Mathematics 1375/OL79, Fall 2020 Exam #2 (Take-home component)
Instructor: Suman Ganguli **Due: Sunday, Nov 22** **Name:** ______________________________

Question:	1	2	3	4	Total
Points:	5	5	10	10	30
Score:					

> In order to receive full credit, you must **show all your work** and simplify your answers. Submit your written solutions by the end of the day Sunday on Blackboard (look for the "Exam #2" Assignment). Please **scan your written answers to a single pdf file**.

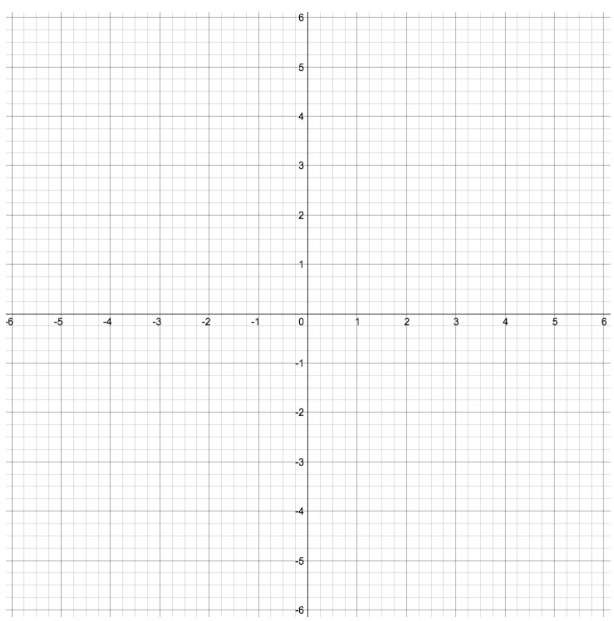
1. (5 points) Consider the quadratic polynomial

$$q(x) = -x^2 + 2x + 1$$

(a) Find the roots of $q(x)$ algebraically, and express them in simplest radical form. (Hint: The function does *not* factor, so use the quadratic formula.)

(b) What are the coordinates of the vertex of the parabola $y = q(x)$? (Recall that for a parabola $y = ax^2 + bx + c$, the x-coordinate of the vertex is given by $x = -\dfrac{b}{2a}$.)

(c) Sketch the graph of $q(x)$, labelling the x-intercepts, the y-intercept, and the vertex with their coordinates:



(d) Use the graph to solve the following inequality; express the solution in interval notation:

$$-x^2 + 2x + 1 > 0$$

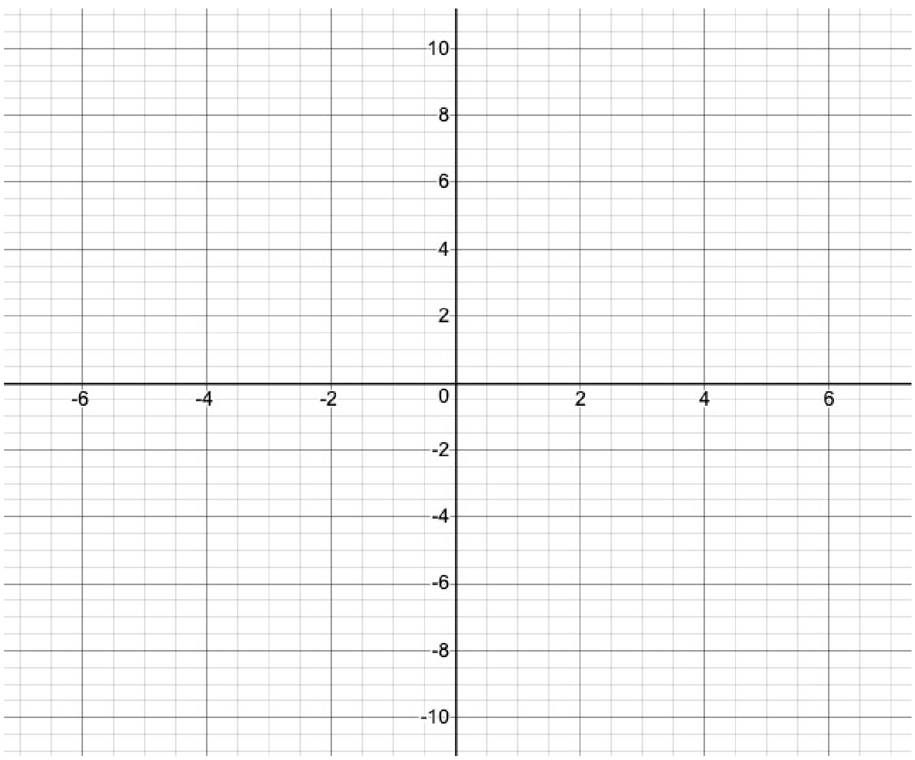
2. (5 points) Consider the function

$$f(x) = (x-2)^3$$

(a) Fill in the blanks:
"The only root of $f(x)$ is $x =$ ______, which is a root of multiplicity ______ ."

