Mathematics 1275/D225, Spring 2024 Exam #2 - makeup exercises
Instructor: Suman Ganguli Mon May 6 - Wed May 15 Name: ____________________

In order to receive full credit, you must **show all your work** and simplify your answers.

1. (10 points) Algebraically find the vertex, y-intercept, and x-intercept(s) for the graph of the quadratic function

$$y = x^2 - 6x + 8$$

and then sketch the graph.

(a) y-intercept (i.e., find value of y when $x = 0$):

Solution:
$y = 8$ when $x = 0$ so the y-intercept is $(0, 8)$

(b) x-intercepts (i.e., find value(s) of x for when $y = 0$):

Solution:
Since $x^2 - 6x + 8 = (x-2)(x-4)$, the solutions of $x^2 - 6x + 8 = 0$ occur for $x - 2 = 0$ and $x - 4 = 0$, i.e., $x = 2$ and $x = 4$. Thus, the x-intercepts of the graph are the points $(2, 0)$ and $(4, 0)$.

(c) Find the vertex of $y = x^2 - 6x + 8$ by either of two methods:
(i) put the function in vertex form : $y = (x - h)^2 + k$
by completing the square (in which case the vertex is at (h, k)); or
(ii) use the vertex formula, which says that the x-coordinate of the vertex of $y = ax^2 + bx + c$ is

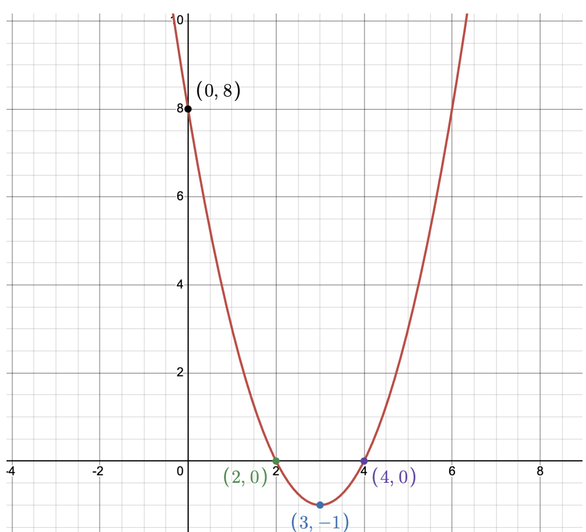
$$x = -\frac{b}{2a}$$

Solution:
(i) completing the square:
$y - 8 = x^2 - 6x \Rightarrow y - 8 + 9 = x^2 - 6x + 9 \Rightarrow y = (x - 3)^2 - 1$
So the vertex is at $(3, -1)$.
(ii) Since $a = 1$ and $b = 6$, the the x-coordinate of the vertex is

$$x = -\frac{-6}{2(1)} = \frac{6}{2} = 3$$

and then the x-coordinate of the vertex is $y = 3^2 - 6(-3) + 8 = 9 - 18 + 8 = -1$. Thus, the vertex is at $(3, -1)$.

(d) Sketch the graph of $y = x^2 - 6x + 9$. Label the vertex, y-intercept, and x-intercepts with their coordinates:



2. (10 points) Simplify the following complex fraction:

$$\frac{\frac{1}{y}+\frac{5}{y^2}}{1-\frac{25}{y^2}} =$$

Solution:

$$\frac{\frac{1}{y}+\frac{5}{y^2}}{1-\frac{25}{y^2}} = \frac{\frac{y}{y}\cdot\frac{1}{y}+\frac{5}{y^2}}{\frac{y^2}{y^2}-\frac{25}{y^2}} = \frac{\frac{y+5}{y^2}}{\frac{y^2-25}{y^2}} = \frac{y+5}{y^2}\cdot\frac{y^2}{y^2-25} = \frac{y+5}{(y+5)(y-5)} = \frac{1}{y-5}$$

3. Use the quadratic formula to solve the given quadratic equations. Simplify the solutions completely.

(a) (5 points) $-x^2+8x+1=0$

Solution: Applying the quadratic formula with $a=-1, b=8, c=1$:

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-8\pm\sqrt{8^2-4(-1)(1)}}{2(-1)} = \frac{-8\pm\sqrt{64+4}}{-2} = \frac{-8\pm\sqrt{68}}{-2} = \frac{-8\pm2\sqrt{17}}{-2} = -4\pm\sqrt{17}$$

(b) (5 points) $x^2-2x+2=0$
(Hint: this quadratic equations has two complex solutions; simplify the solutions into the form $a\pm bi$.)

Solution: Applying the quadratic formula with $a=1, b=-2, c=2$:

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{2\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)} = \frac{2\pm\sqrt{4-8}}{2} = \frac{2\pm\sqrt{-4}}{2} = \frac{2}{2}\pm\frac{2i}{2} = 1\pm i$$

4. Perform the indicated operations on the complex numbers. Write the result in standard complex form, i.e., in the form $a+bi$. (Remember to use the definition of i to simplify: $i^2=-1$.)

(a) (5 points)

$$(5+12i)(5-2i) =$$

Solution: $(5+12i)(5-2i) = 25+60i-10i-24i^2 = 25+50i+24 = 49+50i$

(b) (5 points) Recall that for division of complex numbers, we use the complex conjugate of the denominator:

$$\frac{3+2i}{1-i}$$

Solution:

$$\frac{3+2i}{1-i}\times\frac{1+i}{1+i} = \frac{3++3i+2i+2i^2}{1+i-i-i^2} = \frac{3+5i-2}{1+1} = \frac{1+5i}{2} = \frac{1}{2}+\frac{5}{2}i$$