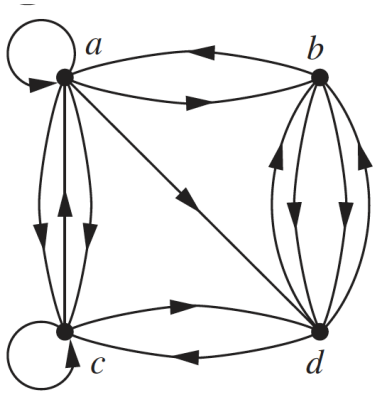
MAT2540/D670 **Exam 2 - Take-home Exercises**
Instructor: Suman Ganguli **Due: Monday, April 28** **Name:** _____

Question	Points	Score
1	10	
2	5	
3	5	
4	0	
Total:	20	

1. (10 points) Consider the following graph:



(a) This is a directed multigraph, and is also pseudograph. Why is that the case (i.e., what does it mean for a graph to be *directed*, to be a *multigraph*, and to be a *pseudograph*)?

(b) Write down the adjacency matrix for this graph. (Since there are 4 vertices, the adjacency matrix will be a 4-by-4 matrix, and the entry in the i-th row and j-th column of the matrix will be m if there m edges originating at the i-th vertex and terminating at the j-th vertex.)

(c) For each vertex $v \in V$, write down the in-degree and out-degree of v ($\deg^-(v)$ and $\deg^+(v)$). (Hint: recall that in a simple graph, $\deg(v) = |N(v)|$ for all $v \in V$, and that since this is not a pseudograph, $v \notin N(v)$.)

$\deg^-(a) =$ $\deg^+(a) =$

$\deg^-(b) =$ $\deg^+(b) =$

$\deg^-(c) =$ $\deg^+(c) =$

$\deg^-(d) =$ $\deg^+(d) =$

(d) Verify that Theorem 3 holds for this graph, i.e., show that $|E| = \sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v)$:

$|E| =$

$\sum_{v \in V} \deg^-(v) =$

$\sum_{v \in V} \deg^+(v) =$

2. (5 points) Choose one of the graph models described in Sec 10.1, i.e., one of Examples 1-14 under the "Graph Models" subsection.

(a) Which graph model/example did you choose?

(b) What do the vertices in the graph represent? What do the edges represent?

(c) Are the graphs in this application simple graphs or multigraphs? Are they undirected or directed? Explain why in 2-3 sentences. You can also draw a sample graph to illustrate.

3. (5 points) Watch the following two YouTube videos by Dr. Trefor Bazett, either on his YouTube channel or posted on the OpenLab site:

   - "Intro to Graph Theory — Definitions & Ex: 7 Bridges of Konigsberg"
   - "Euler Paths & the 7 Bridges of Konigsberg — Graph Theory"

   (a) Do you like Dr. Bazett's teaching style? Explain why or or why not (2-3 sentences):

   (b) Did you learn something new from these videos (e.g., what is an "Euler path")? Did it help you understand something we already covered in class? Give a list of 2-3 topics that you found the videos useful for:

   (c) Draw the graph for the Bridges of Konigsberg problem, and explain what the vertices and edges in the graph represent:

4. (a) What is the "complete bipartite graph" $K_{m,n}$? Give the definition, and draw $K_{2,4}$ to illustrate.

(b) Suppose you are setting up a computer network consisting of m servers and n clients. Explain why $K_{m,n}$ makes sense as a network topology (i.e., graph of connections) for this sort of client-server setup.