(b) What is the y-intercept of the graph of $f(x)$? Show the necessary calculations.

(c) Sketch the graph of $y = f(x)$. Label the x-intercept and y-intercept on your graph.



3. (10 points) Consider the cubic polynomial:

$$p(x) = x^3 + x^2 - x - 1$$

(a) Verify that $c = -1$ is a root of $p(x)$ (i.e., show that $p(-1) = 0$):

(b) Since we know from (a) that $c = -1$ is a root of p, we know by the Factor Theorem that $(x - c) = (x + 1)$ is a factor of $p(x)$. Use long division to compute $\dfrac{p(x)}{x+1}$:

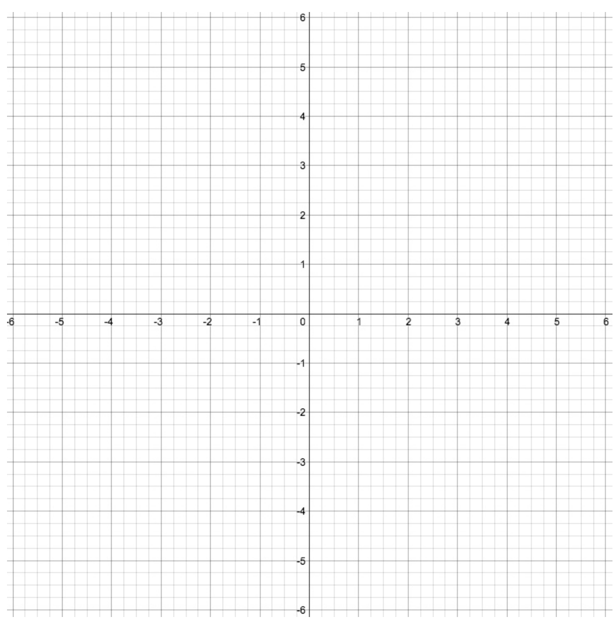
$$x + 1\,\overline{)\,x^3 \quad + \quad x^2 \quad - \quad x \quad - \quad 1}$$

(c) Fill in the blank with your result from (b), and then continue to finish completely factoring $p(x)$:

$$p(x) = x^3 + x^2 - x - 1 = (x + 1)(\underline{\qquad\qquad\qquad}) =$$

(d) What are the roots of $p(x)$?

(e) Sketch a complete graph of the function below (with the help of Desmos or a graphing calculator). Label the x-intercepts and the y-intercept on the graph with their coordinates.



(f) Use the graph to solve the following inequality: **circle the parts of your graph above corresponding to the solution set of the inequality**, and write down the solution set in interval notation:

$$x^3 + x^2 - x - 1 < 0$$

4. (10 points) Consider the rational function: $f(x) = \dfrac{5(x+4)(x-5)}{x^2-9}$

(a) What is the domain of f? Show your calculations, and write the solution in interval notation. (Hint: start by factoring the denominator as a difference of two squares.)

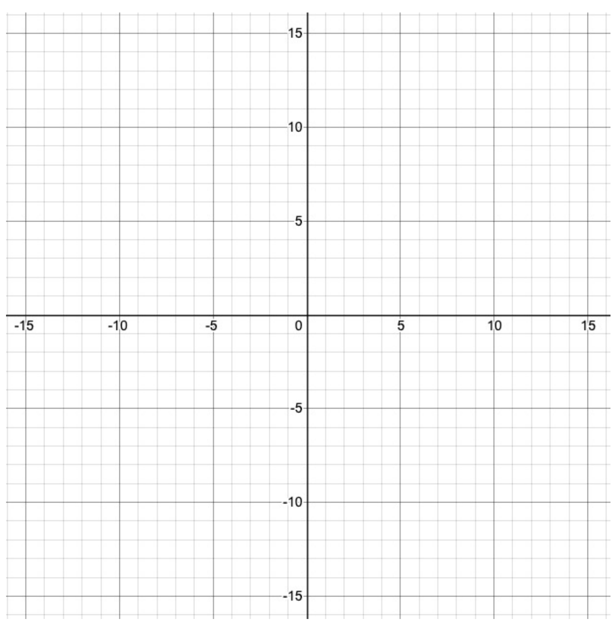
(b) What are the vertical asymptotes of this function?

(c) What is the horizontal asymptote of this function? Show your calculation/reasoning.

(d) Algebraically calculate for the the $x-$intercept(s) and $y-$intercept of the graph of $f(x)$. Again, show the necessary calculations, and write the coordinates of the intercepts in (x, y) form:

(e) Sketch a complete graph of the function below (with the help of Desmos or a graphing calculator):

- Label the x- and y-intercepts with their coordinates
- Draw the vertical asymptotes as dashed lines, and label each with its equation



(f) Use the graph to solve the following inequality: **again, circle the parts of your graph above corresponding to the solution set of the inequality**, and write down the solution set in interval notation.

$$\frac{5(x+4)(x-5)}{x^2-9} \geq 0$$