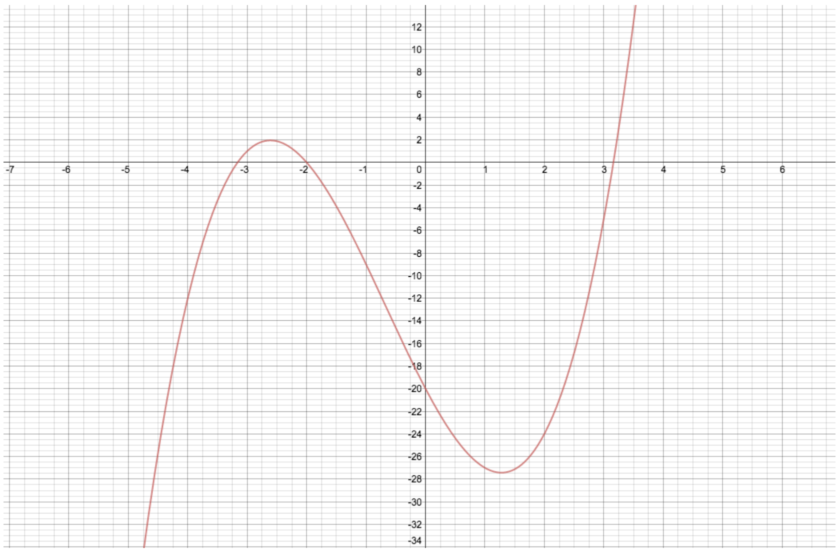
Mathematics 1375/OL79, Fall 2020 **Quiz #3 (take-home)**
Instructor: Suman Ganguli **Due Sunday, Nov 8** **Name:** ______________________

Shown below is the graph of the cubic polynomial $p(x) = x^3 + 2x^2 - 10x - 20$:



1. (2 points) From the graph, it *seems* that $x = -2$ is a root of $p(x)$. Verify that this is the case (i.e., show that $p(-2) = 0$).

> **Solution:** $p(-2) = (-2)^3 + 2(-2)^2 - 10(-2) - 20 = -8 + 8 + 20 - 20 = 0$

2. (6 points) Use the root $c = -2$ to factor the polynomial $p(x)$:

   (a) Since we know from #1 that $c = -2$ is a root of p, we know $(x - c) = (x + 2)$ is a factor of $p(x)$. Use long division to compute $\dfrac{p(x)}{x+2}$:

   $$x + 2 \overline{\big) \, x^3 \quad + \quad 2x^2 \quad - \quad 10x \quad - \quad 20}$$

   > **Solution:**
   >
   > $$\begin{array}{r|l} & x^2 \qquad\quad - 10 \\ \hline x+2 & x^3 + 2x^2 - 10x - 20 \\ & -x^3 - 2x^2 \\ \hline & \qquad\quad - 10x - 20 \\ & \qquad\quad\;\; 10x + 20 \\ \hline & \qquad\qquad\qquad\;\; 0 \end{array}$$

   (b) Fill in the blank with your result from (a) to get the factorization of $p(x)$:

   $$p(x) = x^3 + 2x^2 - 10x - 20 = (x + 2)(\underline{\qquad\qquad\qquad\qquad})$$

   > **Solution:**
   >
   > $$p(x) = x^3 + 2x^2 - 10x - 20 = (x + 2)(x^2 - 10)$$

3. (4 points) Use the factorization from #2(b) to *algebraically* solve for the other two roots of $p(x)$ in radical form (i.e., solve for the roots of the quadratic polynomial that results from factoring $x + 2$ out of $p(x)$). Leave your answers in radical form, i.e., in terms of square roots.

Solution: The roots of $p(x) = x^3 + 2x^2 - 10x - 20 = (x + 2)(x^2 - 10)$ occur when $x + 2 = 0$ or $x^2 - 10 = 0$.

The equation $x + 2 = 0$ yields the root $x = -2$, which was identified from the graph and verified as a root in part (a). We solve the equation $x^2 - 10 = 0$ in order to find the other two roots of $p(x)$. You can use the quadratic formula, but in this case (when $b = 0$, i.e., there's no x term) it's easier to just solve directly:

$$x^2 - 10 = 0 \Longleftrightarrow x^2 = 10 \Longleftrightarrow x = \pm\sqrt{10}$$

4. (4 points) (a) Write down the (x, y) coordinates of the 3 x-intercepts of the graph of $p(x)$, corresponding to the 3 roots:

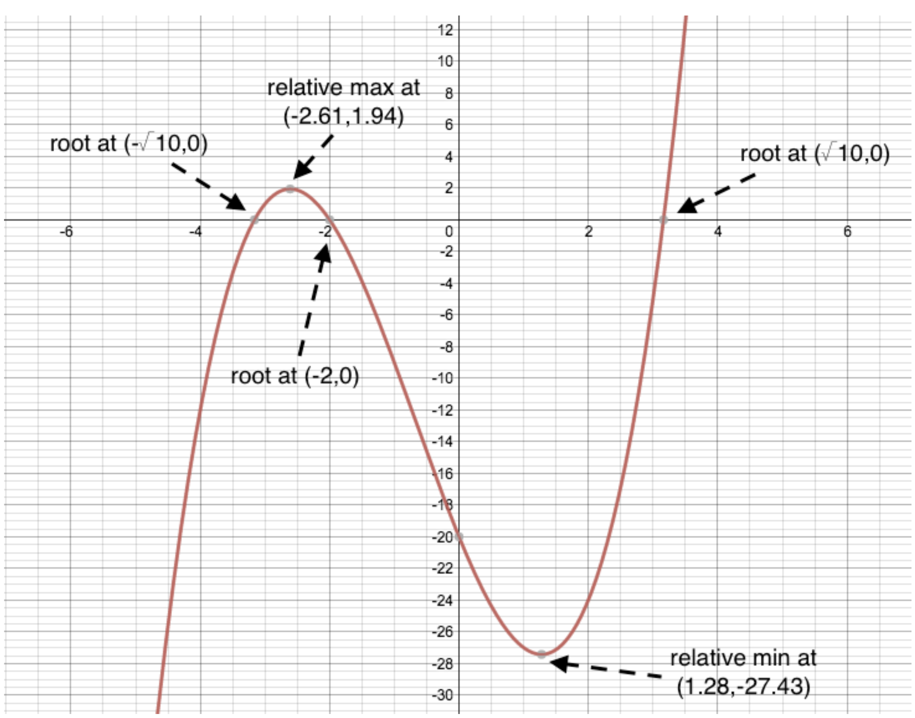
Solution:

$$(-\sqrt{10}, 0), (-2, 0), \sqrt{10}, 0)$$

(b) Algebraically calculate the y-intercept of the graph $y = p(x)$ and write down the coordinates of the y-intercept:

Solution: Since $p(0) = 0^3 + 2(0^2) - 10(0) - 20 = -20$, the y-intercept of the graph is at $(-20, 0)$.

5. (4 points) Label the x-intercepts and the y-intercept on the graph with their (x, y) coordinates (leave the x-coordinates corresponding to the 2 roots you found in #3 in radical form, i.e., in terms of square roots).



Extra credit (up to 3pts): Recreate the graph of $p(x)$ in Desmos, and then click on x-intercepts, the y-intercept, and also the local maximum and the local minimum (so that Desmos displays the coordinates of these 6 points).

Download or screenshot your graph to an image file, and submit with your quiz solutions on Blackboard.