Consider the probability experiment consisting of flipping a coin four times in a row.

1. (3 points) What is the sample space of the experiment?

   Hint: There are $2^4 = 16$ elements in sample space; one of them is the coin coming up heads (H) all four times, which we can represent as “HHHH”. List the other 15 elements of the sample space:

   **Solution:** It can help to visualize/construct the sample space as a tree diagram (see OpenLab).

   $S = \{HHHH, HHTH, HHTT, HHTT, HTTH, HTHT, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT\}$

2. (5 points) Now consider the events

   - $A =$ “getting heads four times in a row”
   - $B =$ “getting heads on the first two coin flips”
   - $C =$ “getting tails on the fourth coin flip”

   For each of the following events, list the outcomes that make up the event (as already written below for $B$):

   **Solution:**

   $A = \{HHHH\}$

   $B = \{HHHH, HHTH, HHTT, HHTT\}$

   $C = \{HHTT, HHTT, HTHT, HTTT, THHT, THTT, TTHT, TTTT\}$

   $A \cap B = \{HHHH\}$

   $A \cap C = \emptyset$

   $B \cap C = \{HHTT, HHTT\}$
3. (5 points) Calculate the probabilities of the given events. Write out each as a fraction and then as a decimal (as shown for $B$):

\begin{solution}
\begin{align*}
P(A) &= \frac{1}{16} = 0.0625 \\
P(B) &= \frac{4}{16} = 0.25 \\
P(C) &= \frac{8}{16} = 0.5 \\
P(A \cap B) &= \frac{1}{16} = 0.0625 \\
P(A \cap C) &= \frac{0}{16} = 0 \\
P(B \cap C) &= \frac{2}{16} = 0.125
\end{align*}
\end{solution}

4. (4 points) Calculate the following conditional probabilities, using the formula for conditional probability, as set up for $P(A|B)$ below (then use your answers from (c) to complete the calculations):

\begin{solution}
\begin{align*}
P(A|B) &= \frac{P(A \cap B)}{P(B)} = \frac{1/16}{4/16} = \frac{1}{4} = 0.25 \\
P(B|A) &= \frac{P(B \cap A)}{P(A)} = \frac{1/16}{1/16} = 1 \\
P(A|C) &= \frac{P(A \cap C)}{P(C)} = \frac{0}{8/16} = 0
\end{align*}
\end{solution}

5. (3 points) Explain verbally (in 1-2 sentences) why it makes sense that $P(A|C) = 0$.
(Hint: recall that $C =$ “getting tails on the fourth coin flip”; if you are given the information that $C$ will occur, what does that tell you about the possibility of $A$ occurring?)

\begin{solution}
If we are given the information that $C$ will occur, i.e., that we will get a tails on the 4th coin flip, then we can conclude that we won’t get heads 4 times in a row. In other words, the event $A$ won’t happen, and so $P(A) = 0$.
\end{solution}