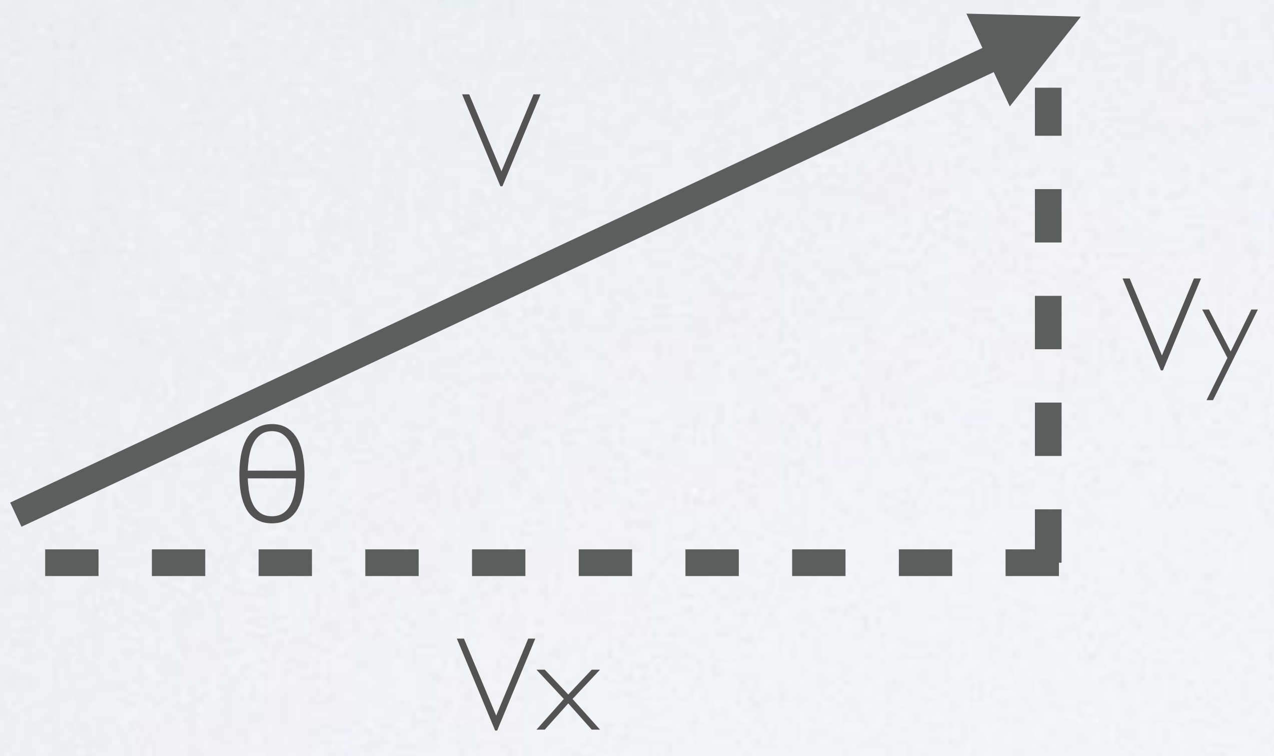
$\cos\theta = X/R$

$\tan\theta = Y/X$

$R^2 = X^2 + Y^2$

ADDING BY COMPONENTS

If instead you have a vector you can always get the x and y components by making a triangle.



