Review RATIONAL FUNCTIONS

$f(x) = \frac{2x^2}{x^2+25}$ $\quad x^2+25=0$, $x^2=-25$ never ↑ NO V.A.

domain $(-\infty, \infty)$

Ex $f(x) = \frac{3x+9}{x-3}$ $\quad$ domain → $x-3 \neq 0$, $x \neq 3$ → $(-\infty, 3) \cup (3, \infty)$

A) Critical values

$$f'(x) = \frac{(3x+9)'(x-3) - (3x+9)(x-3)'}{(x-3)^2} =$$

$$\frac{3(x-3) - (3x+9)(1)}{(x-3)^2} = \frac{3x-9-3x-9}{(x-3)^2}$$

$$= -\frac{18}{(x-3)^2} = \frac{-18}{(x-3)^2}$$

$f'(x) = 0$ $\quad$ numerator $= 0$ $\quad -18 = 0$ NEVER

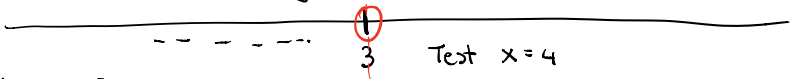
There are no critical points
no local max
no local min

Sign chart for $f'(x) = \frac{-18}{(x-3)^2}$

(solving a rational inequality → Precalculus)

plot zeros of numerator AND zeros of denominator

NONE $\quad x-3=0 \quad x=3$



3 $\quad$ Test $x=4$

Test $x=0$

$f'(0) = \frac{-18}{(0-3)^2} = \frac{-18}{9} = -2 < 0$

$f'(4) = \frac{-18}{(4-3)^2} = \frac{-18}{1^2} = -18 < 0$

Increasing NONE

Decreasing ~~$(-\infty, \infty)$~~ $(-\infty, 3) \cup (3, \infty)$

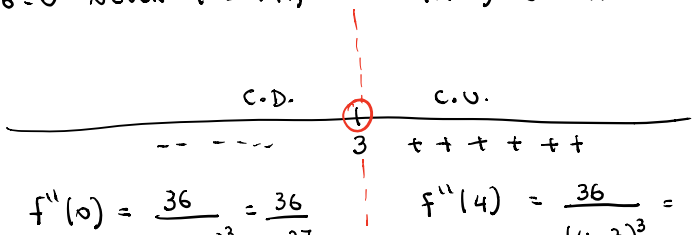
because $x = 3$ is not in domain

Concavity | Inflection points $f''(x)$ sign chart

$$f'(x) = \frac{-18}{(x-3)^2} = -18(x-3)^{-2}$$

$$f''(x) = -18(-2)(x-3)^{-3} = \frac{36}{(x-3)^3}$$

$36 = 0$ NEVER (no I.P.) $(x-3)^3 = 0$ $x = 3$



$$f''(0) = \frac{36}{(0-3)^3} = \frac{36}{-27} < 0$$

$$f''(4) = \frac{36}{(4-3)^3} = \frac{36}{1} > 0$$

Concave up $(3, \infty)$

Concave down $(-\infty, 3)$

Asymptotes $f(x) = \frac{3x+9}{x-3}$

V.A. $x - 3 = 0$ $x = 3$ potential V.A.

$f(x)$ can not be simplified, so $x = 3$ is V.A.

$$\lim_{x \to 3} \frac{3x+9}{x-3} = \frac{9+9}{3-3} = \frac{18}{0}$$ infinite limit

