① Given $5x^2+3x^3y-4y^3=36$, find $\frac{dy}{dx}$ My version of the practice exam

$$10x+(3x^3)'y+3x^3y'-12y^2y'=0$$

$$10x+9x^2y+3x^3y'-12y^2y'=0$$

$$y'(3x^3-12y^2)=-10x-9x^2y$$

$$\boxed{\frac{dy}{dx}=\frac{-10x-9x^2y}{3x^3-12y^2}}$$

slope: At $(2,2)$ $y'=\frac{-10(2)-9(4)(2)}{3(8)-12(4)}=\frac{-20-72}{24-48}=\frac{-92}{-24}=\boxed{\frac{23}{6}}$ slope

$$y=\frac{23}{6}x+b$$

$2=\frac{23}{\cancel{6}\,3}\cdot\cancel{2}+b$ $\quad b=2-\frac{23}{3}=\frac{6}{3}-\frac{23}{3}=\frac{-17}{3}$ $\quad\boxed{y=\frac{23}{6}x-\frac{17}{3}}$ tangent line

② $y=x^{8x}$ $\quad \ln y=8x\ln x$

$(\ln y)'=(8x\ln x)'$

$\frac{y'}{y}=(8x)'\ln x+8x(\ln x)'$

product rule

$\frac{y'}{y}=8\ln x+8x\frac{1}{x}$

$y'=y\,[8\ln x+8]$

$$\boxed{y'=x^{8x}[8\ln x+8]}$$

we used logarithmic differentiation

③ $f(x)=7\tan^{-1}(8x)$

$$f'(x)=7\frac{1}{1+(8x)^2}\times 8=\boxed{\frac{56}{1+64x^2}}$$

$$f'(2)=\frac{56}{1+64(4)}=\boxed{\frac{56}{257}}$$

④ $f(x)=\frac{3\tan x}{x}$

$$f'(x)=\frac{(3\tan x)'x-3\tan x\,(x)'}{x^2}=\frac{3\sec^2 x\cdot x-3\tan x}{x^2}$$

quotient rule

$$f'(4)=\boxed{\frac{3\sec^2(4)\cdot 4-3\tan(4)}{16}}=1.538321$$

exact answer is ok

decimal also accepted

⑤ $y = x\ln x + x$ product rule

$\frac{dy}{dx} = (x)'\ln x + x\ln(x)' + 1$

$= \ln x + x \cdot \frac{1}{x} + 1 = \boxed{\ln x + 2}$

⑥ $f(x) = e^{4x}(x^2+8^x)$ product rule $f'(x) = (e^{4x})'(x^2+8^x) + e^{4x}(x^2+8^x)'$

$f'(x) = \boxed{(4e^{4x})(x^2+8^x) + e^{4x}(2x + 8^x\ln 8)}$

⑦ $\frac{d}{dx}\cos(u(x))$ a) $u(x) = 7-x^2$ b) $u(x) = x^{-2}$ c) $u(x) = \tan(x)$

a) $\frac{d}{dx}\cos(7-x^2) = -\sin(7-x^2)(-2x) = \boxed{2x\sin(7-x^2)}$

b) $\frac{d}{dx}\cos(x^{-2}) = -\sin(x^{-2})(-2x^{-3}) = \boxed{\frac{2\sin(x^{-2})}{x^3}}$

c) $\frac{d}{dx}\cos(\tan x)) = \boxed{-\sin(\tan x)\sec^2 x}$

⑧ $F(x) = f(g(x))$ $g(3) = 6$ $g'(3) = 4$ $f'(3) = 1$ $f'(6) = 7$

$F'(3)$ $F'(x) = f'(g(x))\,g'(x)$

$F'(3) = f'(g(3))\,g'(3) = f'(6)\,g'(3) = 7 * 4 = \boxed{28}$

⑨ $f(u) = \tan u$

$g(x) = x^6$

$f(g(x)) = f(x^6) = \boxed{\tan(x^6)}$

$f'(u) = \boxed{\sec^2 u}$

$f'(g(x)) = \boxed{\sec^2(x^6)}$

$g'(x) = \boxed{6x^5}$

$(f\circ g)'(x) = f'(g(x))g'(x) = \boxed{\sec^2(x^6) * 6x^5}$

(11) $y = -4.9t^2 + 9t + 3.5$ meters y = height

a) $t = 0$ $\boxed{3.5 \text{ meters}}$

b) velocity $= y'(t) = -4.9(2t) + 9 = -9.8t + 9$ $v(t) = -9.8t + 9$

at $t = 0$ $\boxed{V = 9\ m/s}$

c) acceleration $= v'(t) = \boxed{-9.8\ m/sec^2}$

(10) $$y = \frac{1}{3\sin x + 6\cos x}$$ tangent line at $(0, \frac{1}{6})$

$$y' = \frac{(1)'\,(\cancel{3\sin x +} 6\cos x) - 1\,(3\sin x + 6\cos x)'}{(3\sin x + 6\cos x)^2} = \frac{-3\cos x + 6\sin x}{(3\sin x + 6\cos x)^2}$$

when $x = 0$ $\cos 0 = 1$

$\sin 0 = 0$

$$y'(0) = \frac{-3 + 0}{(0 + 6)^2} = \frac{-3}{36} = -\frac{1}{12}$$

$y = -\frac{1}{12}x + b$ plug in $(0, \frac{1}{6})$

$\frac{1}{6} = -\frac{1}{12}(0) + b$ $b = \frac{1}{6}$

$$\boxed{y = -\frac{1}{12}x + \frac{1}{6}}$$

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