Review problem (graphing polynomials)

Given $f(x) = x^3 - 3x^2 - 9x + 4$

a) Find the intervals of increase and decrease

b) Find the relative (local) maxima and minima

→ FIRST DERIVATIVE

c) Find the intervals of concavity

d) Find the points of inflection

→ SECOND DERIVATIVE

e) Find the y-intercept

f) Use the information a) - e) to sketch the graph. On your graph clearly label the points you found.

SHOW ALL YOUR WORK

Notice that these steps are the guidelines for graphing polynomials

domain $(-\infty, \infty)$ for every polynomial

a) and b) $\quad f'(x) = 3x^2 - 6x - 9 = 0$

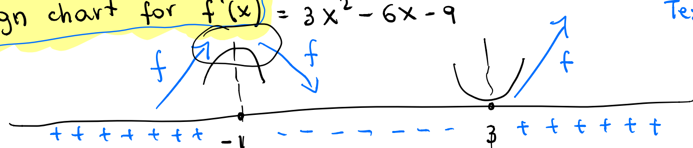
$$3(x^2 - 2x - 3) = 0$$

$$3(x-3)(x+1) = 0$$

critical points $\quad x - 3 = 0 \quad x = 3$

$\quad x + 1 = 0 \quad x = -1$

Sign chart for $f'(x) = 3x^2 - 6x - 9$



Test a number in each interval

Show your work

Test $x = -2$

$f'(-2) = 3(-2)^2 - 6(-2) - 9$

$= 12 + 12 - 9 = 15 > 0$

Test $x = 0$

$f'(0) = -9 < 0$

Test $x = 4$

$f'(4) = 3(16) - 6(4) - 9$

$= 48 - 24 - 9 = 15 > 0$

a) f is increasing in $(-\infty, -1) \cup (3, \infty)$

f is decreasing in $(-1, 3)$

b) There is a local max at $x = -1$ $\qquad f(x) = x^3 - 3x^2 - 9x + 4$

There is a local min at $x = 3$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 4 = -1 - 3 + 9 + 4 = 9$

$f(3) = 27 - 3(9) - 9(3) + 4 = \cancel{27} - \cancel{27} - 27 + 4 = -23$

We have local max when f changes from increasing to decreasing

We have local min when f changes from decreasing to increasing

$(-1, 9)$ local max

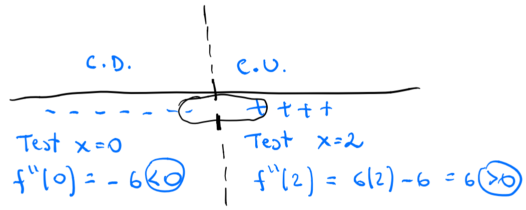
$(3, -23)$ local min

c) and d) Need $f''(x) = 6x - 6$

$f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6 = 0$ $\quad 6x = 6 \quad x = 1$ There is a possible I.P. at $x = 1$

Sign chart for $f''(x)$



c) f is concave up in $(1, \infty)$
f is concave down in $(-\infty, 1)$

Since f changes concavity at $x = 1$ (f'' changes sign at $x = 1$)
there is an inflection point at $x = 1$