$$V_x = V\cos\theta$$

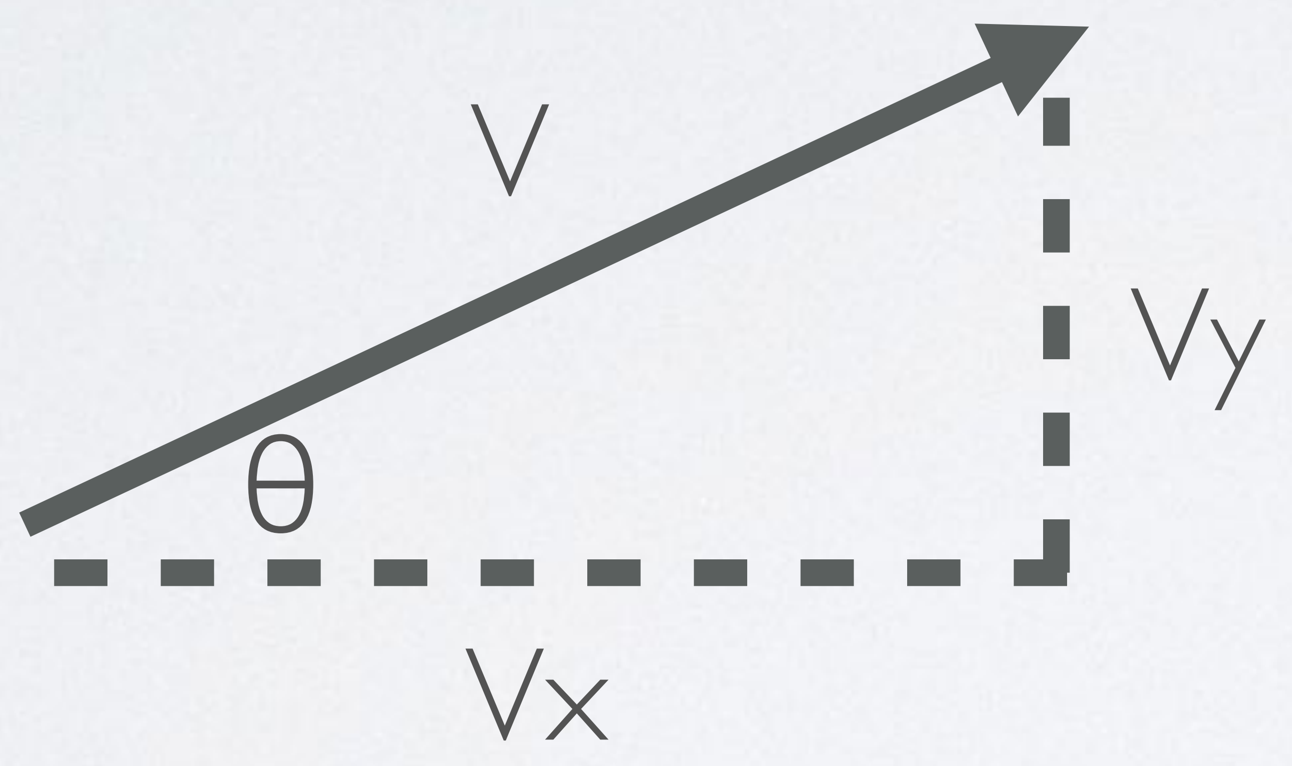
$$V_y = V\sin\theta$$

These are the x and y components of V.

The components can be added like numbers, so once we break vectors into components we can just add the components.

ADDING BY COMPONENTS

If we have added up the components we can get the magnitude and direction of the vector from



$V = \mathrm{sqrt}(Vx^2 + Vy^2)$

$\theta = \tan^{-1}(Y/X)$

Note this will not return degrees unless set on your calculator.

So we can break a vector into components and combine components to get a vector

EXAMPLE 3.3

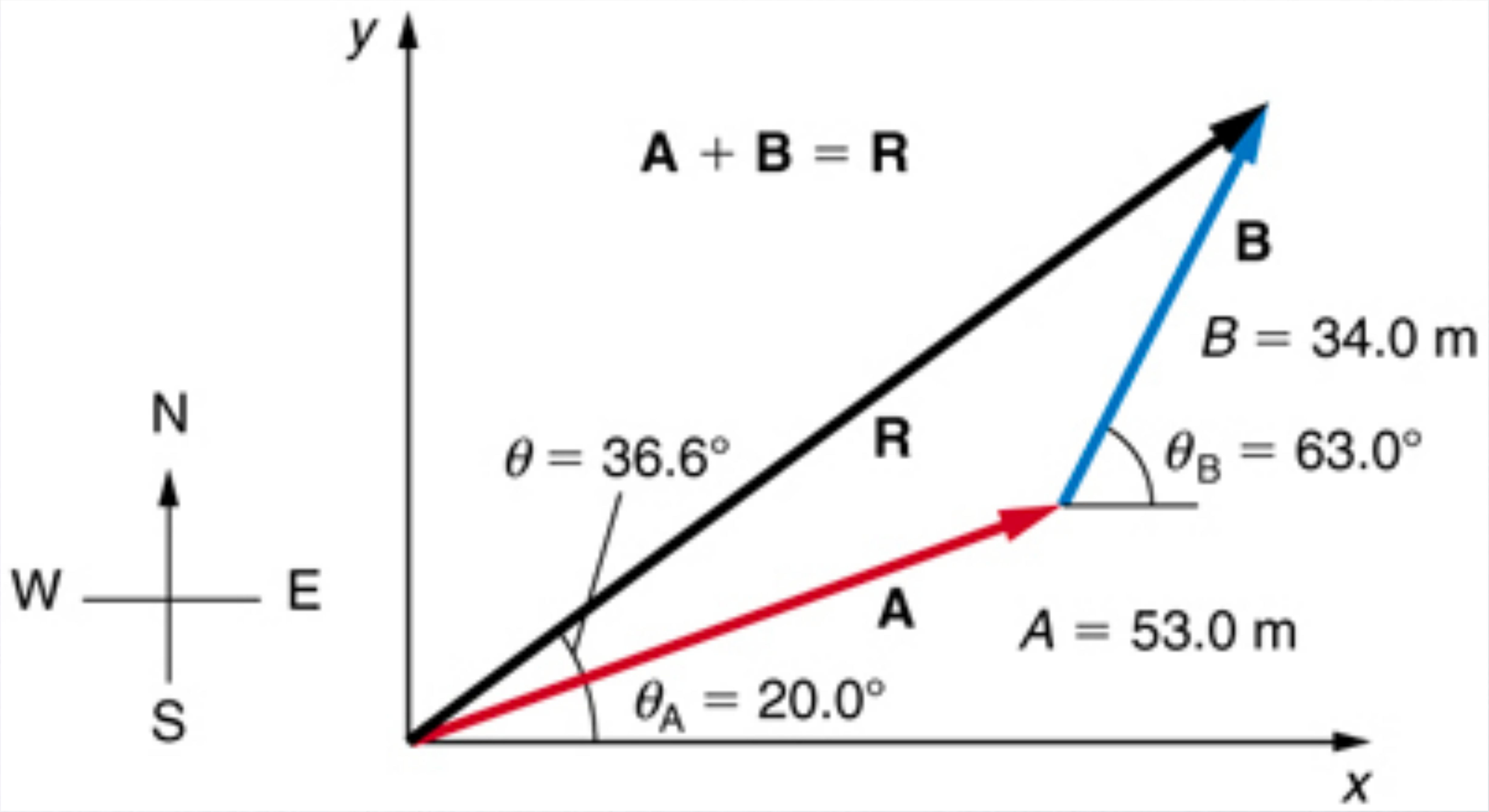
- Add the vector **A** to the vector **B** shown in the Figure, using perpendicular components along the x- and y-axes. The x- and y-axes are along the east–west and north–south directions, respectively. Vector **A** represents the first leg of a walk in which a person walks 53.0 m in a direction 20.0° north of east. Vector **B** represents the second leg, a displacement of 34.0 m in a direction 63.0° north of east.

