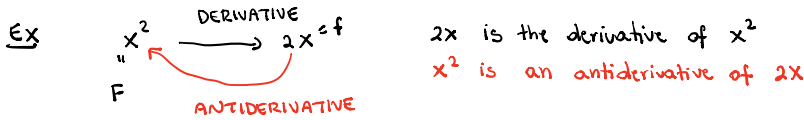


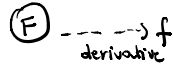
Differentiation Formula	Indefinite Integral
$\frac{d}{dx}(k) = 0$	$\int k dx = \int kx^0 dx = kx + C$
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$
$\frac{d}{dx}(\ln x ) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln x  + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x dx = -\csc x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + C$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x dx = -\cot x + C$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1} x + C$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

This is Table 4.13 in the textbook

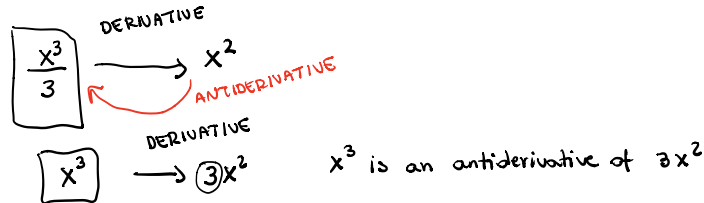
## ANTIDERIVATIVES (INDEFINITE INTEGRALS)



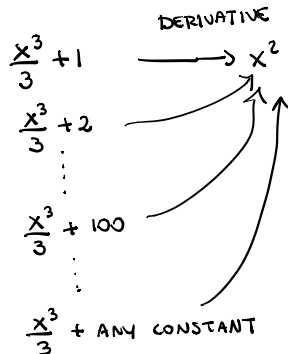
DEF A function  $F$  is an antiderivative of  $f$  if  $F'(x) = f(x)$



EX Find an antiderivative of  $f(x) = x^2$  Answer  $\frac{x^3}{3}$



check  $(\frac{x^3}{3})' = (\frac{1}{3} x^3)' = \frac{1}{3} (3x^2) = x^2$   
 $\frac{1}{3} x^3$



If  $F$  is an antiderivative of  $f$ , then the most general antiderivative of  $f$  is  $F(x) + C$  where  $C$  is an arbitrary constant.

Find the most general antiderivative of  $f(x) = x^2$

Answer  $F(x) = \frac{x^3}{3} + C$  (where  $C$  is a constant)

DEFINITION The set of all antiderivatives of  $f(x)$

is the INDEFINITE INTEGRAL of  $f$ ,

denoted by  $\int f(x) dx$

Ex Find  $\int x^2 dx = \frac{x^3}{3} + C$

*start* (circled  $x^2$ )  $\rightarrow$  *an antiderivative* (circled  $\frac{x^3}{3}$ )  $\rightarrow$  *to take all antiderivatives* (circled  $+ C$ )

Ex Find  $\int x^3 dx = \frac{x^4}{4} + C$

*an antiderivative* (circled  $\frac{x^4}{4}$ )  $\rightarrow$  *all antiderivatives* (circled  $+ C$ )

$\frac{x^4}{4}$   $\xrightarrow{\text{derivative}}$   $x^3$

$\frac{x^4}{4}$   $\xrightarrow{\text{derivative}}$   $\frac{4x^3}{4}$

Ex Find  $\int x^4 dx = \frac{x^5}{5} + C$

$\int x dx = \frac{x^2}{2} + C$

$\int x^1 dx$

POWER FORMULA

$$\int x^N dx = \frac{x^{N+1}}{N+1} + C \quad N \neq -1$$

NOTE  $N = -1$

we can not apply the formula

~~$\int x^{-1} dx = \frac{x^0}{-1+1} = \frac{1}{0} + C$~~

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$$

only  $N = -1$

$\ln x \xrightarrow{\text{DERIVATIVE}}$   $\frac{1}{x}$

$$k=2 \int 2 dx = 2x + C$$

$$k=31 \int 31 dx = 31x + C$$

$$k=1 \int dx = \int 1 dx = x + C$$

$$k=0 \int 0 dx = C$$

$$\int k dx = kx + C$$

k is a constant

Ex  $\int x^{100} dx = \frac{x^{101}}{101} + C \quad N=100$

$$* \int \frac{1}{x^2} dx = \frac{x^{-2+1}}{-2+1} + C = \frac{x^{-1}}{-1} + C = \boxed{-\frac{1}{x} + C}$$

$$\frac{1}{x^2} = x^{-2} \quad N=-2 \quad \text{power formula}$$

$$* \int \frac{1}{x^5} dx = \boxed{-\frac{1}{4x^4} + C}$$

$$\frac{1}{x^5} = x^{-5} \quad N=-5$$

$$\int \frac{1}{x^5} dx = \frac{x^{-5+1}}{-5+1} + C = \frac{x^{-4}}{-4} + C$$

$$= -\frac{1}{4x^4} + C$$

$$* \int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$\sqrt{x} = x^{1/2} \quad N=\frac{1}{2}$$

$$= \frac{x^{3/2}}{3/2} + C = \boxed{\frac{2}{3} x^{3/2} + C} = \boxed{\frac{2}{3} \sqrt{x^3} + C}$$

radical form

$$\frac{1}{3/2} = \frac{1}{3/2} = \frac{1}{1} \times \frac{2}{3} = \frac{2}{3}$$

$$\sqrt[N]{x^N} = x^{\frac{N}{N}}$$

$$\sqrt{x^3} = x^{3/2}$$

POWER FUNCTION

EX  $\int (4 + x^2 - 5x^3) dx = 4x + \frac{x^3}{3} - 5\frac{x^4}{4} + c$

$\uparrow$  DERIVATIVE       $\downarrow$  integral of each term  
 $\boxed{4x + \frac{x^3}{3} - \frac{5x^4}{4} + c}$

