Differentiation Formula	Indefinite Integral
$\frac{d}{dx}(k) = 0$	* $\int k dx = \int kx^0 dx = kx + C$
$\frac{d}{dx}(x^n) = nx^{n-1}$	* $\int x^n dn = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$
$\frac{d}{dx}(\ln\|x\|) = \frac{1}{x}$	* $\int \frac{1}{x} dx = \ln\|x\| + C$
$\frac{d}{dx}(e^x) = e^x$	* $\int e^x dx = e^x + C$
$\frac{d}{dx}(\sin x) = \cos x$	* $\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	* $\int \sin x\, dx = -\cos x + C$
DERIVATIVE $\frac{d}{dx}(\tan x) = \sec^2 x$ ANTIDERIVATIVE	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x\, dx = -\csc x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$
DERIVATIVE $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ ANTIDERIVATIVE	$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1} x + C$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

This is Table 4.13 in the textbook

ANTIDERIVATIVES (INDEFINITE INTEGRALS)

Ex

$$\underset{F}{x^2} \xrightarrow{\text{DERIVATIVE}} 2x = f$$

$$2x \xleftarrow{\text{ANTIDERIVATIVE}} \text{ to } x^2$$

$2x$ is the derivative of x^2

x^2 is an antiderivative of $2x$

DEF A function F is an antiderivative of f if $F'(x) = f(x)$

$$F \xrightarrow{\text{derivative}} f$$

Ex Find an antiderivative of $f(x) = x^2$ Answer $\frac{x^3}{3}$

$$\boxed{\frac{x^3}{3}} \xrightarrow{\text{DERIVATIVE}} x^2$$

ANTIDERIVATIVE

$$\boxed{x^3} \xrightarrow{\text{DERIVATIVE}} 3x^2$$

x^3 is an antiderivative of $3x^2$

check $\left(\frac{x^3}{3}\right)' = \left(\frac{1}{3}x^3\right)' = \frac{1}{\cancel{3}}(\cancel{3}x^2) = x^2$

$\frac{1}{3}x^3$

DERIVATIVE

$$\frac{x^3}{3} + 1 \longrightarrow x^2$$

$\frac{x^3}{3} + 2$

$\vdots$

$\frac{x^3}{3} + 100$

$\vdots$

$\frac{x^3}{3}$ + ANY CONSTANT

If F is an antiderivative of f, then the most general antiderivative of f is $F(x)+C$ where C is an arbitrary constant.

Find the most general antiderivative of $f(x)=x^2$

Answer $F(x) = \frac{x^3}{3} + C$ (where C is a constant)

DEFINITION The set of all antiderivatives of $f(x)$ is the INDEFINITE INTEGRAL of f, denoted by $\int f(x)\,dx$

Ex Find $\int x^2\,dx = \frac{x^3}{3} + C$

start

to take all antiderivatives

an antiderivative

Ex Find $\int x^3\,dx = \frac{x^4}{4} + C$

all antiderivatives

an antiderivative

$$\boxed{\frac{x^4}{4}} \xrightarrow{\text{derivative}} x^3$$

$$\frac{x^4}{4} \xrightarrow{\text{derivative}} \frac{4x^3}{4}$$

Ex Find $\int x^4\,dx = \frac{x^5}{5} + C$

$$\int x\,dx = \frac{x^2}{2} + C$$

$$\shortparallel$$

$$\int x^1\,dx$$

POWER FORMULA $\boxed{\int x^N\,dx = \frac{x^{N+1}}{N+1} + C \quad N \neq -1}$

NOTE $N=-1$ we can not apply the formula

$$\cancel{\int x^{-1}\,dx = \frac{x^0}{-1+1} = \frac{1}{0} + C}$$

$$\int x^{-1}\,dx = \boxed{\int \frac{1}{x}\,dx = \ln|x| + C}$$

ONLY $N=-1$

$$\ln x \xrightarrow[\text{DERIVATIVE}]{} \frac{1}{x}$$

$K=2 \quad \int 2\,dx = 2x + C$

$K=31 \quad \int 31\,dx = 31x + C$

$k=1 \quad \int dx = \int 1\,dx = x + C$

$K=0 \quad \int 0\,dx = C$

$$\boxed{\int k\,dx = kx + C}$$

k is a constant

Ex $\int x^{100}\,dx = \frac{x^{101}}{101} + C \qquad N = 100$

* $\int \frac{1}{x^2}\,dx = \frac{x^{-2+1}}{-2+1} + C = \frac{x^{-1}}{-1} + C = \boxed{-\frac{1}{x} + C}$

$\frac{1}{x^2} = x^{-2} \qquad N=-2 \qquad$ power formula

* $\int \frac{1}{x^5}\,dx = \boxed{-\frac{1}{4x^4} + C}$

$\frac{1}{x^5} = x^{-5} \qquad N=-5$

$\int \frac{1}{x^5}\,dx = \frac{x^{-5+1}}{-5+1} + C = \frac{x^{-4}}{-4} + C$

$= -\frac{1}{4x^4} + C$

* $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$\sqrt{x} = x^{1/2} \qquad N = \frac{1}{2}$

radical form

$= \frac{x^{3/2}}{3/2} + C = \frac{2}{3}x^{3/2} + C = \boxed{\frac{2}{3}\sqrt{x^3} + C}$

