Review

Find

$$\lim_{x\to 0} \frac{e^{4x}-1}{7\sin x} \overset{H}{=} \lim_{x\to 0} \frac{4e^{4x}}{7\cos x} \underset{x=0}{=} \frac{4e^{4(0)}}{7\cos(0)} = \frac{4}{7}$$

functions are not equal but $\lim_{x\to 0}$ is the same

$e^0 = 1$

$$\frac{e^{4(0)}-1}{7\sin(0)} = \frac{e^0-1}{7\cdot 0} = \frac{1-1}{0} = \frac{0}{0}$$

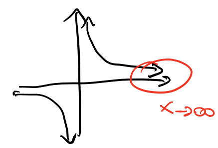
$$\lim_{t\to 0} \frac{e^t-1}{t} \overset{H}{=} \lim_{t\to 0} \frac{e^t}{1} \underset{t=0}{=} \frac{e^0}{1} = \frac{1}{1} = 1$$

$$\frac{e^0-1}{0} = \frac{1-1}{0} = \frac{0}{0}$$

$$* \lim_{x\to\infty} \frac{\ln x}{x} \overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$

$\frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{1}{x} = 0$$



$$\lim_{x\to 0} \frac{x^9-1}{x^5-1} \underset{x=0}{=} \frac{0-1}{0-1} = \frac{-1}{-1} = 1$$ no H rule

$$\lim_{x\to 1} \frac{x^9-1}{x^5-1} = \frac{1-1}{1-1} = \frac{0}{0} \quad H \quad \lim_{x\to 1} \frac{9x^8}{5x^4} \underset{x=1}{=} \frac{9}{5}$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} \overset{H}{=} \lim_{x\to 0} \frac{-(-\sin x)}{2x} = \lim_{x\to 0} \frac{\sin x}{2x} = \lim_{x\to 0} \frac{\cos x}{2}$$

$$\frac{1-\cos(0)}{0} = \frac{1-1}{0} = \frac{0}{0}$$

H again

$$\frac{\sin(0)}{2(0)} = \frac{0}{0}$$

"plug in $x=0$"

$$\frac{\cos(0)}{2} = \boxed{\frac{1}{2}}$$

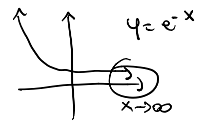
H RULE $\left(\frac{0}{0} \quad \frac{\infty}{\infty}\right)$

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

MOST POPULAR

OTHER INDETERMINATE FORMS (require some algebra)

① INDETERMINATE PRODUCTS $0 \cdot \infty$ or $0 \cdot (-\infty)$

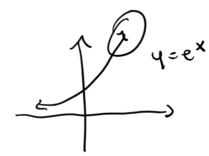


Ex $\lim\limits_{x \to \infty} x\, e^{-x}$ with $x \to \infty$, $e^{-x} \to 0$

$e^{-x} = \dfrac{1}{e^x}$

$= \lim\limits_{x \to \infty} \dfrac{x}{e^x} \underset{\frac{\infty}{\infty}}{\overset{H}{=}} \lim\limits_{x \to \infty} \dfrac{1}{e^x} = 0$, with $e^x \to \infty$

$\dfrac{1}{\infty} = 0$



$x e^{-x} = \dfrac{x}{e^x}$ algebra needed

② INDETERMINATE DIFFERENCES $\infty - \infty$

Ex $\lim\limits_{x \to \infty} x - \ln x$ with $\infty - \infty$ $= \lim\limits_{x \to \infty} x\left(1 - \dfrac{\ln x}{x}\right) = \infty(1-0) = \infty$, with $\dfrac{\ln x}{x} \to 0$

factor out a term

$\lim\limits_{x \to \infty} \dfrac{\ln x}{x} = 0$ (above notes)

③ OTHER INDETERMINATE FORMS

$0^0 \quad 1^\infty \quad \infty^0$

RATIONAL FUNCTIONS - ASYMPTOTES

$f(x) = \dfrac{\text{polynomial}}{\text{polynomial}}$

VERTICAL ASYMPTOTES

NOTE If you have a rational function, $x = c$ is a V.A if

① The denominator is zero at $x = c$

* ② $\lim\limits_{x \to c} f(x)$ is an infinite limit $\left(\dfrac{\text{NUMBER}}{0}\right)$

$\lim\limits_{x \to 2} \dfrac{x-4}{x-2} = \dfrac{-2}{0}$ = infinite limit

so $x = 2$ is V.A.

