$$f(x) = \begin{cases} \dfrac{x^2 - 3x - 4}{x^2 - 9x - 10}, & x \neq -1 \\ C, & x = -1 \end{cases}$$ $f(-1) = C$

What value of C would make $f(x)$ continuous at $x = -1$? $\frac{5}{11}$

- Decimal approximations are not allowed for this problem.
- Compute the exact value for C and express your answer algebraically.

$\lim_{x \to -1} f(x) = f(-1)$ $\quad$ f continuous at $x = -1$

$f(-1) = C$

close to $x=-1$
but $x \neq -1$

$$\lim_{x \to -1} \frac{x^2-3x-4}{x^2-9x-10} = \lim_{x \to -1} \frac{(x-4)\cancel{(x+1)}}{(x-10)\cancel{(x+1)}} = \lim_{x \to -1} \frac{x-4}{x-10} =$$

plug in $x=-1$

check: plug in $x=-1$ $\quad \dfrac{(-1)^2-3(-1)-4}{(-1)^2-9(-1)-10} = \dfrac{0}{0}$ COMMON FACTOR

$$\frac{-5}{-11} = \boxed{\frac{5}{11}} = C$$

you can also solve with H rule

$$\lim_{x \to -1} \frac{x^2-3x-4}{x^2-9x-10} \underset{\frac{0}{0}\ H}{=} \lim_{x \to -1} \frac{2x-3}{2x-9} = \frac{2(-1)-3}{2(-1)-9} = \frac{-5}{-11} = \frac{5}{11}$$

[redacted] A conical water tank with vertex down has a radius of 12 feet at the top and is 21 feet high. If water flows into the tank at a rate of 20 ft^3/min, how fast is the depth of the water increasing when the water is 16 feet deep?

↓ volume

The volume of a circular cone is

↓ At $h = 16$

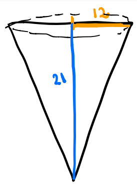
$$V = \frac{1}{3}\pi r^2 h,$$

where r if the radius of the base and h is the height of the cone.

The depth of the water is increasing at [ ] ft/min. → 0.0761581

preview answers

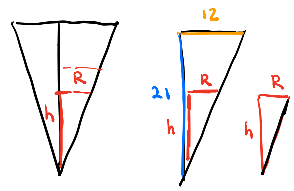
We want $\frac{dh}{dt}$



$\frac{dV}{dt} = 20$

$h = h(t) =$ depth

$R = R(t)$



SIMILAR TRIANGLES

$$h \quad \boxed{\frac{R}{h} = \frac{12}{21}} \quad h$$ FACT

$V = \frac{1}{3}\pi R^2 h$

We wish to write R as a function of h

$R = \frac{12}{21} h$

$R = \frac{4}{7} h$

$$V = \frac{1}{3}\pi \left(\frac{4}{7} h\right)^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{16}{49} h^2\right) h = \underbrace{\frac{1}{3}\pi\frac{16}{49}}_{\text{NUMBER}} h^3 = \underbrace{\frac{16\pi}{147}}_{\text{NUMBER}} h^3$$

$$\underbrace{\frac{dV}{dt}}_{= 20} = \frac{16\pi}{147}\underbrace{(3h^2)}_{h=16}\boxed{\frac{dh}{dt}}$$

need to find

$$20 = \underbrace{\frac{16\pi}{147} 3(16)^2}_{= 262.6115002} \frac{dh}{dt}$$

$$\frac{20}{(\quad)} = \frac{(262.6115002)}{(\quad)}\boxed{\frac{dh}{dt}}$$

$$\frac{dh}{dt} = \frac{20}{262.6115002} =$$

$$\boxed{0.0761581 \text{ ft/min}}$$

Answer

Part 1: Limit of a difference quotient

Suppose $f(x) = \dfrac{4}{x-3}$. Evaluate the limit by using algebra to simplify the difference quotient (in first answer box) and then evaluating the limit (in the second answer box).

$$\lim_{h\to 0}\left(\frac{f(6+h)-f(6)}{h}\right) = \lim_{h\to 0}\left(\square\right) = \square.$$

Part 2: Interpreting the limit of a difference quotient

The limit of the difference quotient, [redacted], from Part 1 above is (select all that apply).

- **A.** the instantaneous rate of change of f at $x = 6$.
- **B.** the slope of the secant line to the graph of $y = f(x)$ at $x = 6$.
- **C.** $f'(6)$
- **D.** $f(6)$.
- **E.** the slope of the tangent line to the graph of $y = f(x)$ at $x = 6$.
- **F.** the average rate of change of f at $x = 6$.