$$A_x = A\cos(\theta_A) \quad = 53.0m\cos(20) = 49.8m$$

$$A_y = A\sin(\theta_A) \quad = 53.0m\sin(20) = 18.1m$$

$$B_x = B\cos(\theta_B) \quad = 34.0m\cos(63) = 15.4m$$

$$B_y = B\sin(\theta_B) \quad = 34.0m\sin(63) = 30.3m$$



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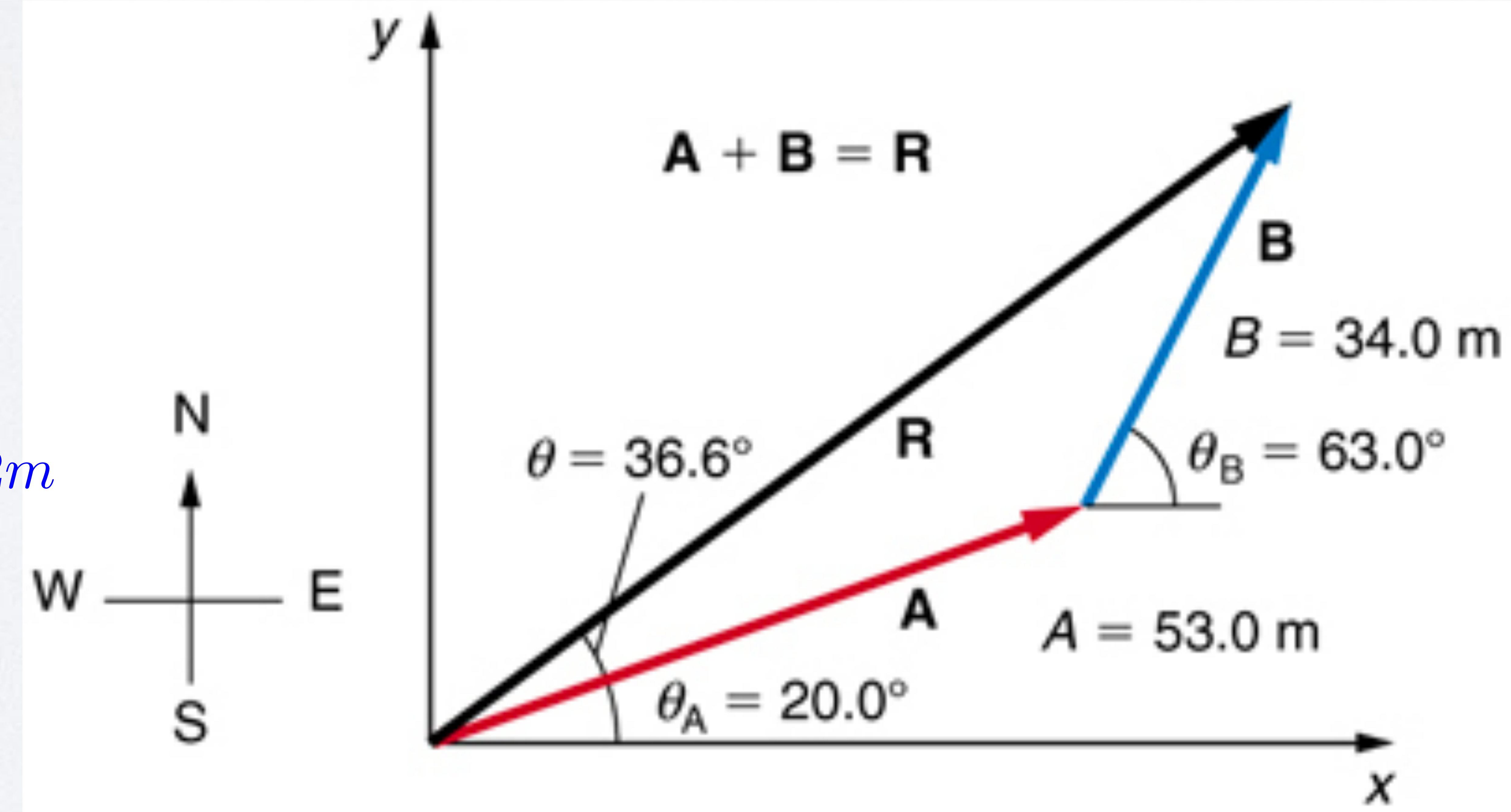
$$R_x = A_x + B_x \quad = 49.8m + 15.4m = 65.2m$$

$$R_y = A_y + B_y \quad = 18.1m + 30.3m = 48.4m$$

$$R = \sqrt{R_x^2 + R_y^2} \quad = \sqrt{(65.2m)^2 + (48.4m)^2} = 81.2m$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} \quad = \tan^{-1}\frac{48.4m}{65.2m} = 36.6^\circ$$



KINEMATICS IN 2D

- The key thing about physics in 2D, is that the components can be treated separately.
- Not only addition, but all physics apply separately to the components,
- So for example an object moving at constant velocity in 2D would have an x and y velocity. Its position as a function of time would be given by

$$x = x_0 + v_x t \quad \text{and} \quad y = y_0 + v_y t$$

- This applies for all physics. If the quantity is a vector we can write an x component and y component version of the equation.

PROJECTILE MOTION

- Projectile motion is the study of objects that have constant acceleration in one direction and an initial velocity.

- This corresponds to lots of things in the real world; a cannon ball, a tennis ball, jumping out of a plane, golf, a water fountain, the high jump, a coyote running off of a cliff.

- The most common case of projectile motion is the acceleration comes from gravity in which case there is only y acceleration and $a_x = 0$.