$f(x) = x^3 - 3x^2 - 9x + 4$

$f(1) = 1 - 3 - 9 + 4 = -7$

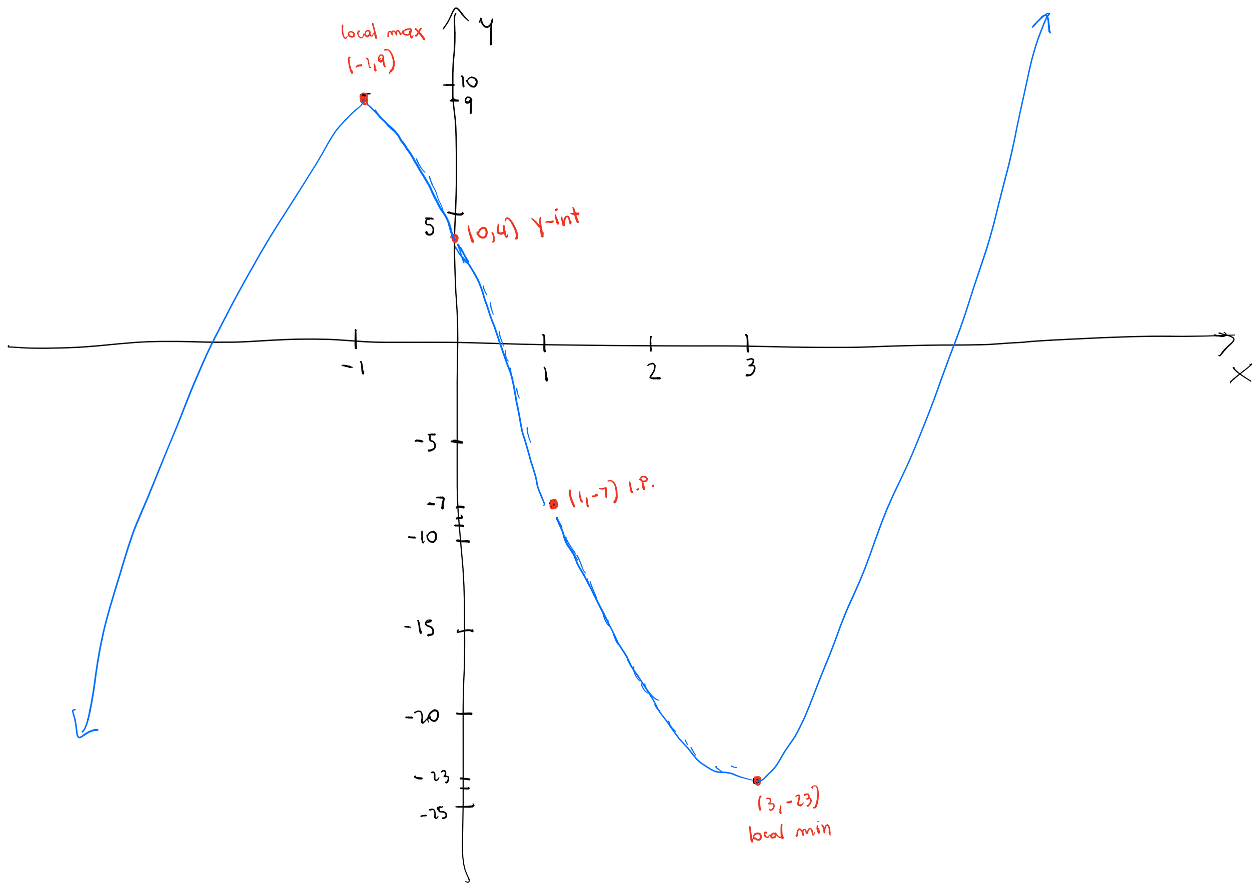
d) $(1, -7)$ is the inflection point

e) $f(0) = 4$

$x = 0$

$(0, 4)$ y-intercept

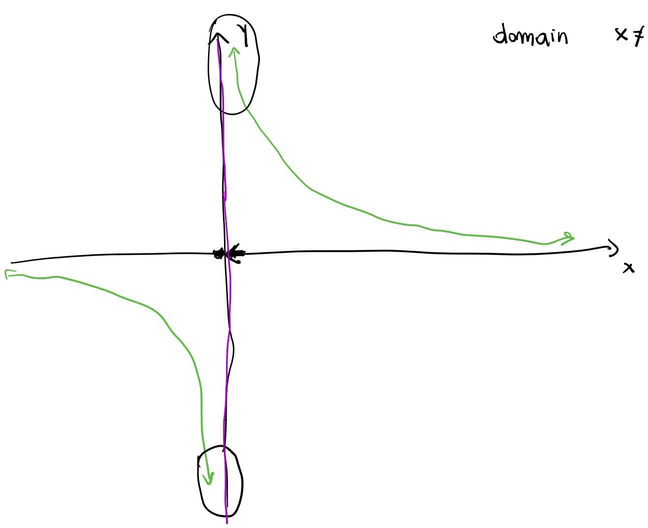
GRAPH



GRAPHING OTHER FUNCTIONS

We need more tools (LIMITS)

