MAT2540/D670 **Exam #3 - Take-home Exercises**
Instructor: Suman Ganguli **Due: Monday, May 19** **Name:** ____________________

Question	Points	Score
1	5	
2	10	
3	5	
4	5	
Total:	25	

1. (5 points) Tree structures are often used to represent organizational hierarchies in companies. Shown here is (part of) an example "org chart" for a company:

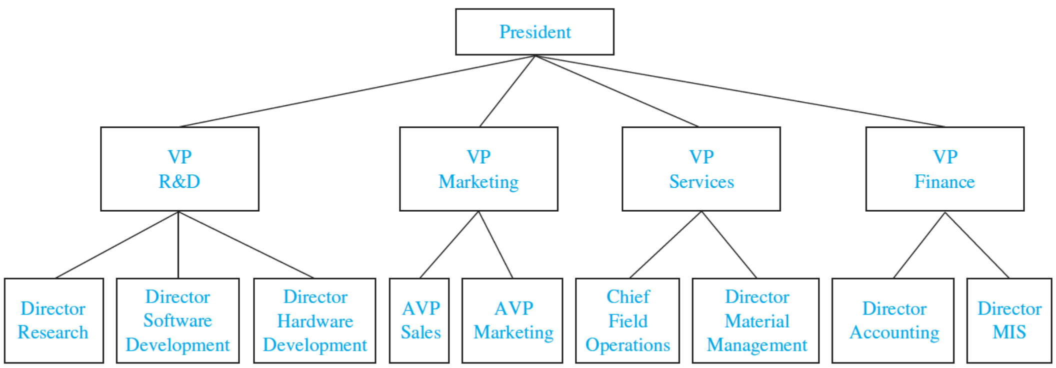


FIGURE 10 **An organizational tree for a computer company.**

(a) Which position is the root of this tree?

(b) Which positions in the company are "children" vertices of the VP of Marketing?

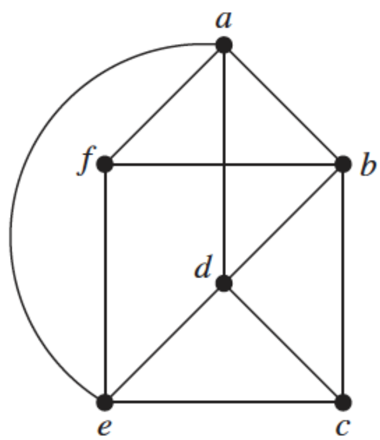
(c) What do the edges in this sort of "org chart" tree represent? In other words, what is the relationship within the company between a given vertex and its "parent" vertex and, or between a given vertex and its "children" vertices?

(d) For the tree shown above, we say that:

- The CEO is at level 0 of the tree
- The 4 Vice Presidents are at level 1
- The Directors and AVPs (Assistant Vice Presidents) are at level 2

Suppose there are additional employees in the company at level 3 in the org chart. In particular, suppose that each vertex at level 2 has 4 "children" at level 3, and that the level 3 employees are the leaves of the tree. Add vertices to the tree above showing these level 3 employees, and calculate the total number of employees in the company:

2. (10 points) Consider the following simple graph:



(a) For each vertex $v \in V$, write down the neighborhood of v ($N(v)$) and the degree of v ($\deg(v)$). (Hint: recall that in a simple graph, $\deg(v) = |N(v)|$ for all $v \in V$, and that since this is not a pseudograph, $v \notin N(v)$.)

$N(a) =$ $\deg(a) =$

$N(b) =$ $\deg(b) =$

$N(c) =$ $\deg(c) =$

$N(d) =$ $\deg(d) =$

$N(e) =$ $\deg(e) =$

$N(f) =$ $\deg(f) =$

(b) Using Theorems 1 and 2 in Section 10.5 of the textbook, explain why this graph has an Euler path but does not have an Euler circuit. Which two vertices must be the starting and ending points of any such Euler circuit?

(c) Write down two distinct Euler paths through this graph:

3. (5 points) Use mathematical induction to prove that the statement

$$P(n) : 3 + 6 + 9 + \ldots + 3n = \frac{3n(n+1)}{2}$$

is true for all natural numbers n.

Proof (by induction):

- **Base case** ($n = 1$)**:**

- **Inductive step:**
  (Hint: start by assuming and stating the inductive hypothesis, and use it to prove $P(k+1)$. What is the inductive hypothesis in "weak" induction?)

4. (5 points) (a) Suppose you have to create a license plate consisting of 3 letters followed by 4 digits, where letters can be repeated but the first digit cannot be 0. For example, one possible license plate is "ABC1234"

   How many different such license plates are possible? Explain your calculation in terms of the Product Rule for counting.

   (b) Now suppose the license plate can be either 3 letters followed by 3 digits, or 4 letters followed by 2 digits, with the same restriction that the first digit cannot be 0. How many such license plates are possible? Explain your calculation in terms of the Product Rule and Sum Rule for counting.