1. $f(x) = x^3 + 12x^2 - 27x + 11$

critical points $f'(x) = 3x^2 + 24x - 27 = 0$

$3(x^2 + 8x - 9) = 0$

$3(x+9)(x-1) = 0$ $\quad x = 1$

$x = -9$

a) In $[-10, 0]$ we have the critical point $x = -9$. We need to compare $f(-9), f(-10), f(0)$

| $x$ | $f(x)$ |
|---|---|
| $-9$ | $f(-9) = (-9)^3 + 12(-9)^2 - 27(-9) + 11 = 497$ |
| $-10$ | $f(-10) = (-10)^3 + 12(-10)^2 - 27(-10) + 11 = -1000 + 1200 + 270 + 11 = 481$ |
| $0$ | $f(0) = 11$ |

Absolute minimum $= \boxed{11}$

Absolute maximum $= \boxed{497}$

b) In $[-7, 2]$ we have the critical point $x = 1$. We need to compare $f(-7), f(2), f(1)$

| $x$ | $f(x)$ |
|---|---|
| $1$ | $f(1) = 1 + 12 - 27 + 11 = -3$ |
| $-7$ | $f(-7) = (-7)^3 + 12(-7)^2 - 27(-7) + 11 = 445$ |
| $2$ | $f(2) = 2^3 + 12(2^2) - 27(2) + 11 = 8 + 48 - 54 + 11 = 13$ |

Absolute minimum $= \boxed{-3}$

Absolute maximum $= \boxed{445}$

c) In $[-10, 2]$ we have both critical points. We compare $f(1), f(-9), f(-10), f(2)$

$f(1) = -3$, $f(-9) = 497$, $f(-10) = 481$, $f(2) = 13$

(All these were computed above)

Absolute minimum $= \boxed{-3}$

Absolute maximum $= \boxed{497}$

2. $y = e^{x/4}$ $\quad dy = f'(x)\,dx$

$f(x) = e^{x/4}$ $\quad f'(x) = e^{x/4}\left(\frac{1}{4}\right) = \frac{1}{4}e^{x/4}$ $\quad$ [ $\frac{1}{4}$ is the derivative of the exponent $\frac{x}{4}$ ]

$dy = \frac{1}{4}e^{x/4}\,dx$

when $x = 4$ and $dx = 0.5$ $\quad dy = \frac{1}{4}e^{4/4}(0.5) = \boxed{.125\,e} = .3397852286$

exact answer

when $x = 4$ and $dx = 0.02$ $\quad dy = \frac{1}{4}e^{4/4}(0.02) = \boxed{.005\,e} = .0135914091$

3. $f(x) = x^2+4x \qquad x_0 = 3 \qquad dx = 0.01$

a) $\Delta f = f(x_0+dx) - f(x_0) = f(3+0.01) - f(3) = f(3.01) - f(3) = 21.1001 - 21 = \boxed{.1001}$

$f(3.01) = (3.01)^2 + 4(3.01) = 21.1001$

$f(3) = 9+12 = 21$

b) $df = f'(3)\,dx \qquad f'(x) = 2x+4$

$f'(3) = 6+4 = 10$

$df = 10(.01) = \boxed{.1}$

c) $.1001 - .1 = \boxed{.001}$

4. $f(x) = \frac{1}{x}$ in $[4,7]$

A) $\dfrac{f(7)-f(4)}{7-4} = \dfrac{\frac{1}{7}-\frac{1}{4}}{3} = \dfrac{\frac{4-7}{28}}{\frac{3}{1}} = -\frac{3}{28}\cdot\frac{1}{3} = \boxed{-\frac{1}{28}}$

B) $f(x) = x^{-1} \qquad f'(x) = -\frac{1}{x^2}$

$-\frac{1}{x^2} = -\frac{1}{28} \qquad -x^2 = -28 \qquad x^2 = 28 \qquad x = \pm\sqrt{28}$

in $[4,7]$ we only have $c = \boxed{\sqrt{28}}$

5. $\lim\limits_{x\to 0} \dfrac{x+\tan x}{-10\sin x} = \lim\limits_{x\to 0} \dfrac{1+\sec^2 x}{-10\cos x} = \dfrac{1+\sec^2(0)}{-10\cos(0)} = \dfrac{1+1}{-10} = \dfrac{2}{-10} = \boxed{-\frac{1}{5}}$

$\frac{0}{0}$ we apply H rule; plug in $x=0$

[Note $\sec(x) = \frac{1}{\cos(x)}$ so $\sec(0) = \frac{1}{\cos(0)} = \frac{1}{1} = 1$]