Ex $f(x) = \frac{1}{x}$ RATIONAL FUNCTION



domain $x \neq 0$ $(-\infty, 0) \cup (0, \infty)$

$\cancel{f(0)} = \cancel{\frac{1}{0}}$

$\lim_{x \to 0} \frac{1}{x}$ = DNE $\frac{1}{0}$

$\frac{\text{NUMBER}}{0}$ infinite limit

REVIEW

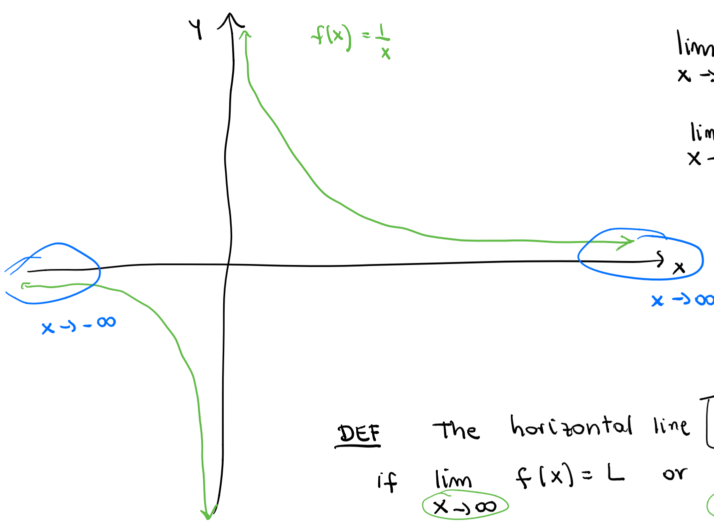
$$\lim_{x \to 0^-} \frac{1}{x} = -\infty \qquad \frac{1}{\text{NEGATIVE}} = (-)\,\infty$$

$$\lim_{x \to 0^+} \frac{1}{x} = \infty \qquad \frac{1}{\text{POSITIVE}} = \infty \quad (+\infty)$$

The line $x = 0$ (y-axis) is a vertical asymptote

DEF If the limit of $f(x)$ as x approaches c from either the left or right (or both) is ∞ or $-\infty$, we say that $f(x)$ has a VERTICAL ASYMPTOTE at c

$\boxed{\text{The line } x = c \text{ is a VERTICAL ASYMPTOTE}}$ → INFINITE LIMITS



$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to -\infty} \frac{1}{x} = 0$$

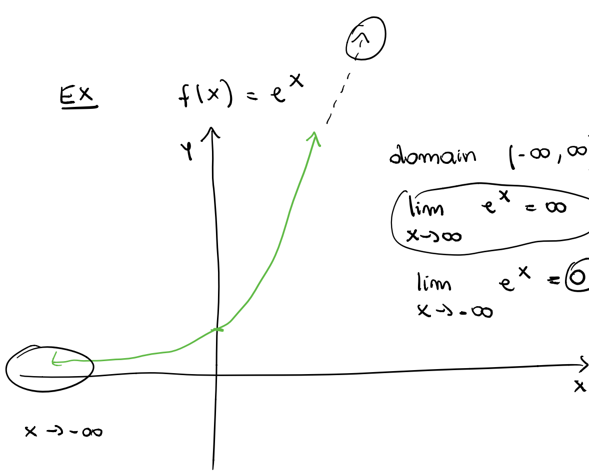
The line $y = 0$ (x-axis) is a HORIZONTAL ASYMPTOTE

DEF The horizontal line $\boxed{y = L \text{ is a HORIZONTAL ASYMPTOTE}}$

if $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$ (or both)

LIMITS AT INFINITY $\lim_{x \to \infty}$ $\lim_{x \to -\infty}$

EX $f(x) = e^x$

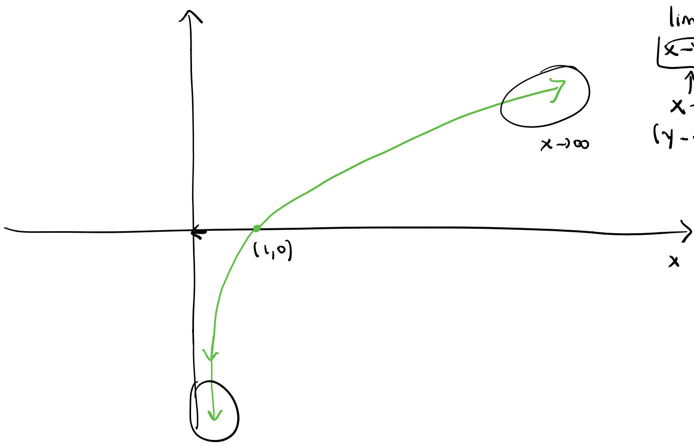


domain $(-\infty, \infty)$ no V.A.

$$\boxed{\lim_{x \to \infty} e^x = \infty}$$

$$\lim_{x \to -\infty} e^x = 0 \qquad y = 0 \ (x\text{-axis}) \text{ is H.A}$$

Ex $f(x) = \ln x$



domain $(0,\infty)$

$\lim_{\boxed{x \to 0^+}} \ln x = -\infty$

$\uparrow$

$x = 0$ is V.A
(y-axis)

$\lim_{x \to \infty} \ln x = \infty$

no H.A.

