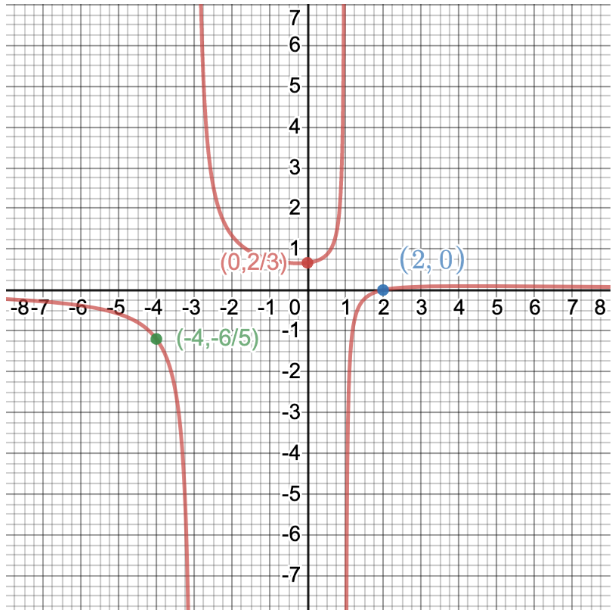
Mathematics 1375/OL79, Fall 2020 **Exam #1**
Instructor: Suman Ganguli **Due: Sunday, October 11** **Name:** ______________________

---

1. (10 points) Shown is the graph of the function $f(x) = \dfrac{x-2}{x^2+2x-3}$:



(a) Compute the following values of f (show your calculations), and **label the corresponding points with their coordinates** on the graph above:

> **Solution:**
>
> - $f(0) = \dfrac{0-2}{0+0-3} = \dfrac{2}{3}$ (so plot $(0, 2/3)$ on the graph)
> - $f(2) = \dfrac{2-2}{4+4-3} = 0$ (so plot $(2, 0)$ on the graph)
> - $f(-4) = \dfrac{-4-2}{16-8-3} = -\dfrac{6}{5}$ (so plot $(-4, 6/5)$ on the graph)

(b) What is the domain of f? For full credit, show your work, and write the solution in interval notation. (Hint: Start by factoring the denominator.)

> **Solution:** Since the denominator of f is $x^2+2x-3 = (x+3)(x-1)$, the function is undefined for $x = -3$ and $x = 1$. Hence, the domain of f is
>
> $$(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$$

(c) Briefly describe what happens to the graph of the function near the points which are *not* in the domain.

> **Solution:** Near the two points which are not in the domain, i.e., $x = -3$ and $x = 1$, we see on the graph that the y-values of the function go off to ∞ or $-\infty$. We will study this behavior later in the course; we will call $x = -3$ and $x = 1$ "vertical asymptotes."

2. (10 points) Solve each of the following inequalities algebraically, and
   - write the solution set in interval notation
   - graph the solution set on the given number line

(a) $|3 - 2x| > 7$

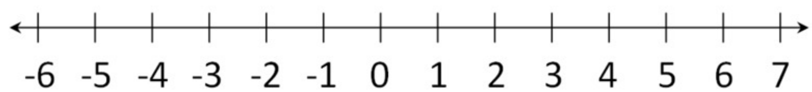
Solution:

$3 - 2x < -7 \quad \text{or} \quad 3 - 2x > 7$

$-2x < -10 \quad \text{or} \quad -2x > 4$

$x > 5 \quad \text{or} \quad x < -2$

$(-\infty, -2) \cup (5, \infty)$



(b) $|4x - 3| \leq 5$

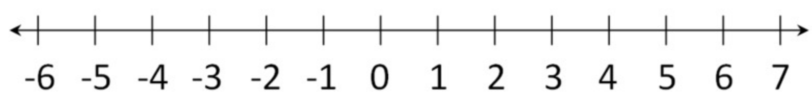
Solution:

$-5 \leq 4x - 3 \leq 5$

$-2 \leq 4x \leq 8$

$-\frac{1}{2} \leq x \leq 2$

$\left[-\frac{1}{2}, 2\right]$



3. (10 points) We discussed in class that we can interpret $|x|$ as the distance of x from 0.

(a) Hence, the solution set of the inequality $|x| < d$ should correspond to the set of numbers less than distance d from 0. What is the solution set of $|x| < d$ in interval notation?

Solution: The solution set of $|x| < d$ is $-d < x < d$, i.e., $(-d, d)$.

(b) Now solve the inequality $|x - a| < d$ (for arbitrary constants a and d). Write the solution set in interval notation.

Solution: We solve $|x - a| < d$ as follows:

$-d < x - a < d$

$a - d < x < a + d$

$(a - d, a + d)$

(c) Sketch the solution set from (b) on a number line, and then verbally describe the solution set in terms of distance d and the point a.

4. (10 points) Write down **and simplify** the following for $g(x) = x^2 - 7x - 20$:

(a) $g(x+h) =$

Solution: $g(x+h) = (x+h)^2 - 7(x+h) - 20 = x^2 + 2xh + h^2 - 7x - 7h - 20$

(b) $g(x+h) - g(x) =$

Solution: $g(x+h) - g(x) = (x^2 + 2xh + h^2 - 7x - 7h - 20) - (x^2 - 7x - 20) = 2xh + h^2 - 7h$

(c) $\dfrac{g(x+h) - g(x)}{h} =$

Solution: $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2xh + h^2 - 7h}{h} = 2x + h - 7$

5. (10 points) Let $f(x) = 4x - 1$ and $g(x) = \sqrt{x}$. Write down and simplify expressions for the following functions, and find their respective domains.

(a) $\left(\dfrac{f}{g}\right)(x) =$

domain of $\left(\dfrac{f}{g}\right)$:

Solution: $\left(\dfrac{f}{g}\right)(x) = \dfrac{4x-1}{\sqrt{x}}$ domain: $(0, \infty)$

(b) $\left(\dfrac{g}{f}\right)(x) =$

domain of $\left(\dfrac{g}{f}\right)$:

Solution: $\left(\dfrac{g}{f}\right)(x) = \dfrac{\sqrt{x}}{4x-1}$ domain: $[0, 1/4) \cup (1/4, \infty)$

(c) $(f \circ g)(x) =$

domain of $(f \circ g)$:

Solution: $(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 4\sqrt{x} - 1$ domain: $[0, \infty)$

(d) $(g \circ f)(x) =$

domain of $(g \circ f)$:

Solution: $(g \circ f)(x) = g(f(x)) = g(4x-1) = \sqrt{4x-1}$ domain: $[1/4, \infty)$