$\lim\limits_{x \to 2} \dfrac{x^2-4}{x-2} \underset{\frac{0}{0}}{=} \lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = 4$

$x = 2$ is <u>NOT</u> a V.A.

NOTE 2 $x = c$ is a V.A. if

The denominator is zero at $x = c$ AFTER the simplification

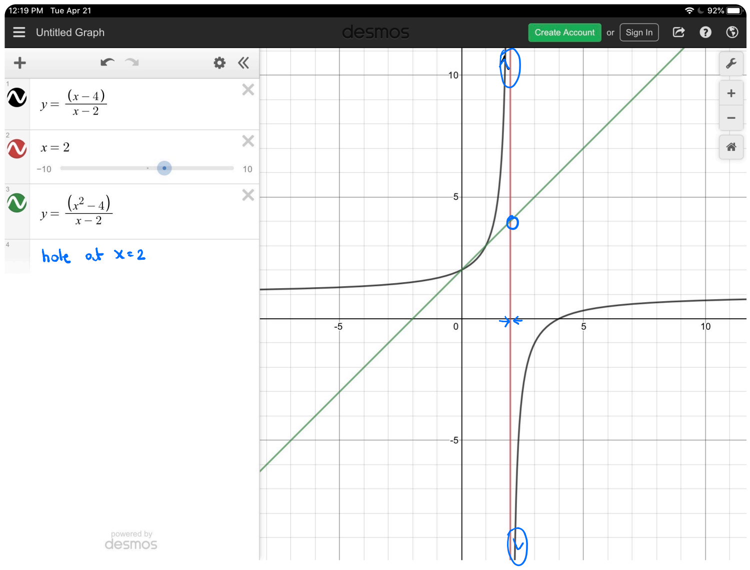
① $y = \frac{x-4}{x-2}$

$x-2=0$ $\boxed{x=2}$ V.A.

② $y = \frac{x^2-4}{x-2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = x+2$ with a "hole" at $x=2$

reduce

no V.A.



HORIZONTAL ASYMPTOTES

$\lim_{x \to \infty} f(x)$ $\lim_{x \to -\infty} f(x)$

① $f(x) = \frac{3x-4}{x+1}$

* $\lim_{x \to \infty} \frac{3x-4}{x+1} \overset{H}{=} \frac{\infty}{\infty}$ $\lim_{x \to \infty} \frac{3}{1} = \lim_{x \to \infty} 3 = 3$

same work $\lim_{x \to -\infty} \frac{3x-4}{x+1} = 3$

$\boxed{y=3 \text{ is a H.A.}}$

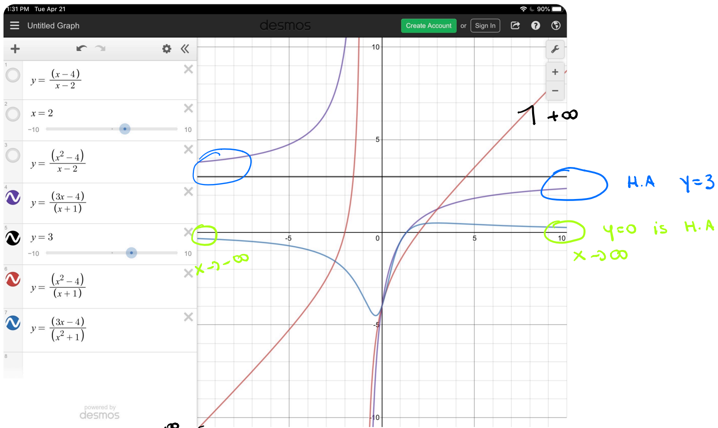
$\frac{3x^1-4}{x^1+1}$ ↗ degree 1, → degree 1 $\approx \frac{3x}{1x} = 3$

* If degree of numerator = degree of denominator (from MAT 1375)

$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = \frac{\text{coefficient of highest power of } x}{\text{coefficient of highest power of } x} = L$

$y = L$ is H.A.