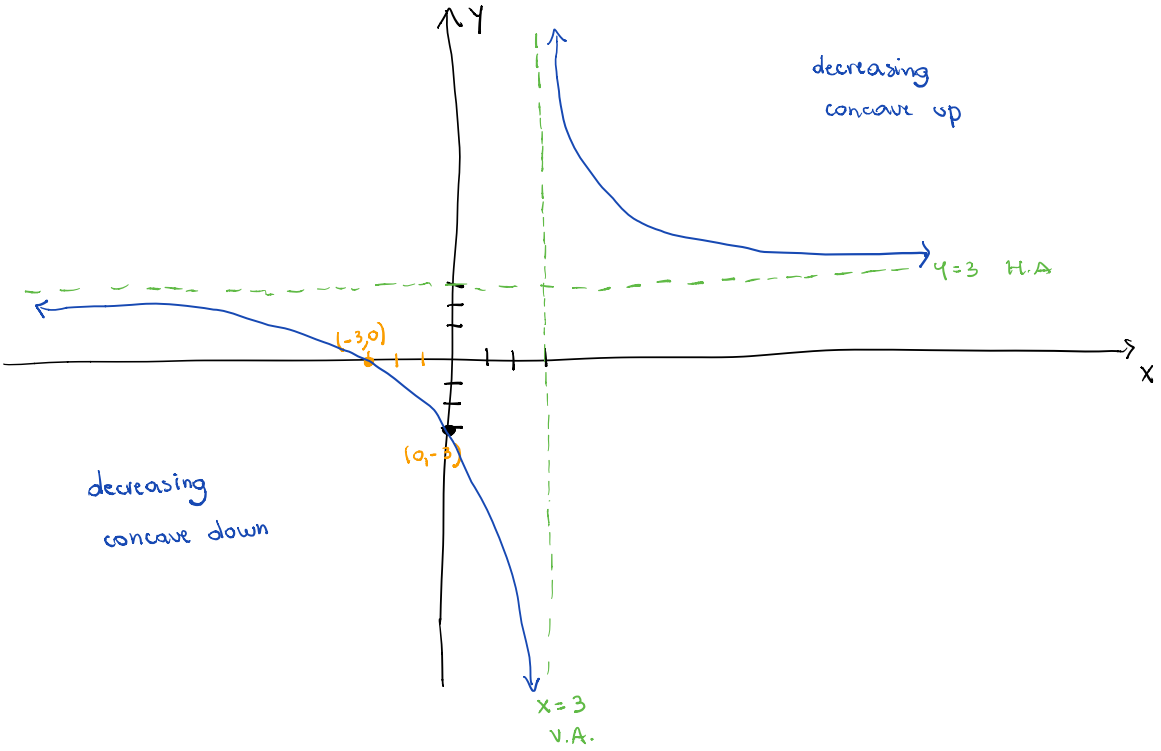
so $x = 3$ is V.A.

vertical line

H.A. $\lim_{x \to \pm\infty} \frac{3x+9}{x-3} \overset{H}{=} \lim_{x \to \pm\infty} \frac{3}{1} =$

$\lim_{x \to \pm\infty} 3 = 3$ $\quad y = 3$ is H.A.

horizontal line



$f(x) = \frac{3x+9}{x-3}$ $\qquad y = \frac{3x+9}{x-3}$

y-int $\quad x = 0 \quad f(0) = \frac{9}{-3} = -3 \quad (0,-3)$

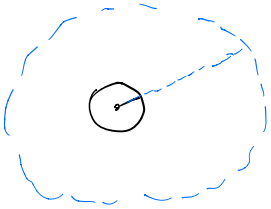
x-int $\quad y = 0 \quad$ numerator $= 0 \quad 3x+9 = 0$

$\frac{3x}{3} = \frac{-9}{3} \quad x = -3 \quad (-3,0)$

Ex (Related Rates)

The radius of a circle is increasing at a rate of 2 METERS/MINUTE (DERIVATIVE; ↓ time)

How fast is the area of the circle increasing when the radius is 22 METERS?



$R(t)$ $A(t)$

$$\frac{dR}{dt} = 2$$

we want $\frac{dA}{dt}$

① $A = \pi R^2$ relation between A and R

② $\frac{dA}{dt} = \pi (2R) \frac{dR}{dt}$ related RATES (relation between derivatives)

③ plug in $R = 22$, $\frac{dR}{dt} = 2$

$$\frac{dA}{dt} = \pi (44) 2 = \boxed{88\pi} \text{ METERS}^2/\text{MINUTE}$$

exact