Note: To check you can check

$$\left(4x + \frac{x^3}{3} - \frac{5x^4}{4} + c\right)' = 4 + x^2 - 5x^3$$

$$4 + \frac{1}{3}(3x^2) - \frac{5}{4}4x^3 + 0 = 4 + x^2 - 5x^3 \quad \checkmark$$

PROPERTIES  $\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$

$$\int c f(x) dx = c \int f(x) dx$$

$c$  constant

Ex  $\int 5x^3 dx = 5 \int x^3 dx = 5 \frac{x^4}{4} + c$

EX Find  $f(x)$  described by the initial value problem

$f'(x) = 4x^3 - 3x^2$        $f(-1) = 9$

Find  $f$   $\xrightarrow{\text{DERIVATIVE GIVEN}}$   $f'(x) = 4x^3 - 3x^2$

$\xleftarrow{\text{ANTIDERIVATIVE}}$

$$f(x) = \int (4x^3 - 3x^2) dx = 4\frac{x^4}{4} - 3\frac{x^3}{3} + c =$$

$f(x) = x^4 - x^3 + c$       Use  $f(-1) = 9$  to find the specific  $f$

$$f(x) = x^4 - x^3 + c$$

$$9 = f(-1) = (-1)^4 - (-1)^3 + c = 1 + 1 + c$$

$\downarrow$  plug in  $x = -1$

$$9 = 2 + c \quad c = 7$$

$-2 \quad -2$

$$\boxed{f(x) = x^4 - x^3 + 7}$$

Ex  
(product)  $\int (1-t)(2+t^2) dt = \int 2 + t^2 - 2t - t^3 dt =$

FoIL

**NOTE**  $\int fg$  is NOT  $\int f * \int g$   
because  $(fg)' \neq f' * g'$

$$2t + \frac{t^3}{3} - 2\frac{t^2}{2} - \frac{t^4}{4} + C$$

$$= 2t + \frac{t^3}{3} - t^2 - \frac{t^4}{4} + C$$

Ex  
(quotient)  $\int \frac{x^5 - x^3}{x^4} dx = \int x - \frac{1}{x} dx = \frac{x^2}{2} - \ln|x| + C$

rewrite

**NOTE**  $\int \frac{f}{g}$  is NOT  $\frac{\int f}{\int g}$

$\int \frac{x^5 - x^3}{x^4}$  is NOT  $\frac{\int x^5 - x^3}{\int x^4}$

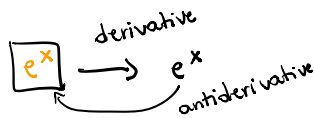
$$\frac{x^5 - x^3}{x^4} = \frac{x^5}{x^4} - \frac{x^3}{x^4}$$

$$= x - \frac{1}{x}$$

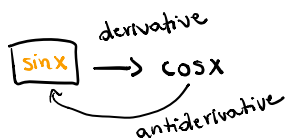
IMPORTANT When you have product or quotient you need to rewrite the function

### INTEGRALS OF OTHER FUNCTIONS

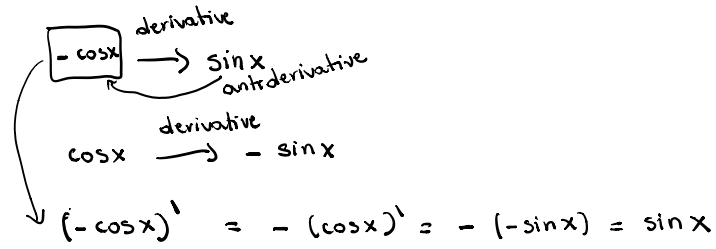
$$\int e^x dx = e^x + C$$



$$\int \cos x dx = \sin x + C$$



$$\int \sin x \, dx = -\cos x + C$$



Ex  
 (www)

$$\int \frac{du}{6\sqrt{u}} \stackrel{\substack{\text{rewrite as} \\ \text{power function}}}{=} \int \frac{1}{6} u^{-1/2} du \stackrel{\text{POWER RULE } N = -\frac{1}{2}}{=} \int \frac{1}{6\sqrt{u}} du$$

$$\frac{1}{\sqrt{u}} = \frac{1}{u^{1/2}} = u^{-1/2}$$

OPTIMIZATION (word problems) due Monday  
 It takes time!

$$= \frac{1}{6} \frac{u^{-1/2+1}}{-\frac{1}{2}+1} + C = \frac{1}{6} \frac{u^{1/2}}{\frac{1}{2}} + C$$

$$\frac{1}{6} * \frac{2}{1} u^{1/2} + C = \boxed{\frac{1}{3} \sqrt{u} + C}$$