EXAMPLE 3.4

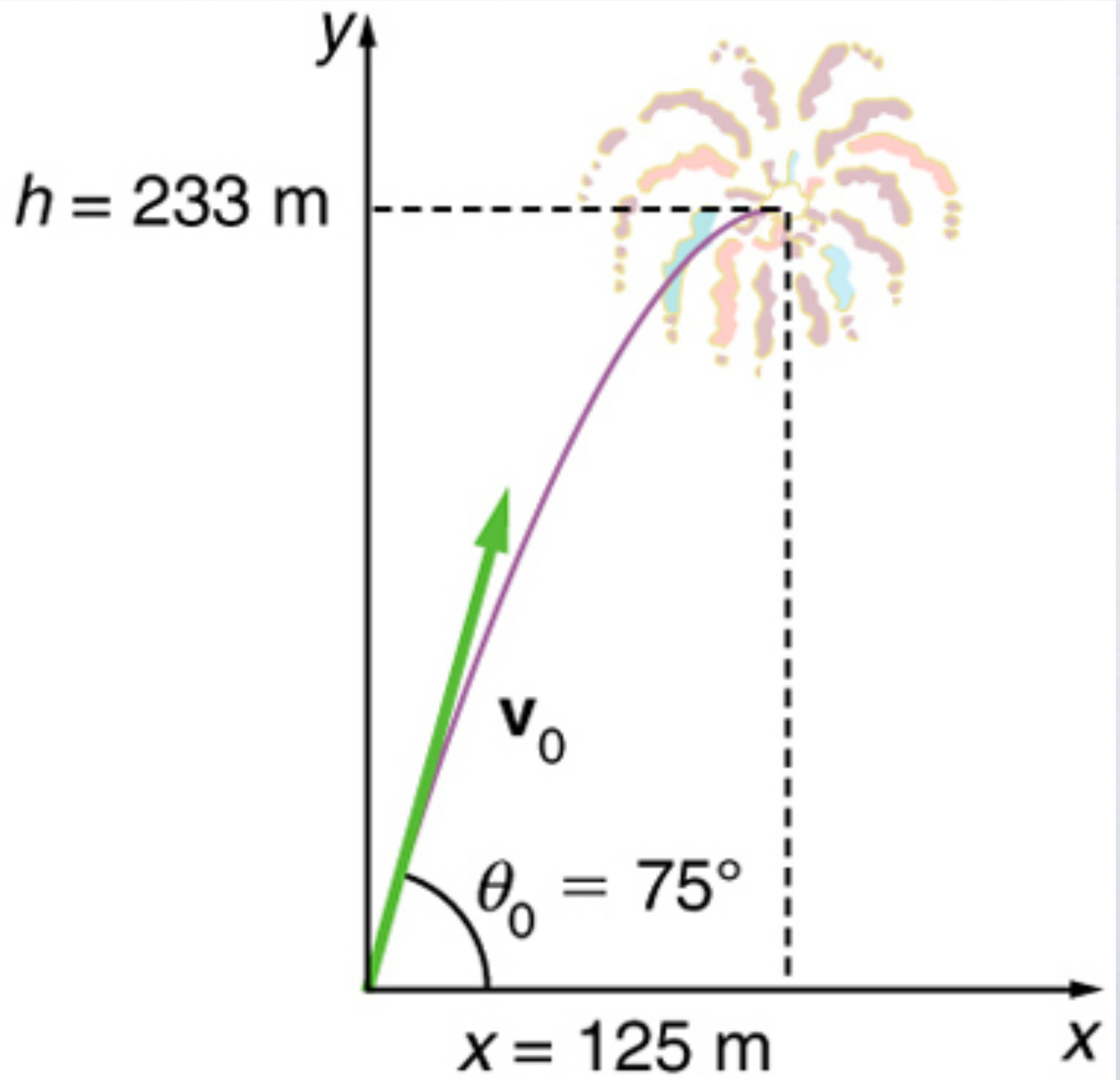
- During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0° above the horizontal, as illustrated in the Figure. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

$v_0 = 70.0$ m/s
$\theta = 75$
$a_x = 0$
$a_y = -9.81$ m/s^2

$y = ?$
$t = ?$
$x = ?$

$$v_{0x} = v_0 \cos\theta = (70.0m/s)\cos 75 = 18.1m/s$$

$$v_{0y} = v_0 \sin\theta = (70.0m/s)\sin 75 = 67.6m/s$$



a) $$v_y^2 = v_{0y}^2 + 2a(y - y_0)$$ $v_y = 0$

$$v_{0y}^2 = 2gy \qquad y = \frac{v_{0y}^2}{2g} = \frac{(67.6m/s)^2}{2(9.8m/s^2)} = 233m$$

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$$v_{0x} = v_0 \cos\theta = (70.0 m/s) \cos 75 = 18.1 m/s$$

