Relationship between the limit and one-sided limits

$\lim_{x\to a} f(x) = L \Rightarrow \lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$ $\qquad \lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L \Rightarrow \lim_{x\to a} f(x) = L$

$\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x) \Rightarrow \lim_{x\to a} f(x)$ Does Not Exist

Properties

Assume $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist and c is any number then,

1. $\lim_{x\to a}\left[cf(x)\right] = c\lim_{x\to a} f(x)$
2. $\lim_{x\to a}\left[f(x)\pm g(x)\right] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
3. $\lim_{x\to a}\left[f(x)g(x)\right] = \lim_{x\to a} f(x)\ \lim_{x\to a} g(x)$
4. $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ provided $\lim_{x\to a} g(x) \neq 0$
5. $\lim_{x\to a}\left[f(x)\right]^n = \left[\lim_{x\to a} f(x)\right]^n$
6. $\lim_{x\to a}\left[\sqrt[n]{f(x)}\right] = \sqrt[n]{\lim_{x\to a} f(x)}$

Derivatives

Definition and Notation

If $y = f(x)$ then the derivative is defined to be $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

L'Hospital's Rule

If $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$ then,

$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ a is a number, ∞ or $-\infty$

Function	Rule to Use	Derivative
y = c (constant)	The derivative of a constant is zero	y' = 0
$y = x^n$	Power Rule	$y' = nx^{n-1}$
y = f(x)g(x)	Product Rule	y' = f '(x)g(x) + f(x)g'(x)
$y = \frac{f(x)}{g(x)}$	Quotient Rule	$y' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
$y = (f(x))^n$	Chain Rule	$y' = n(f(x))^{n-1}(f'(x))$
y = sin u	Trig Derivative	y' = u' cos u
y = cos u	" "	y' = – u' sin u
y = tan u	" "	$y' = u' \sec^2 u$
y = cot u	" "	$y' = -u' \csc^2 u$
y = sec u	" "	y' = u' sec u tan u
y = csc u	" "	y' = – u' csc u cot u
y = ln u	Natural Log Rule	$y' = \frac{u'}{u}$
$y = e^u$	Exponential Rule	$y' = u' e^u$
$y = a^u$ (a is a constant)	Exponential Rule	$y' = a^u u' \ln(a)$

y = arcsin u / y = $\sin^{-1}$ u	Inverse Trig Func Rule	y' = $\frac{u'}{\sqrt{1-u^2}}$
y = arccos u / y = $\cos^{-1}$ u	Inverse Trig Func Rule	y' = $-\frac{u'}{\sqrt{1-u^2}}$
y = arctan u / y = $\tan^{-1}$ u	Inverse Trig Func Rule	y' = $\frac{u'}{\sqrt{1+u^2}}$

***Note: "u" = "f(x)" instead of u being a single variable, it is considered a function.

Implicit Differentiation

Find y' if $\mathbf{e}^{2x-9y} + x^3y^2 = \sin(y) + 11x$. Remember $y = y(x)$ here, so products/quotients of x and y will use the product/quotient rule and derivatives of y will use the chain rule. The "trick" is to differentiate as normal and every time you differentiate a y you tack on a y' (from the chain rule). After differentiating solve for y'.

Additional Things to Remember:

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$$\frac{\sin x}{\cos x} = tanx \qquad \frac{\cos x}{\sin x} = \cot x$$

$$cscx = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x}$$