Ex $f(x) = \frac{5x^3-2x+11}{3x^3+15}$ ↗ degree 3, ↓ degree 3 H.A. $y = \frac{5}{3}$



(2) $y=\dfrac{x^2-4}{x+1}$ deg 1

$\lim_{x\to\infty}\dfrac{x^2-4}{x+1} \overset{H}{=}_{\frac{\infty}{\infty}} \lim_{x\to\infty}\dfrac{2x}{1} = \infty$ NO H.A.

$2>1$

$\lim_{x\to-\infty}\dfrac{x^2-4}{x+1} = \lim_{x\to-\infty}\dfrac{2x}{1} = -\infty$

$\dfrac{x^2}{x} = x \to \infty$

* FROM MAT 1375 If degree of numerator > degree of denominator $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ are both infinite and there is no H.A.

(3) $y=\dfrac{3x-4}{x^2+1}$ deg 1, deg 2

$\lim_{x\to\infty}\dfrac{3x-4}{x^2+1} \overset{H}{=}_{\frac{\infty}{\infty}} \lim_{x\to\infty}\dfrac{3}{2x} = 0$

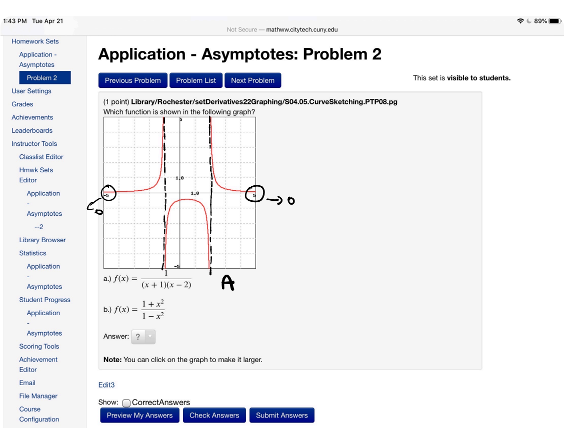
$\dfrac{\text{NUMBER}}{\infty} = 0$

same $\lim_{x\to-\infty}\dfrac{3x-4}{x^2+1} = 0$

$\dfrac{x}{x^2} = \dfrac{1}{x} \xrightarrow{x\to\infty} 0$

$y=0$ (x-axis) is H.A.

* FROM MAT 1375 If degree of numerator < degree of denominator $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$ so $y=0$ is H.A.



$f(x) = \frac{1}{(x+1)(x+2)}$ degree 0 $x^0 = 1$

degree 2

$0 < 2$

V.A. $x=-1$ $x=-2$

H.A. $y=0$

$\lim_{x \to \pm\infty} \frac{1}{(x+1)(x+2)} = 0$ $y=0$

$\frac{1}{\infty}$

$f(x) = \frac{1+x^2}{1-x^2}$ can not be simplified

V.A.

$1 - x^2 = 0$ $1 = x^2$

$x = \pm\sqrt{1}$ $\boxed{\begin{matrix} x=1 \\ x=-1 \end{matrix}}$ V.A.

$\lim_{x \to 1} f(x) = \frac{2}{1-1} = \frac{2}{0}$ infinite limit $x=1$ V.A.

$\lim_{x \to -1} f(x) = \frac{1+(-1)^2}{1-(-1)^2} = \frac{1+1}{1-1} = \frac{2}{0}$ $x=-1$ V.A.

Method 1 $\lim_{x \to \pm\infty} f(x) \underset{H}{=} \lim_{x \to \pm\infty} \frac{2x}{-2x} = \lim_{x \to \pm\infty} -1 = -1$ $y=-1$ is H.A.

$\frac{\infty}{\infty}$

Method 2 $\frac{1+x^2}{1-x^2}$ $\frac{1x^2}{-1x^2} = \frac{1}{-1} = -1$ $\boxed{y=-1 \text{ is H.A.}}$

EXAM 3 Monday May 4