

**Problem 1.**

1. (10 points) Library/Union/setDerVOptimization/4-5-23.pg

Find the absolute maximum and absolute minimum values of the function

$$f(x) = x^3 + 12x^2 - 27x + 11$$

over each of the indicated intervals.

(a) Interval =  $[-10, 0]$ .

1. Absolute maximum = \_\_\_\_\_
2. Absolute minimum = \_\_\_\_\_

(b) Interval =  $[-7, 2]$ .

1. Absolute maximum = \_\_\_\_\_
2. Absolute minimum = \_\_\_\_\_

(c) Interval =  $[-10, 2]$ .

1. Absolute maximum = \_\_\_\_\_
2. Absolute minimum = \_\_\_\_\_

Correct Answers:

- 497
- 11
- 445
- -3
- 497
- -3

**Problem 2.**

2. (10 points) Library/UVA-Stew5e/setUVA-Stew5e-C03S11-LinAppr

ox/3-11-22.pg

Let  $y = e^{x/4}$ .

Find the differential  $dy$  when  $x = 4$  and  $dx = 0.5$  \_\_\_\_\_

Find the differential  $dy$  when  $x = 4$  and  $dx = 0.02$  \_\_\_\_\_

Correct Answers:

- 0.339785228557381
- 0.0135914091422952

**Problem 3.**

3. (10 points) Library/Hope/Calc1/03-11-Differentials/Differen

tials-04/Differentials-04.pg

The figure shows how a function  $f(x)$  and its linear approximation (i.e., its tangent line) change value when  $x$  changes from  $x_0$  to  $x_0 + dx$ .

Suppose  $f(x) = x^2 + 4x$ ,  $x_0 = 3$  and  $dx = 0.01$ . Your answers below need to be very precise, so use many decimal places.

(a) Find the change  $\Delta f = f(x_0 + dx) - f(x_0)$ .

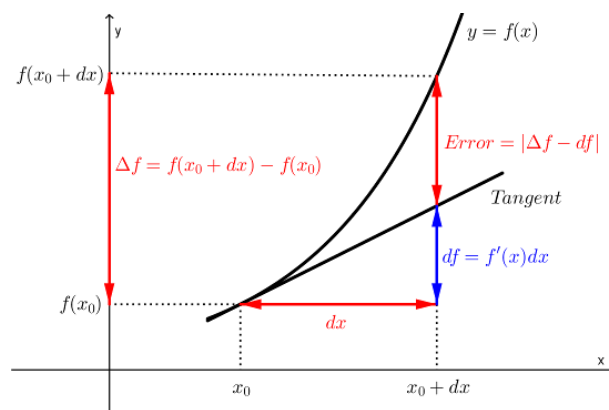
$\Delta f =$  \_\_\_\_\_

(b) Find the estimate (i.e., the differential)  $df = f'(x_0) dx$ .

$df =$  \_\_\_\_\_

(c) Find the approximation error  $|\Delta f - df|$ .

Error = \_\_\_\_\_



(Click on graph to enlarge)

Correct Answers:

- 0.1001
- 0.1
- 0.0001

**Problem 4.**

4. (10 points) Library/UVA-Stew5e/setUVA-Stew5e-C04S02-MeanVal

Thm/4-2-13a.pg

Consider the function  $f(x) = \frac{1}{x}$  on the interval  $[4, 7]$ .

(A) Find the average or mean slope of the function on this interval.

Average Slope = \_\_\_\_\_

(B) By the Mean Value Theorem, we know there exists a  $c$  in the open interval  $(4, 7)$  such that  $f'(c)$  is equal to this mean slope. Find all values of  $c$  that work and list them (separated by commas) in the box below.

List of values: \_\_\_\_\_

Correct Answers:

- -0.0357142857142857
- 5.29150262212918

**Problem 5.**

5. (5 points) Library/UCSB/Stewart5\_4\_4/Stewart5\_4\_4\_10.pg

Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist.

$$\lim_{x \rightarrow 0} \frac{x + \tan x}{-10 \sin x} = \underline{\hspace{2cm}}$$

Correct Answers:

- -0.2

**Problem 6.**

6. (5 points) Library/WHFreeman/Rogawski\_Calculus\_Early\_Transcendentals\_Second\_Edition/4\_Applications\_of\_the\_Derivative/4.5\_LHopitals\_Rule/4.5.22.pg

Use L'Hôpital's Rule (possibly more than once) to evaluate the following limit

$$\lim_{x \rightarrow \infty} \left( \frac{14x^3 + 5x^2}{12x^3 - 11} \right) = \underline{\hspace{2cm}}$$

If the answer equals  $\infty$  or  $-\infty$ , write INF or -INF in the blank.