Note: *You can earn partial credit on this problem.*

[redacted]

[redacted] Find $\frac{dy}{dx}$ by implicit differentiation.

$$4 + 7x = \sin(xy^2) + y$$

Answer: $\frac{dy}{dx} =$ ☐

Find the equation of the tangent line to the curve at the point $(0, 4)$.

$y =$ ☐ $-9x+4$

Note: *You can earn partial credit on this problem.*

[redacted]

[redacted] Find $g'(4)$ given that $f(4) = -2$, $f'(4) = 9$, and $g(x) = \sqrt{x} f(x)$.

Answer: ☐ 17.5 (work at the end of the file)

Find $\frac{dy}{dx}$ $\quad (4+7x)' = (\sin(xy^2) + y)'$

$y' = \frac{dy}{dx}$

Derivative notation

$f'(x)$

$$7 = \cos(xy^2)\,(xy^2)' + y'$$

product rule

$$7 = \cos(xy^2)\,[(x)'y^2 + x\,(y^2)'] + y'$$

$(y^2)' = 2y\,y'$

because y is a function of x

$$7 = \cos(xy^2)\,[y^2 + x\,(2y\,y')] + y'$$

Solve for y' (algebra)

$$7 = \cos(xy^2)\,y^2 + \cos(xy^2)\,2xy\,y' + y'$$

MOVE

$$\frac{7 - \cos(xy^2)\,y^2}{\cos(xy^2)\,2xy + 1} = \frac{y'\,[\cos(xy^2)\,2xy + 1]}{\cos(xy^2)\,2xy + 1}$$

$$\frac{dy}{dx} = y' = \boxed{\frac{7 - \cos(xy^2)\,y^2}{2xy\cos(xy^2) + 1}}$$

slope: plug in $x=0$ $y=4$

$\cos(0 \cdot 16) = \cos(0) = 1$

Tangent line at $(0,4)$ $\quad$ slope $= \frac{7-16}{0+1} = -9$ $\quad$ $y = -9x + b$

$4 = -9(0) + b$ $\quad$ $b = 4$ $\quad$ $\boxed{y = -9x+4}$ tangent line

(a) Find the equation of the tangent line to $f(x) = \sqrt[3]{x}$ at $a = 125$.

$y =$ ☐

(b) Use your answer to part (a) to estimate the value of $\sqrt[3]{125.1}$.

$\sqrt[3]{125.1} \approx$ ☐

Suppose that $f(x) = x^3 - x^2 - 80x + 1$.

(A) List all the critical values of $f(x)$. Note: If there are no critical values, enter 'NONE'.

(B) Use interval notation to indicate where $f(x)$ is increasing.
Note: Use 'INF' for ∞, '-INF' for $-\infty$, and use 'U' for the union symbol.
Increasing:

(C) Use interval notation to indicate where $f(x)$ is decreasing.
Decreasing:

(D) List the x values of all local maxima of $f(x)$. If there are no local maxima, enter 'NONE'.
x values of local maximums =

(E) List the x values of all local minima of $f(x)$. If there are no local minima, enter 'NONE'.
x values of local minimums =

Apply L'Hôpital's Rule to evaluate the following limit. It may be necessary to apply it more than once.

$$\lim_{x \to e} \frac{e - x}{1 - e^{x-e}} = \square$$

[redacted]

Given that $f'(x) = 7\cos(x)$ and $f(\frac{3\pi}{2}) = 4$, find $f(x)$.

$f(x) = \square$

The top and bottom margins of a poster are 6 cm and the side margins are each 2 cm. If the area of printed material on the poster is fixed at 384 square centimeters, find the dimensions of the poster with the smallest area.

<u>Ex</u> $g(x) = \sqrt{x}\ f(x)$

Find $g'(4)$ $\quad f(4) = -2 \quad f'(4) = 9 \quad g(x) = \sqrt{x}\ f(x)$

$$g'(x) = (\sqrt{x})'\ f(x) + \sqrt{x}\ f'(x)$$

product rule

$$= \frac{1}{2\sqrt{x}} f(x) + \sqrt{x}\ f'(x)$$

$$g'(4) = \frac{1}{2\sqrt{4}} f(4) + \sqrt{4}\ f'(4) =$$

$$\frac{1}{4}(-2) + 2(9) = -\frac{1}{2} + 18 = -\frac{1}{2} + \frac{36}{2} = \frac{35}{2} = \boxed{17.5}$$