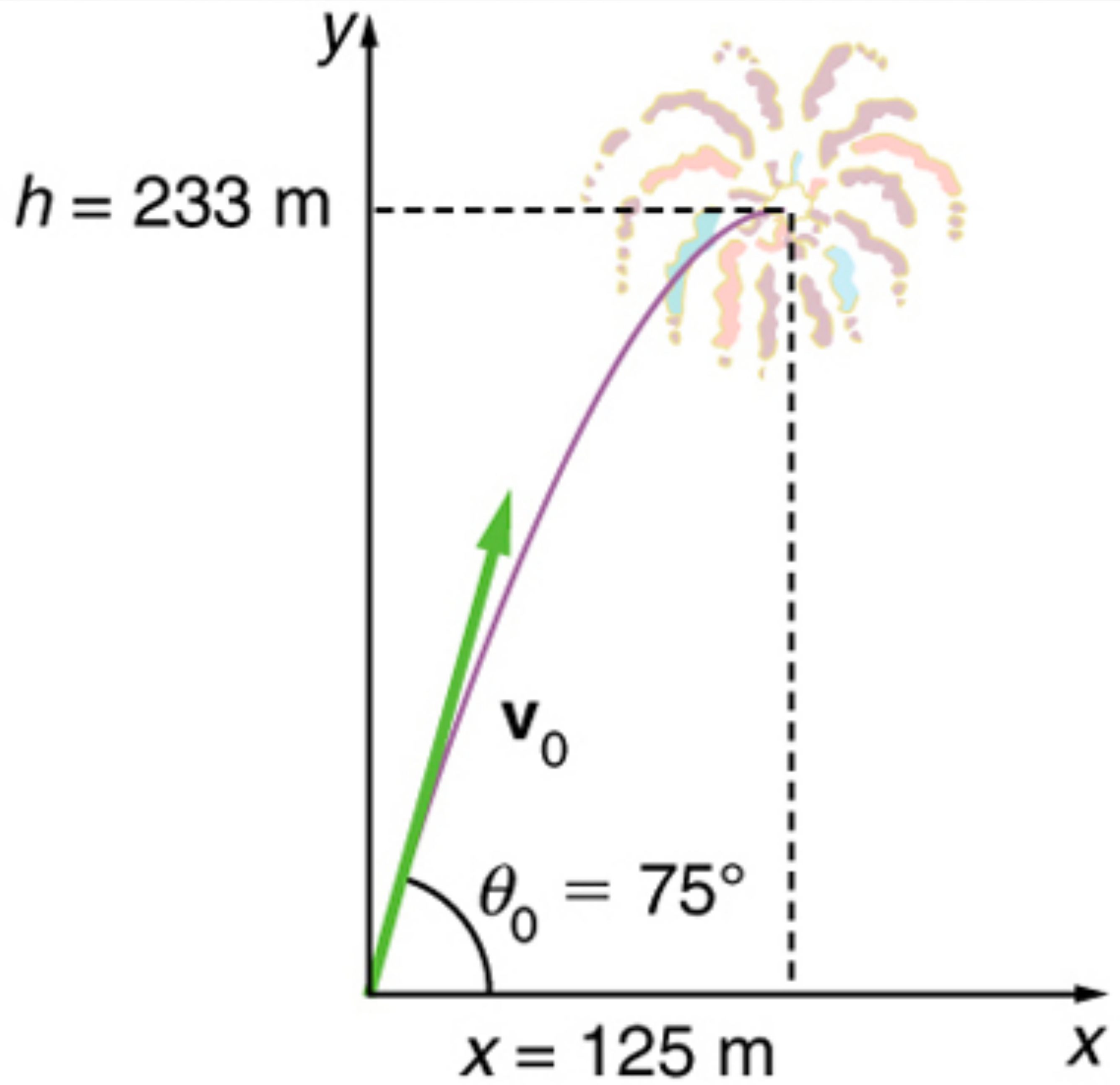
$$v_{0y} = v_0 \sin\theta = (70.0 m/s) \sin 75 = 67.6 m/s$$

c) $$x = x_0 + v_{0x} t$$

$$= 18.1 m/s (6.90 s) = 125 m$$

b) $$v_y = v_{0y} + at$$ $v_y = 0$

$$v_{0y} = gt \qquad t = \frac{v_{0y}}{g} = \frac{67.6 m/s}{9.81 m/s^2} = 6.90 s$$



EXAMPLE 3.5

Calculating Projectile Motion: Hot Rock Projectile
Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle 35.0° above the horizontal, as shown in the Figure. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?

