Professor K. Poirier
ALC Math Department Liaison
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Atrium Learning Center Math Tutors' Cheat Sheet
MAT 1190 - Reasoning and Logic

This document summarizes material from Sections 1-1, 3-1, and 3-2 of Sobecki and Bluman's *Math in Our World* (3rd Edition, McGraw-Hill) as is relevant for MAT 1190. It should be used by ALC math tutors to familiarize themselves with the basics in order to tutor MAT 1190 students.

Section 1-1: The Nature of Mathematical Reasoning

Chapter 1 focuses on problem solving; only Sections 1-1 and 1-2 are covered in MAT 1190. Since the material in Section 1-2 should already be familiar to tutors, we focus here only on Section 1-1.

The primary focus of this section is the distinction between *inductive* and *deductive* reasoning. Tutors should already be aware of the distinction though they may not know the formal language.

- *Inductive reasoning* is the process of reasoning that arrives at a general conclusion based on the observation of specific examples (think: generalizing or predicting by recognizing a pattern or a trend).
- *Deductive reasoning* is the process of reasoning that arrives at a general conclusion based on previously accepted general statements (think: generalizing using formal proofs). It does not rely on specific examples.

Often, one might use inductive reasoning to recognize a pattern in order to form a conjecture and then use inductive reasoning to prove the conjecture. Question #4 on the final exam review sheet, copied here, is an example of these uses.

Pick a number, subtract 5, multiply the answer by 4, divide the answer by 2, and subtract 2 times the original number

(a) *Use inductive reasoning to find a rule that relates the number selected to the final answer.*

(b) *Use deductive reasoning to prove your conjecture.*

To answer (a), we will select a few numbers (somewhat at random, though small positive integers will be easiest to work with), apply the operations, and see if we can find a pattern. If we don't see a pattern right away, we can select a few more numbers.

- Input: 1; output: $\frac{(1-5)\times 4}{2} - 2 \times 1 = -10$
- Input: 2; output: $\frac{(2-5)\times 4}{2} - 2 \times 2 = -10$

- Input: 3; output: $\frac{(3-5)\times 4}{2} - 2 \times 3 = -10$

Since the output for three different inputs is always -10, an obvious guess is that the output is *always* -10, no matter what the input is. We can make this a precise conjecture using a variable as an input; here, we let a represent any real number. Then our conjecture is:

$$\frac{(a-5)\times 4}{2} - 2 \times a = -10.$$

To prove this conjecture for (b), we perform the operations step-by-step to the input a as follows:

$$\begin{aligned} a \\ a-5 \\ (a-5)\times 4 &= 4a-20 \\ \frac{4a-20}{2} &= 2a-10 \\ (2a-10)-2a &= -10 \end{aligned} \tag{1}$$

This shows us that no matter what number the input a is, the output will always be -10.

Chapter 3: Logic

Chapter 3 focuses on logic; only Sections 3-1 and 3-2 are covered in MAT 1190. Section 3-1 lays out basic definitions and rules and Section 3-2 covers truth tables.

> A *statement* (also called a *proposition* in other texts) is a declarative sentence that is either true or false, but not both.

That is, a statement must have a *truth value*, usually indicated as T or F. Here are some examples.

(a) "$3 > 5$" is an example of a statement because we can check its truth value.

(b) "$x > 5$" is not an example of a statement because we cannot check its truth value since we don't know what the value of x is.

(c) "What time does the movie start?" is not an example of a statement because it is a question; it cannot have a truth value.

(d) "That movie was great," is not an example of a statement because it is an opinion; it does not have an objective truth value.

(e) "Janelle Monáe is in the movie *Hidden Figures*," is an example of a statement because we can check its truth value.

Of the two examples that are statements, (a) is false and (b) is true.

Often we use *propositional variables* to represent specific propositions. This helps when we want to form *compound propositions* (new propositions created from old ones) and check their truth values. Common variables are p and q.

Given propositions p and q, relevant examples of compound propositions are:

- *negation* $\sim p$ (sometimes denoted $\neg p$), read as "not p,"
- *conjunction* $p \wedge q$, read as "p and q,"
- *disjunction* $p \vee q$, read as "either p or q (or both),"
- *conditional* $p \to q$, read as "if p, then q," and
- *buconditional* $p \leftrightarrow q$, read as "p, if and only if q" (equivalent to $(p \to q) \wedge (q \to p)$).

For example, if p represents the statement "$3 > 5$" and q represents the statement "Janelle Monáe is in the movie *Hidden Figures*," then

- $\sim p$ represents "3 is not greater than 5" (equivalently, "$3 \leq 5$"),
- $p \wedge q$ represents "$3 > 5$ and Janelle Monáe is in the movie *Hidden Figures*,"
- $p \vee q$ represents "either $3 > 5$ or Janelle Monáe is in the movie *Hidden Figures* (or both),"
- $p \to q$ represents "if $3 > 5$ then Janelle Monáe is in the movie *Hidden Figures*,"
- $p \leftrightarrow q$ represents "$3 > 5$ if and only if Janelle Monáe is in the movie *Hidden Figures*."

Truth tables help us determine truth values of compound statements in terms of the truth values that make them up. For the relevant examples above, we have the following table:

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Often students struggle the most with understanding the $p \to q$ column. It can be helpful to think of $p \to q$ as meaning "if p is true then q must also be true." Therefore, $p \to q$ is true if p and q are both true and false if p is true but q is not true. The next two rows in this column might present more of a challenge. One way to think of it is like this. This statement does not make *any* claim at all unless p is true. Therefore, if p is false, it doesn't matter what the truth value of q is; it is impossible for $p \to q$ to be false, therefore it must be true.

Using these truth values, we can make truth tables for other compound statements, building up step-by-step. The statement $\sim p \wedge (p \to q)$ from Question #3 of the final exam review sheet has the following truth table.

p	q	$\sim p$	$p \to q$	$\sim p \wedge (p \to q)$
T	T	F	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T