6. $\lim\limits_{x\to\infty} \left(\dfrac{14x^3+5x^2}{12x^3-11}\right) = \lim\limits_{x\to\infty} \dfrac{42x^2+10x}{36x^2} = \lim\limits_{x\to\infty} \dfrac{84x+10}{72x} =$

$\frac{\infty}{\infty}$ we apply H rule; $\frac{\infty}{\infty}$ H again; $\frac{\infty}{\infty}$ H again

$\lim\limits_{x\to\infty} \frac{84}{72} = \frac{84}{72} = \boxed{\frac{7}{6}}$

Note: correct limit notation is very important

7. The radius of a spherical balloon is increasing at a rate of 2 cm/min. How fast is the surface area changing when the radius is 12 cm?

$S = 4\pi R^2 \qquad S = S(t) \qquad R = R(t)$

$\frac{dS}{dt} = 4\pi(2R)\frac{dR}{dt} = 8\pi R \frac{dR}{dt}$ these are the related rates

plug in $dR/dt = 2 \quad R = 12$

Note: correct notation is very important here!

$\frac{dS}{dt} = 8\pi(12)(2) = \boxed{192\pi}\ \text{cm}^2/\text{min}$

Note: Just enter number without units

8. $f(x) = \frac{2x^2}{x^2+25}$ domain is $(-\infty, \infty)$ because the denominator is never zero

(A) $f'(x) = 0$ to find critical points

$f'(x) = \frac{(2x^2)'(x^2+25) - 2x^2(x^2+25)'}{(x^2+25)^2} = \frac{(4x)(x^2+25) - 2x^2(2x)}{(x^2+25)^2}$

quotient rule

$= \frac{\cancel{4x^3} + 100x - \cancel{4x^3}}{(x^2+25)^2} = \frac{100x}{(x^2+25)^2}$

$100x = 0$ critical point $x = \boxed{0}$

For B-C-D-E we use a $\boxed{\text{sign chart for } f'(x)}$

f decreasing ↘ 0 ↗ f increasing

$- - - -$ | $+ + + +$

$f'(-1) = \frac{-100}{(26)^2} < 0 \qquad f'(1) = \frac{100}{(26)^2} > 0$

Note: Make sure you show which point you are testing

(B) f is increasing in $\boxed{(0,\infty)}$

(C) f is decreasing in $\boxed{(-\infty,0)}$

(D) There is no local max $\boxed{\text{NONE}}$

(E) $x = \boxed{0}$ f changes from decreasing to increasing at $x=0$ so there is a local min at $x=0$

For F-G-H we use a sign chart for $f''(x)$

$$f'(x) = \frac{100x}{(x^2+25)^2}$$

$$\underset{\substack{\downarrow \\ \text{quotient} \\ \text{rule}}}{f''(x)} = \frac{(100x)'(x^2+25) - 100x\left((x^2+25)^2\right)'}{(x^2+25)^4} =$$

$$\frac{100(x^2+25)^2 - 100x\left(2(x^2+25)(2x)\right)}{(x^2+25)^4} = \frac{100(x^2+25)^2 - 400x^2(x^2+25)}{(x^2+25)^4} =$$

factor out $100(x^2+25)$

$$\frac{100(x^2+25)\left[x^2+25-4x^2\right]}{(x^2+25)^4} = \frac{100(25-3x^2)}{(x^2+25)^3}$$

$100(25-3x^2)=0 \qquad (25-3x^2)=0 \qquad 25 = 3x^2 \qquad x^2 = \frac{25}{3} \qquad x = \pm\sqrt{\frac{25}{3}} = \pm\frac{5}{\sqrt{3}} \approx \pm 2.9$

| C.D | | C.U. | | C.D. |
|---|---|---|---|---|
| - - - - - | $-\frac{5}{\sqrt{3}}$ | + + + + + | $\frac{5}{\sqrt{3}}$ | - - - - - - - |
| $f''(-4) = \frac{100(25-48)}{(16+25)^3} < 0$ | | $f''(0) = \frac{100(25)}{25^3} > 0$ | | $f''(4) = \frac{100(25-48)}{(16+25)^3} < 0$ |

Note: Make sure you show which points you are testing

F) concave up in $\boxed{\left(-\frac{5}{\sqrt{3}}, \frac{5}{\sqrt{3}}\right)}$

G) concave down in $\boxed{\left(-\infty, -\frac{5}{\sqrt{3}}\right) \cup \left(\frac{5}{\sqrt{3}}, \infty\right)}$