Correct Answers:

- 1.1666666666666667

**Problem 7.**

7. (10 points) Library/ASU-topics/setImplicitDerivatives/5-5-14.pg

The radius of a spherical balloon is increasing at a rate of 2 centimeters per minute. How fast is the surface area changing when the radius is 12 centimeters?  
Hint: The surface area is  $S = 4\pi r^2$ .

Rate of change of surface area =  $\underline{\hspace{2cm}}$

Correct Answers:

- 603.18578948928

**Problem 8.**

8. (20 points) Library/ASU-topics/setSecondDerivative/4-4-72.pg

Suppose that

$$f(x) = \frac{2x^2}{x^2 + 25}$$

(A) List all the critical values of  $f(x)$ . Note: If there are no critical values, enter 'NONE'.

(B) Use interval notation to indicate where  $f(x)$  is increasing.

**Note:** Use 'INF' for  $\infty$ , '-INF' for  $-\infty$ , and use 'U' for the union symbol. If there is no interval, enter 'NONE'.

Increasing:  $\underline{\hspace{2cm}}$

(C) Use interval notation to indicate where  $f(x)$  is decreasing.

Decreasing:  $\underline{\hspace{2cm}}$

(D) List the  $x$  values of all local maxima of  $f(x)$ . If there are no local maxima, enter 'NONE'.

$x$  values of local maximums =  $\underline{\hspace{2cm}}$

(E) List the  $x$  values of all local minima of  $f(x)$ . If there are no local minima, enter 'NONE'.

$x$  values of local minimums =  $\underline{\hspace{2cm}}$

(F) Use interval notation to indicate where  $f(x)$  is concave up.

Concave up:  $\underline{\hspace{2cm}}$

(G) Use interval notation to indicate where  $f(x)$  is concave down.

Concave down:  $\underline{\hspace{2cm}}$

(H) List the  $x$  values of all the inflection points of  $f$ . If there are no inflection points, enter 'NONE'.

$x$  values of inflection points =  $\underline{\hspace{2cm}}$

(I) Find all horizontal asymptotes of  $f$ , and list the  $y$  values below. If there are no horizontal asymptotes, enter 'NONE'

$y$  values of horizontal asymptotes =  $\underline{\hspace{2cm}}$

(J) Find all vertical asymptotes of  $f$ , and list the  $x$  values below. If there are no vertical asymptotes, enter 'NONE'

$x$  values of vertical asymptotes =  $\underline{\hspace{2cm}}$

(K) Use all of the preceding information to sketch a graph of  $f$ . When you're finished, enter a "1" in the box below.

Graph complete:  $\underline{\hspace{2cm}}$

Correct Answers:

- 0
- (0, infinity)
- (-infinity, 0)
- none
- 0
- (-2.88675134594813, 2.88675134594813)
- (-infinity, -2.88675134594813) U (2.88675134594813, infinity)
- -2.88675134594813, 2.88675134594813
- 2
- none
- 1

**Problem 9.**

9. (20 points) Library/ASU-topics/setSecondDerivative/4-4-50.pg

Suppose that  $f(x) = x^4 - 3x^3$ .

(A) List all the critical values of  $f(x)$ . Note: If there are no critical values, enter 'NONE'.

(B) Use interval notation to indicate where  $f(x)$  is increasing.

**Note:** Use 'INF' for  $\infty$ , '-INF' for  $-\infty$ , and use 'U' for the union symbol.

Increasing:  $\underline{\hspace{2cm}}$

(C) Use interval notation to indicate where  $f(x)$  is decreasing.

Decreasing: \_\_\_\_\_

(D) List the  $x$  values of all local maxima of  $f(x)$ . If there are no local maxima, enter 'NONE'.

$x$  values of local maximums = \_\_\_\_\_

(E) List the  $x$  values of all local minima of  $f(x)$ . If there are no local minima, enter 'NONE'.

$x$  values of local minimums = \_\_\_\_\_

(F) Use interval notation to indicate where  $f(x)$  is concave up.

Concave up: \_\_\_\_\_

(G) Use interval notation to indicate where  $f(x)$  is concave down.

Concave down: \_\_\_\_\_

(H) List the  $x$  values of all the inflection points of  $f$ . If there are no inflection points, enter 'NONE'.

$x$  values of inflection points = \_\_\_\_\_

(I) Use all of the preceding information to sketch a graph of  $f$ . When you're finished, enter a "1" in the box below.

\_\_\_\_\_

*Correct Answers:*

- 0, 2.25
- (2.25, infinity)
- (-infinity, 2.25)
- none
- 2.25
- (-infinity, 0) U (1.5, infinity)
- (0, 1.5)
- 0, 1.5
- 1