$v_0 = 25.0$ m/s

$\theta = 35$

$y = -20.0$ m

$a_y = -g = -9.81$ m/s^2

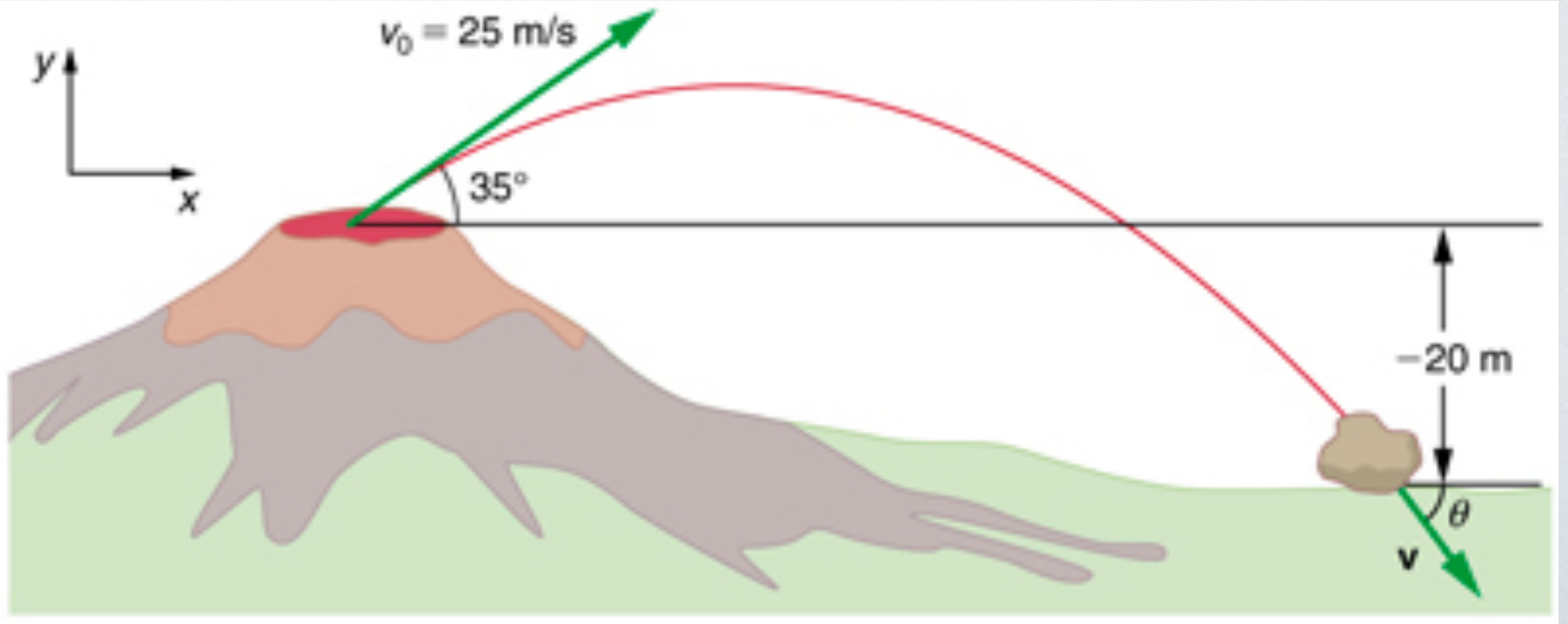
$v_{0y} = v0 \sin\theta = 14.3$ m/s

$$y = y_0 + v_{0y}t + \frac{1}{2}at^2$$

$$(4.9m/s^2)t^2 - (14.3m/s)t - 20.0m = 0$$

$$at^2 + bt + c = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 3.96s$$



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$\theta = 35$

$y = -20.0$ m

$t = 3.96$s

$a_y = -g = -9.81$ m/s^2

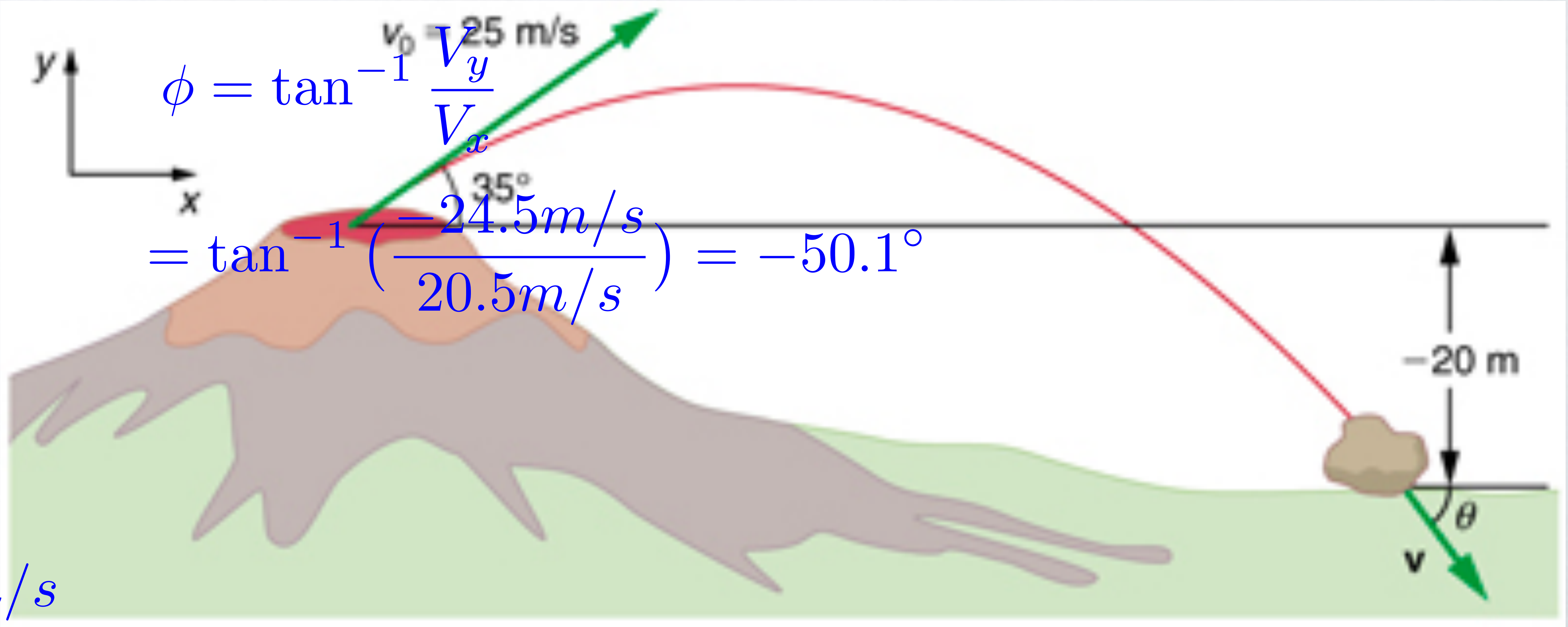
$v_{0y} = v_0 \sin\theta = 14.3$ m/s

