Mathematics 1375/OL79, Fall 2020 **Exam #2 (In-class component)**
Instructor: Suman Ganguli **Monday, Nov 23** **Name:** ____________________

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1. (10 points) Consider the quadratic polynomial

$$q(x) = x^2 - 3x - 4$$

(a) Find the roots of $q(x)$ algebraically. (Hint: Either factor $q(x)$ or use the quadratic formula.)

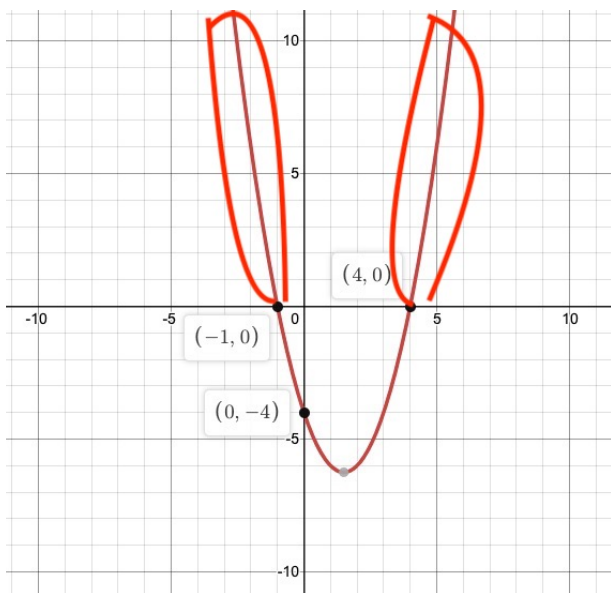
> **Solution:** To find the roots, we solve the equation $q(x) = 0$. Since $q(x) = x^2 - 3x - 4 = (x-4)(x+1)$, the roots are $x = -1$ and $x = 4$.

(b) What is the y-intercept of $q(x)$? (Hint: evaluate $q(0)$.)

> **Solution:** Since $q(0) = -4$, the y-intercept of $q(x)$ is at $(0, -4)$.

(c) Sketch a rough graph of $q(x)$, labelling the x-intercepts and the y-intercept with their coordinates:

> **Solution:** From #1, we know the x-intercepts occur at $(-1, 0)$ and $(4, 0)$, and from #2 we know the y-intercept is at $(0, -4)$.



(d) Use your graph to solve the following inequality: **circle the parts of your graph corresponding to the solution of the inequality** and then write down the solution set in interval notation:

$$x^2 - 3x - 4 \geq 0$$

> **Solution:** From the graph (or really just from the fact that we know the graph of $y = x^2 - 3x - 4$ is an parabola opening upward, with x-intercepts at $x = -1$ and $x = 4$), we see that the solution set of $x^2 - 3x - 4 \geq 0$ is
>
> $$(-\infty, -1] \cup [4, \infty)$$

2. (10 points) Consider the rational function: $f(x) = \dfrac{3x-2}{x+2}$

(a) What is the domain of f? Show your calculations, and write the solution in interval notation.

Solution: Since the denominator of f is $x+2$, the function is undefined for $x=-2$. Hence, the domain of f is $(-\infty,-2)\cup(-2,\infty)$

(b) What is the equation of the vertical asymptote of this function?

Solution: The vertical asymptotes occur at the x-values at which the denominator is 0, i.e., the vertical line $x=-2$.

(c) What is the equation of the horizontal asymptote of this function? Show your calculation.

Solution: The horizontal asymptote is given by the ratio of the leading terms, which for this function is:

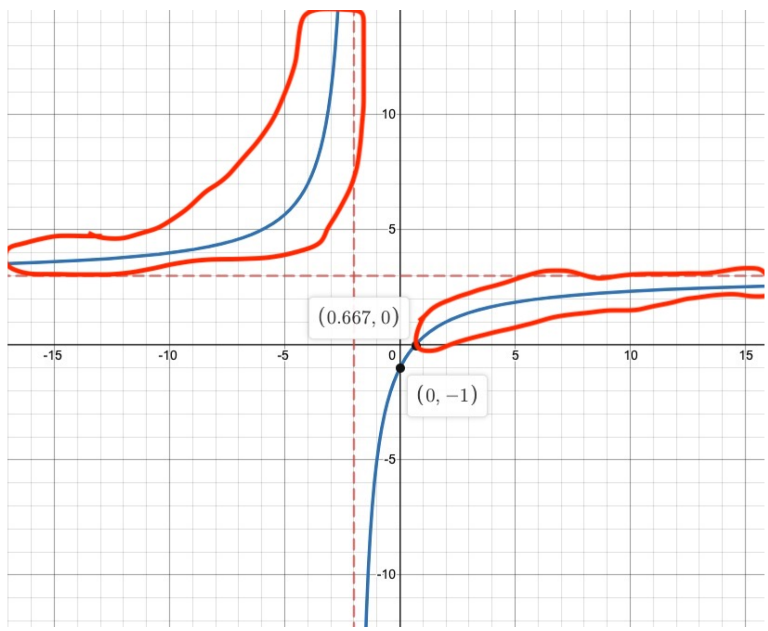
$$y = \frac{3x}{x} = 3$$

(d) Algebraically calculate the $x-$intercept(s) and $y-$intercept of the graph of $f(x)$. Again, show the necessary calculations, and write the coordinates of the intercepts in (x,y) form:

Solution:
The $x-$intercept is given by the roots of the numerator, which is at $3x-2=0$, i.e., $x=\frac{2}{3}$. Thus, the single $x-$intercept is at the point $(\frac{2}{3},0)$.
The $y-$intercept occurs at $f(0)=\dfrac{3(0)-2}{0+2}=\dfrac{-2}{2}=-1$, i.e., at the point $(0,-1)$.

(e) On the given graph of the function:

- Label the x- and y-intercepts with their coordinates
- Draw the vertical and horizontal asymptotes as dashed lines, and label each with its equation



(f) Use the graph (and the value of the root and the vertical asymptote) to solve the following inequality: **again, circle the parts of the graph corresponding to the solution set of the inequality**, and write down the solution set in interval notation.

$$\frac{3x-2}{x+2} \geq 0$$

Solution: We circle the parts of the graph at or above the x-axis. We see that this corresponds to the following solution set of the inequality:

$$(-\infty, -2) \cup [\frac{2}{3}, \infty)$$