

1. Solve the following system of equations. Find **the row echeleon form** and the **row-reduced echeleon form** using elementary row operations and then solve it. **Do not use calculator** to solve this problem. You can check your solution using calculator.

(a) $w - x - y + 2z = 1$, $2w - 2x - y + 3z = 3$, $-w + x - y = -3$

(b) $x + y + z = 3$, $x + 2y + 2z = 5$, $3x + 4y + 4z = 12$

2. Determine whether the system is consistent, inconsistent, or dependent.

(a) $3x + 2y = 15$, $6x + 4y = 30$

(b) $3x + 2y - 5z = 4$, $x + y - 2z = 1$, $5x + 3y - 8z = 6$

3. Find the unique value of t for which the following system has a solution. Determine the basic and free variable, if applicable.

- $x_1 + x_3 - x_4 = 3$, $2x_1 + 2x_2 - x_3 - 7x_4 = 1$, $4x_1 - x_2 - 9x_3 - 5x_4 = t$, $3x_1 - x_2 - 8x_3 - 6x_4 = 1$.

4. Show that the following set spans \mathbb{R}^3 and then write the vector \mathbf{b} as a linear combination of these vectors.

$\{[1 \ -1 \ 0]^T, [1 \ 1 \ 1]^T, [1 \ 0 \ 1]^T\}$; $\mathbf{b} = [-1 \ 2 \ 2]^T$

5.

Let $A = \begin{bmatrix} 1 & 5 & -2 & 0 \\ -3 & 1 & 9 & -5 \\ 4 & -8 & -1 & 7 \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 3 \\ -2 \\ 0 \\ -4 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} -7 \\ 9 \\ 0 \end{bmatrix}$. It can be shown

that \mathbf{p} is a solution of $A\mathbf{x} = \mathbf{b}$. Use this fact to exhibit \mathbf{b} as a specific linear combination of the columns of A .

6. Let $v_1 = (-2, 0, 1)$, $v_2 = (1, -1, 2)$ and $v_3 = (4, -2, 3)$. Are these vectors linearly dependent or independent? Justify your answer.
7. Go through the assignment problems on Concept Checking/True and False – Note just by writing true or false will not give you any point. You need to justify your conclusion i.e. you need to provide explanation why you think the following statement is true or false.