$v_{0x} = v_0 \cos\theta = 20.5$ m/s

$$v_y = v_{0y} + at$$

$$v_y = 14.3m/s - 9.8m/s^2(3.96s) = -24.5m/s$$

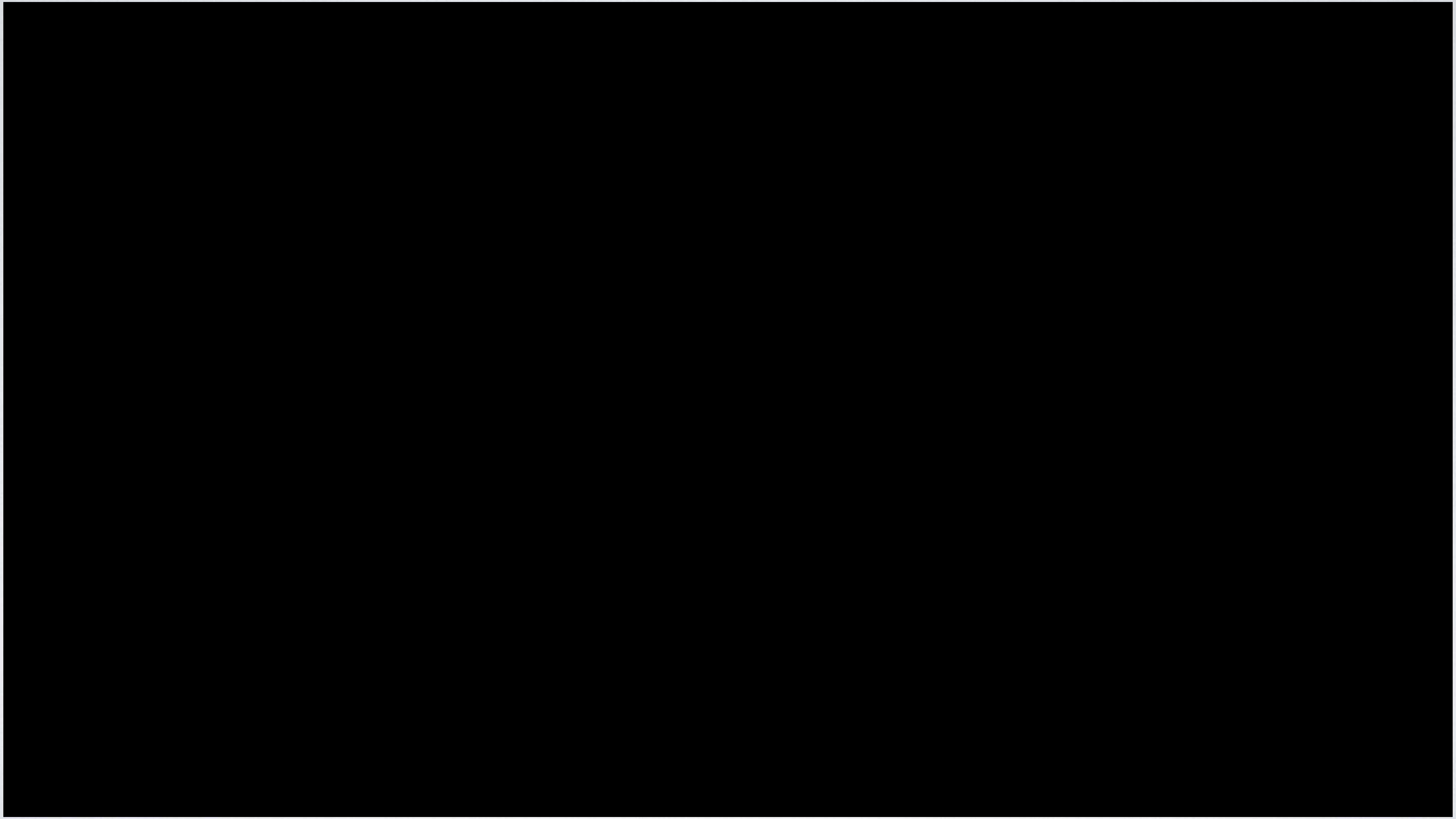
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(14.3m/s)^2 + (-24.5m/s)^2} = 31.9m/s$$



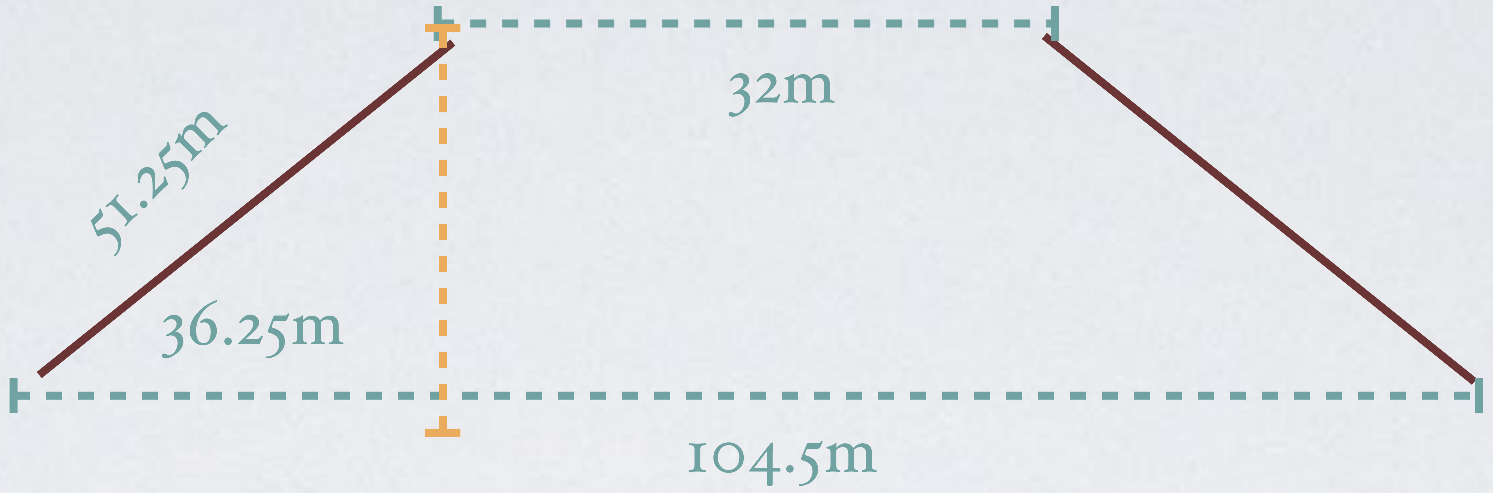
$$\phi = \tan^{-1}\frac{V_y}{V_x}$$

$$= \tan^{-1}\left(\frac{-24.5m/s}{20.5m/s}\right) = -50.1°$$

The Blues Brothers



This is the 95th St. Bridge in Chicago which we can look up and find has a length of 104.5m



