NEW YORK CITY COLLEGE OF TECHNOLOGY

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Atrium Learning Center Math Tutors' Cheat Sheet
MAT 1190 - Probability

This document summarizes material from Sections 11-3 and 11-4 of Sobecki and Bluman's *Math in Our World* (3rd Edition, McGraw-Hill) as is relevant for MAT 1190. It should be used by ALC math tutors to familiarize themselves with the basics in order to tutor MAT 1190 students.

Chapter 11: Probability and Counting Techniques

Chapter 11 focuses on probability and counting techniques; only Sections 11-3 and 11-4 are covered in MAT 1190. Sections 11-1 and 11-2 introduce elementary counting principles that you should be familiar with.

- Consider a sequence of n procedures. Assume that the ith procedure can occur in k_i ways. Then the total number of ways the sequence of procedures can occur is $k_1 \cdot k_2 \cdot \cdots \cdot k_n$.
- The number of permutations of n objects (where order of objects matters) is $n!$.
- The number of combinations of r objects chosen from a total of n objects (where order of objects does not matter) is $\frac{n!}{(n-r)!r!}$, denoted ${}_nC_r$ (and often $\binom{n}{r}$ in other texts).

You will likely need only the first of these for solving MAT 1190 problems, but you should know the others as well.

Section 11-3: The Basic Concepts of Probability

A *probability experiment* (or *trial*) is a procedure that can be repeated and leads to a well-defined set of possible outcomes. An *outcome* is the result of a single trial of a probability experiment. A *sample space* for a probability experiment is the set of all possible outcomes. An *event* for a probability experiment is any subset of the sample space.

Section 11-2 focuses on conducting one probability experiment at a time.

For MAT 1190, *classical probability* studies the results of theoretical experiments where each outcome in a sample space is equally likely. If E is an event in the sample space S, $n(E)$ is the number of outcomes in E and $n(S)$ is the number of outcomes in S, then the probability of E is

$$P(E) = \frac{n(E)}{n(S)}.$$

(Notice that $P(E)$ is always less than or equal to 1 and $P(S)$ is always equal to 1.)

For example, one roll of a usual 6-sided die is a probability experiment. The sample space is the set of possible outcomes of a roll: $\{1, 2, 3, 4, 5, 6\}$, whose size is 6. The probability of any one outcome is therefore $\frac{1}{6}$.

Question 11(a) on the final exam review sheet asks what the probability of rolling a 1 or a 2 is. Here, the event is $\{1, 2\}$, whose size is 2. Therefore the probability of rolling a 1 or a 2 is $\frac{2}{6} = \frac{1}{3}$. Notice that this also means that the probability of rolling something *other* than a 1 or a 2, that is, the probability of rolling a 3, 4, 5, or 6, is $1 - \frac{1}{3} = \frac{2}{3}$.

Empirical probability, which uses data from experiments rather than counting, is also discussed in this section. If the *observed frequency* f is the number of times that a certain event has occurred in a given number of trials n, then the probability of an event E is

$$P(E) = \frac{f}{n}.$$

Question 11(b) on the final exam review sheet uses a survey of 105 students asking each to select their favorite breed of cat. The results of the survey appear in the following table:

Cat	Siamese	Abyssinian	Sphynx	Himalayan	Tonkinese
Number of students	26	15	35	22	7

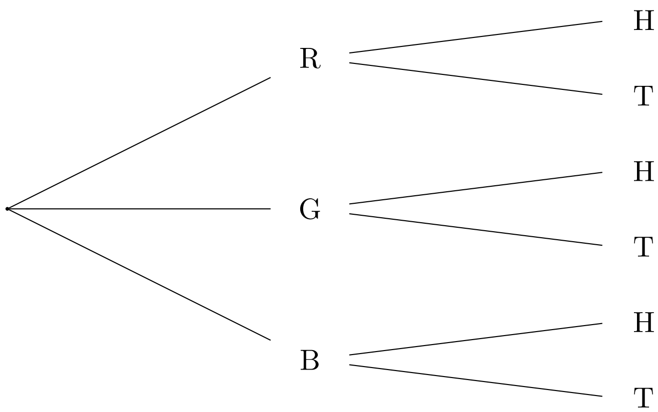
To determine the probability that a student will select a Siamese or a Himalayan cat we need to know the observed frequency f and the number of trials n. Since 26 students chose Siamese and 22 students chose Himalayan, f is equal to $26 + 22 = 48$. Since 105 students took the survey, n is equal to 105. Therefore the probability that a student will select a Siamese or a Himalayan cat is $\frac{48}{105} = \frac{16}{35}$.

Section 11-3: Tree Diagrams, Tables, and Sample Spaces

Now that we have established our formal language, we can revisit the first the elementary counting technique described above for conducting a sequence of probability experiments. If there are n experiments and there are k_i possible outcomes for the ith experiment, then the number of outcomes for the sequence of experiments is $k_1 \cdot k_2 \cdot \cdots \cdot k_n$.

Tree diagrams help us keep track of outcomes for sequences of experiments in a visual way. These are rooted trees where each node represents a probability experiment and the children of a node represent the outcomes for that experiment.

Question #5 on the final exam review sheet provides a good example. Here, the first probability experiment involves spinning a spinner that is labeled with the colors red (R), green (G), and blue (B). The second probability experiment involves tossing a coin so the outcomes are heads (H) and tails (T). Since the first experiment has 3 outcomes (each with probability $\frac{1}{3}$) and the second has 2 outcomes (each with probability $\frac{1}{2}$, the sequence has 6 possible outcomes. The tree diagram on the next page organizes these outcomes.



From the tree diagram, we see that the 6 possible outcomes are $\{RH, RT, GH, GT, BH, BT\}$. Computing probability with this sample space is the same as it was in Section 11-1.

The probability of any one of the outcomes is $\frac{1}{6}$ so, for example, the probability of spinning green on the spinner and flipping heads on the coin is $P(\{GH\}) = \frac{1}{6}$. The probability of spinning red or green on the spinner and flipping tails on the coin is $P(\{RT, GT\}) = \frac{2}{6} = \frac{1}{3}$.