H) Since $f$ changes concavity at $x = -\frac{5}{\sqrt{3}}$ and $x = \frac{5}{\sqrt{3}}$ there are inflection points at $x = -\frac{5}{\sqrt{3}}$ and $x = \frac{5}{\sqrt{3}}$ $\boxed{-\frac{5}{\sqrt{3}}, \frac{5}{\sqrt{3}}}$

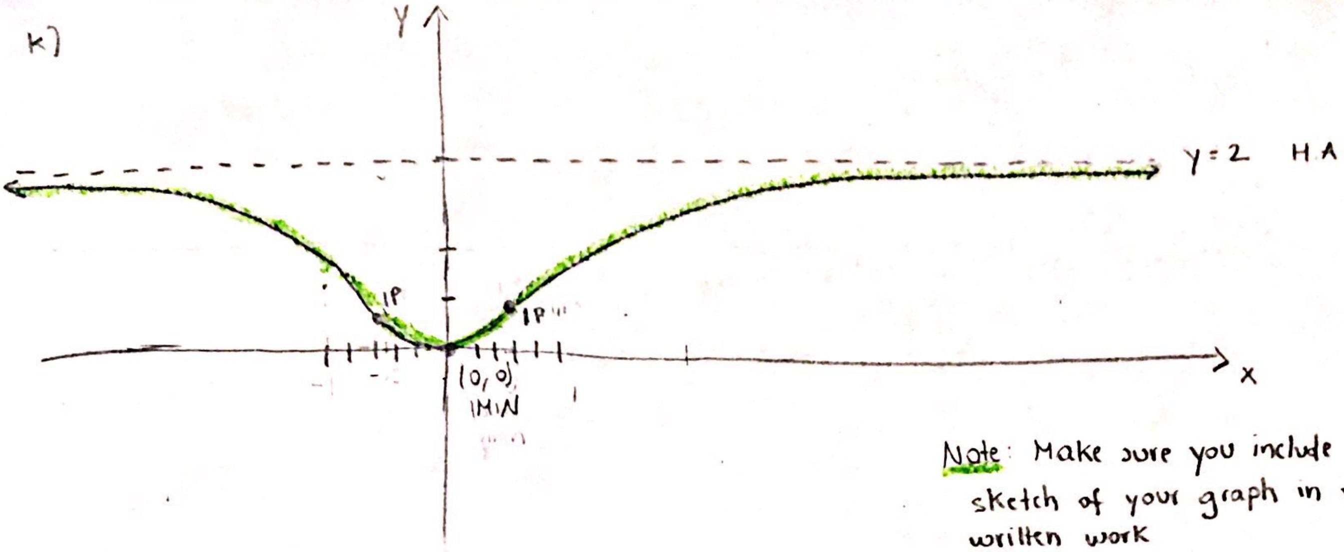
I) $$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{2x^2}{x^2+25} \underset{\substack{\frac{\infty}{\infty} \\ \text{H rule}}}{=} \lim_{x\to\pm\infty} \frac{4x}{2x} \underset{\substack{\downarrow \\ \text{simplify}}}{=} \lim_{x\to\pm\infty} 2 = 2$$

$y = \boxed{2}$ is H.A.

J) Since $x^2+25=0$ has no solution there are no vertical asymptotes

$\boxed{\text{NONE}}$

k)




**Note:** Make sure you include a sketch of your graph in your written work

(9) $f(x) = x^4 - 3x^3$

A) $f'(x) = 4x^3 - 9x^2 = 0$

$x^2(4x - 9) = 0 \quad x = 0 \quad x = 9/4$ are critical points $\boxed{0, 9/4}$

For B - C - D - E we use a sign chart for $f'(x)$

↘ | ↘ | ↗

$f'(x)$: - - - - - - 0 - - - - $9/4 = 2.25$ + + + +

$f'(-1) = 4(-1)^3 - 9(-1)^2 = -4 - 9 = -13 < 0$

$f'(1) = 4 - 9 = -5 < 0$

$f'(3) = 4(27) - 9(9) = 108 - 81 = 27 > 0$

**Note:** Make sure you show which points you are testing

B) f is increasing in $\boxed{(9/4, \infty)}$

C) f is decreasing in $\boxed{(-\infty, 9/4)}$

D) Local max $\boxed{\text{NONE}}$

E) Local min at $x = \boxed{9/4}$ since the function changes from decreasing to increasing at $x = 9/4$