* NEED TO REVIEW LIMITS AND STUDY A NEW WAY TO COMPUTE SOME OF THEM

$\boxed{\text{L' HÔPITAL'S RULE}}$

Review: $\lim_{x \to 3} \dfrac{x^2 - 2x}{x^2 - 3x + 2} = \dfrac{9-6}{9-9+2} = \dfrac{3}{2}$

plug in $x = 3$

$\lim_{x \to 2} \dfrac{x^2 - 2x}{x^2 - 3x + 2} = \dfrac{4-4}{4-6+2} = \dfrac{0}{0}$ indeterminate form

need work!

plug in $x = 2$

$= \lim_{x \to 2} \dfrac{x\cancel{(x-2)}}{(x-1)\cancel{(x-2)}} = \lim_{x \to 2} \dfrac{x}{x-1} = \dfrac{2}{2-1} = \dfrac{2}{1} = 2$

$\downarrow$ factor and simplify

$\downarrow$ plug in $x = 2$

New: $\lim_{x \to 1} \dfrac{\ln x}{x-1} = \dfrac{\ln 1}{1-1} = \dfrac{0}{0}$ indeterminate form

To solve this we need:

L' HÔPITAL'S RULE

Suppose that f and g are differentiable and $g'(x) \neq 0$ near a (except possibly at a) suppose that

$\lim_{x\to a} f(x) = 0$ and limit $\lim_{x\to a} g(x) = 0$ → we have $\frac{0}{0}$

OR

$\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$ → we have $\frac{\infty}{\infty}$

Then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

this is either $\frac{0}{0}$ or $\frac{\infty}{\infty}$

<u>Ex</u> $\lim_{x\to 1} \frac{\ln x}{x-1} \underset{H}{=} \lim_{x\to 1} \frac{(\ln x)'}{(x-1)'} = \lim_{x\to 1} \frac{\frac{1}{x}}{1} = \lim_{x\to 1} \frac{1}{x} = \frac{1}{1} = 1$

$\frac{0}{0}$ Note: this is not a quotient rule

plug in $x=1$

Note: H rule does <u>not</u> say that $\frac{\ln x}{x-1} = \frac{1}{x}$

These 2 functions are different as you can see graphing in DESMOS

It says $\lim_{x\to 1} \frac{\ln x}{x-1} = \lim_{x\to 1} \frac{1}{x}$

<u>Ex</u> $\lim_{x\to 2} \frac{x^2-2x}{x^2-3x+2} \underset{H}{=} \lim_{x\to 2} \frac{2x-2}{2x-3} = \frac{2(2)-2}{2(2)-3} = \frac{2}{1} = 2$

$\frac{0}{0}$ done before with factoring

plug in $x=2$

<u>Ex</u> $\lim_{x\to\infty} \frac{x^2-2}{3x^2+1} \underset{H}{=} \lim_{x\to\infty} \frac{2\cancel{x}}{6\cancel{x}} = \lim_{x\to\infty} \frac{2}{6} = \frac{2}{6} = \frac{1}{3}$

$\frac{\infty}{\infty}$

constant

$\lim_{x\to\infty} x^2-2 = \infty$

$\lim_{x\to\infty} 3x^2+1 = \infty$

__Ex__ $\lim_{x \to \infty} \frac{e^x}{x^3} \underset{\frac{\infty}{\infty}\ H}{=} \lim_{x \to \infty} \frac{e^x}{3x^2} \underset{\frac{\infty}{\infty}\ H}{=} \lim_{x \to \infty} \frac{e^x}{6x} \underset{\frac{\infty}{\infty}\ H}{=} \lim_{x \to \infty} \frac{e^x}{6} = \frac{\infty}{6} = \boxed{\infty}$

$\lim_{x \to \infty} e^x = \infty$

$\lim_{x \to \infty} x^3 = \infty$

you may have to apply H rule several times

__Ex__ $\lim_{x \to 0} \frac{\sin x}{x} \underset{\frac{0}{0}\ H}{=} \lim_{x \to 0} \frac{\cos x}{1} = \cos(0) = 1$

$\lim_{x \to 0} \frac{\cos x - 1}{x} \underset{\frac{0}{0}\ H}{=} \lim_{x \to 0} \frac{-\sin x}{1} = -\sin(0) = 0$