Then the angle is cos⁻¹(36.25/51.25) = 45°. We then have

$$x = 32m \qquad v_{x0} = v_0 \cos(45) \qquad a_x = 0$$

$$y = 0 \qquad v_{y0} = v_0 \sin(45) \qquad a_y = g = -9.81 m/s^2$$

$$y = v_{y0}t + \frac{1}{2}a_y t^2 = 0 \quad => v_{y0} = -\frac{1}{2}a_y t \quad => v_0 = -\frac{1}{2}\frac{a_y}{\sin(45)}t$$

$$x = v_{x0}t \quad => t = \frac{x}{v_{x0}} = \frac{x}{v_0 \cos(45)} \quad => v_0 = -\frac{1}{2}\frac{a_y}{\sin(45)}\frac{x}{v_0\cos(45)}$$

$$=> v_0^2 = -\frac{1}{2}\frac{-9.81m/s^2}{\sin(45)}\frac{32m}{\cos(45)} = 314 m^2/s^2 \quad => v_0 = 17.7 m/s$$

or 40mph

2 Fast 2 Furious

ADDING VELOCITIES

- Velocities and accelerations are also vectors and therefore must be added by components.
- This is most common with velocities where you have something moving which is on something else that is moving.
- For example the velocity of a person walking on a train which is also moving or a boat that is moving in water which itself is moving.

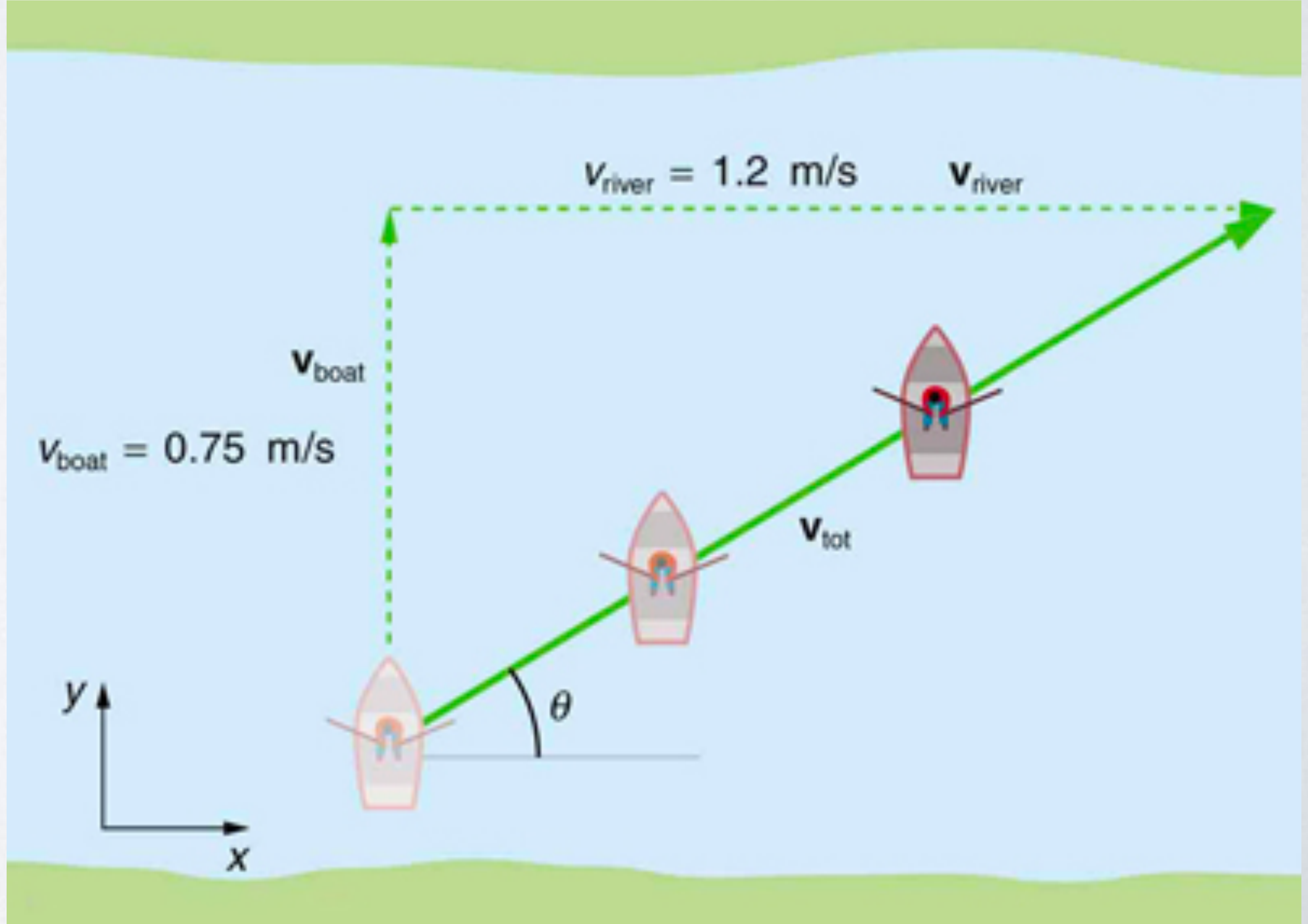
EXAMPLE 3.6

- **Adding Velocities: A Boat on a River.** Refer to the Figure, which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, $\mathbf{v}_{tot}$, The velocity of the boat, $\mathbf{v}_{boat}$ is 0.75 m/s in the y-direction relative to the river and the velocity of the river, $\mathbf{v}_{river}$ is 1.20 m/s to the right.

v_{boat} = 0.75 m/s = v_y
v_{river} = 1.20 = v_x

$$v_{tot} = \sqrt{v_{boat}^2 + v_{river}^2} \quad = \sqrt{(0.75m/s)^2 + (1.20m/s)^2} = 1.42m/s$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} \quad = \tan^{-1}\frac{0.75}{1.20} = 32.0^\circ$$



HOME WORK

- Chap 3 - 16, 20, 27, 46, 53, 57