$\frac{1}{3/2} = \frac{\frac{1}{1}}{3/2} = \frac{1}{1} * \frac{2}{3} = \frac{2}{3}$

$\boxed{\sqrt[M]{x^N} = x^{\frac{N}{M}}}$ POWER FUNCTION

$\sqrt[2]{x^3} = x^{3/2}$

Ex $$\int (4+x^2-5x^3)\,dx = 4x + \frac{x^3}{3} - 5\,\frac{x^4}{4} + C$$

integral of each term

DERIVATIVE

$$= \boxed{4x + \frac{x^3}{3} - \frac{5x^4}{4} + C}$$

Note: To check you can check

$$\left(4x + \frac{x^3}{3} - \frac{5x^4}{4} + C\right)' = 4 + x^2 - 5x^3$$

$$4 + \frac{1}{\cancel{3}}(\cancel{3}x^2) - \frac{5}{\cancel{4}}\cancel{4}x^3 + 0 = 4 + x^2 - 5x^3 \quad \checkmark$$

PROPERTIES

$$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int c\,f(x)\,dx = c \int f(x)\,dx$$

c constant

Ex $$\int 5x^3\,dx = 5\int x^3\,dx = 5\,\frac{x^4}{4} + C$$

Ex Find $f(x)$ described by the initial value problem

$$f'(x) = 4x^3 - 3x^2 \qquad f(-1) = 9$$

GIVEN

$$\text{Find } f \xrightarrow{\text{DERIVATIVE}} f'(x) = 4x^3 - 3x^2$$

$$f \xleftarrow{\text{ANTIDERIVATIVE}} f'(x)$$

$$f(x) = \int (4x^3 - 3x^2)\,dx = 4\,\frac{x^4}{4} - 3\,\frac{x^3}{3} + C =$$

$f(x) = x^4 - x^3 + C$ Use $f(-1) = 9$ to find the specific f

$$f(x) = x^4 - x^3 + C$$

$$9 = f(-1) = (-1)^4 - (-1)^3 + C = 1 + 1 + C$$

plug in $x = -1$

$$9 = 2 + C \qquad C = 7$$
$$-2 \quad\; -2$$

$$\boxed{f(x) = x^4 - x^3 + 7}$$

Ex (product)

$$\int (1-t)(2+t^2)\,dt \underset{\text{FOIL}}{=} \int 2 + t^2 - 2t - t^3 \,dt = 2t + \frac{t^3}{3} - 2\frac{t^2}{2} - \frac{t^4}{4} + C$$

NOTE $\int fg$ is NOT $\int f * \int g$

because $\cancel{(fg)' = f' * g'}$

$$= \boxed{2t + \frac{t^3}{3} - t^2 - \frac{t^4}{4} + C}$$

Ex (quotient)

$$\int \frac{x^5 - x^3}{x^4}\,dx \underset{\text{rewrite}}{=} \int x - \frac{1}{x}\,dx = \boxed{\frac{x^2}{2} - \ln|x| + C}$$

NOTE $\int \frac{f}{g}$ is NOT $\frac{\int f}{\int g}$

$\int \frac{x^5 - x^3}{x^4}$ is NOT $\frac{\int x^5 - x^3}{\int x^4}$

$$\frac{x^5 - x^3}{x^4} = \frac{x^5}{x^4} - \frac{x^3}{x^4}$$

$$= x - \frac{1}{x}$$

IMPORTANT When you have product or quotient you need to rewrite the function

INTEGRALS OF OTHER FUNCTIONS

$$\int e^x\,dx = e^x + C$$

$$\boxed{e^x} \underset{\text{antiderivative}}{\overset{\text{derivative}}{\rightleftarrows}} e^x$$

$$\int \cos x\,dx = \sin x + C$$

$$\boxed{\sin x} \underset{\text{antiderivative}}{\overset{\text{derivative}}{\rightleftarrows}} \cos x$$

$\int \sin x \, dx = -\cos x + C$

$\boxed{-\cos x} \xrightarrow{\text{derivative}} \sin x$

antiderivative

$\cos x \xrightarrow{\text{derivative}} -\sin x$

$(-\cos x)' = -(\cos x)' = -(-\sin x) = \sin x$

rewrite as power function

POWER RULE $N = -\frac{1}{2}$

Ex

(ww)

$\int \frac{du}{6\sqrt{u}} = \int \frac{1}{6} u^{-1/2} \, du =$

$\int \frac{1}{6\sqrt{u}} \, du \qquad \frac{1}{\sqrt{u}} = \frac{1}{u^{1/2}} = u^{-1/2}$

OPTIMIZATION (word problems) due Monday

It takes time!

$= \frac{1}{6} \frac{U^{-1/2+1}}{-\frac{1}{2}+1} + C = \frac{1}{6} \frac{U^{1/2}}{\frac{1}{2}} + C$

$\frac{1}{6} * \frac{2}{1} u^{1/2} + C = \boxed{\frac{1}{3}\sqrt{u} + C}$