$f''(x) = 12x^2 - 18x = 0 \qquad x(12x - 18) = 0 \qquad x = 0$

$x = \frac{18}{12} = \frac{3}{2} = 1.5$

Sign chart for $f''(x)$

C.U | C.D | C.U

+ + + + 0 - - - - 1.5 + + +

$f''(-1) = 12(-1)^2 - 18(-1) = 12 + 18 = 30 > 0$

$f''(1) = 12 - 18 = -6 < 0$

$f''(2) = 12(4) - 18(2) = 48 - 36 = 12 > 0$

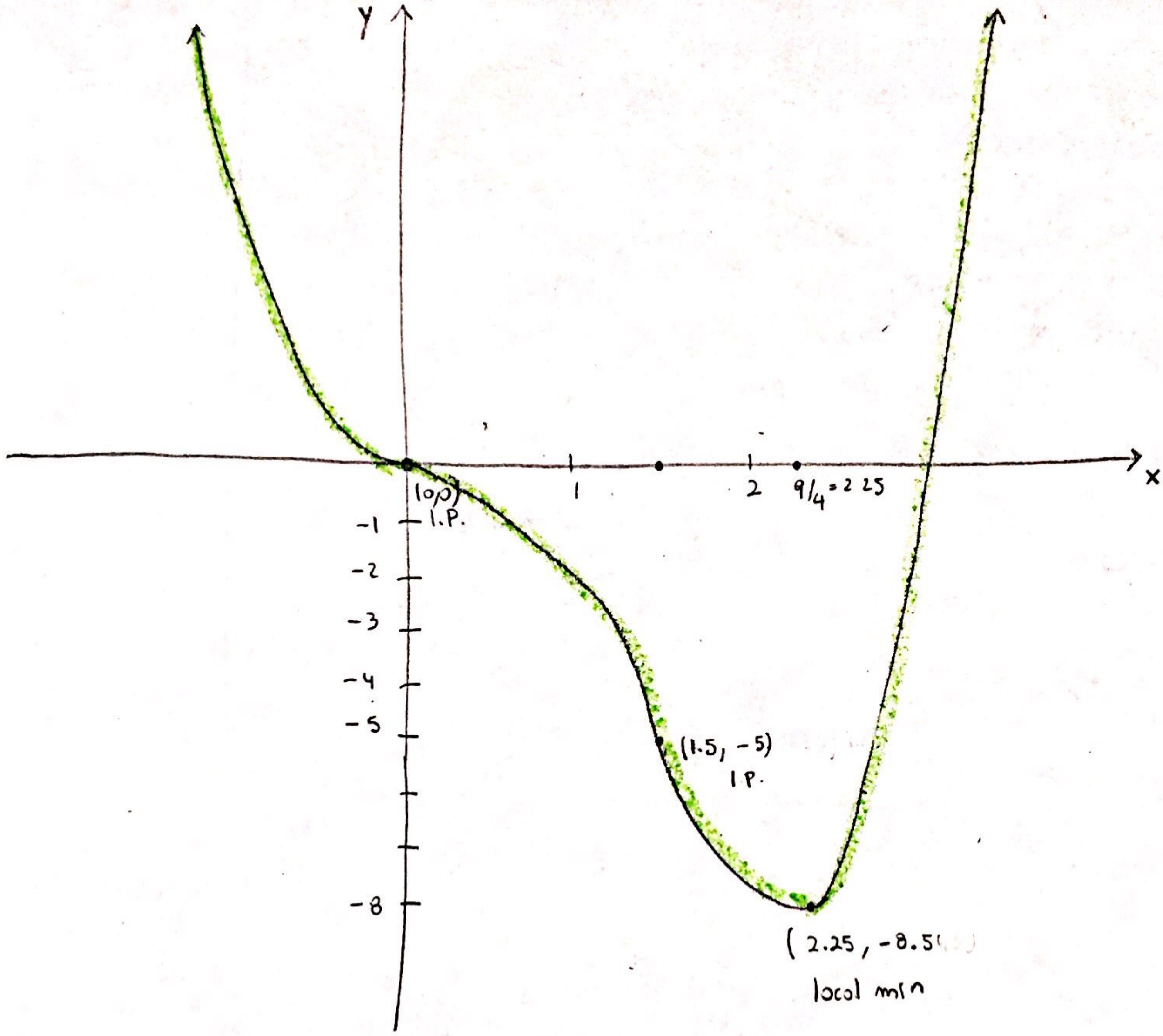
**Note:** Make sure you show which points you are testing

F) Concave up in $\boxed{(-\infty, 0) \cup (1.5, \infty)}$

G) Concave down in $\boxed{(0, 1.5)}$

H) Since f changes concavity at $x = 0$, $x = 1.5$ there are inflection points at $x = 0$, $x = 1.5$

$\boxed{0, 1.5}$




$f(1.5) \approx -5$

$f(2.25) \approx -8.5$

Note Make sure you include a sketch of your graph in your written work