New York City College of Technology

MAT 1475 - Prof. Ghezzi

Exam 3 - Version ~~A~~ B - Total Points: 105

NAME: Key

Instructions: Write your solutions in the space provided after each question. You may use the back of each page for any scratch work that you need to do. To receive (partial) credit you must **show all your work in a clear and organized manner.**

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1. (10 points) Find the extreme values of the function $f(x) = 3\sin(x)$ on $[\pi/6, \pi]$.

$f'(x) = 3\cos x = 0 \qquad x = \frac{\pi}{2}$ (only solution in $[\frac{\pi}{6}, \pi]$)

$f(\frac{\pi}{2}) = 3\sin(\frac{\pi}{2}) = 3$

$f(\frac{\pi}{6}) = 3\sin(\frac{\pi}{6}) = 3(\frac{1}{2}) = \frac{3}{2}$

$f(\pi) = 3\sin(\pi) = 0$

Absolute minimum $= f(\pi) = \boxed{0}$

Absolute maximum $= f(\frac{\pi}{2}) = \boxed{3}$

2. (10 points) The radius of a sphere is increasing at a rate of 5 centimeters/second. How fast is the volume increasing when the radius is 70 centimeters? (Hint: Use the formula $V = \frac{4}{3}\pi R^3$).

$V(t)$ = volume at time t

$R(t)$ = radius at time t

$V = \frac{4}{3}\pi R^3$

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi(\cancel{3}R^2)\frac{dR}{dt} = 4\pi R^2 \frac{dR}{dt}$

$\frac{dV}{dt} = 4\pi(70)^2 5 = 20\pi(4900) = \boxed{98{,}000\pi \text{ centimeters}^3/\text{second}}$ exact answer

$\simeq \boxed{307876.08 \text{ cm}^3/\text{sec}}$

3. (30 points) Given $f(x) = x^3 + 3x^2 - 9x - 2$:

a) Find the intervals of increase or decrease. Show all your work.

b) Find local (relative) maxima and mimima. Show all your work.

c) Find the intervals of concavity. Show all your work.

d) Find the points of inflection. Show all your work.

e) Find the y-intercept.

f) Use the information from a)–e) to sketch the graph. On your graph clearly label local maxima, local minima, inflection points, y-intercept.

WARNING: Due to the nature of this problem only limited partial credit is available. Make sure you work carefully and check your work with a graphing calculator.

$f'(x) = 3x^2 + 6x - 9 = 0$

$3(x^2+2x-3) = 0$

$3(x+3)(x-1) = 0$

$x = -3$, $x = 1$ critical numbers

Sign chart for $f'(x) = 3x^2+6x-9$

$+++$ -3 $-----$ 1 $++++$

$f'(-4) = 48 - 24 - 9 = 15 > 0$

$f'(0) = -9 < 0$

$f'(2) = 12 + 12 - 9 = 15 > 0$

a) $\boxed{\begin{array}{l} f \text{ is increasing in } (-\infty,-3) \cup (1,\infty) \\ f \text{ is decreasing in } (-3,1) \end{array}}$

b) By the first derivative test there is a local maximum at $x=-3$ and a local minimum at $x=1$

$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) - 2 = -27 + 27 + 27 - 2 = 25$

$f(1) = 1 + 3 - 9 - 2 = -7$

$\boxed{\begin{array}{l} (-3,25) \text{ local max} \\ (1,-7) \text{ local min} \end{array}}$

$f''(x) = 6x + 6 = 0 \quad x = -1$

Sign chart for $f''(x) = 6x+6$

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$-----$ -1 $+++++$

$f''(-2) = -6 < 0$ $\quad$ $f''(0) = 6 > 0$

c) $\boxed{\begin{array}{l} f \text{ is concave down in } (-\infty,-1) \\ f \text{ is concave up in } (-1,\infty) \end{array}}$

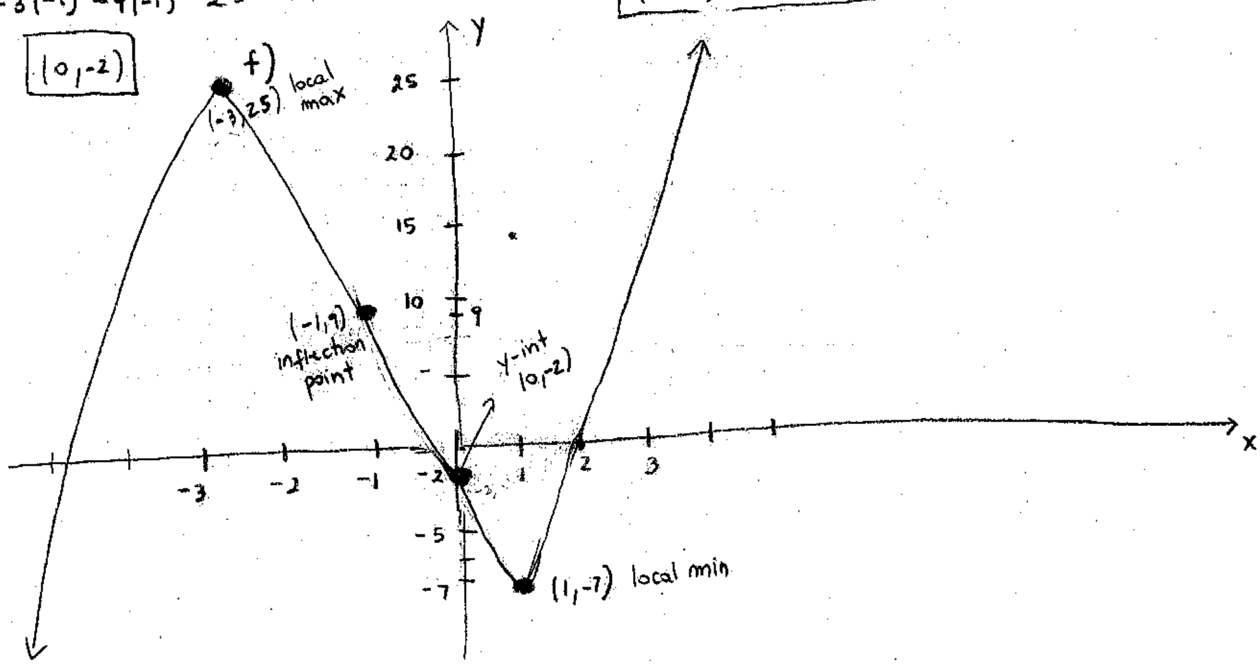
Since f changes concavity at $x=-1$, there is an inflection point at $x=-1$

$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) - 2 = -1 + 3 + 9 - 2 = 9$

d) $\boxed{(-1,9) \text{ inflection point}}$

e) y-intercept $\boxed{(0,-2)}$

f)



4. (30 points) Given $f(x) = \dfrac{3x+9}{x-3}$:

a) Find the domain of f. Write your answer in interval notation.

b) Find the x-intercept and the y-intercept.

c) Find the asymptotes **by setting up and calculating the appropriate limits.** Show all your work.

d) Find the intervals of increase or decrease. Show all your work.

e) Find local (relative) maxima and mimima. Show all your work.

f) Use the information from a)–e) to sketch the graph. On your graph clearly label local maxima, local minima, asymptotes, intercepts.

WARNING: Due to the nature of this problem only limited partial credit is available. Make sure you work carefully and check your work with a graphing calculator.

a) $x-3=0 \quad x=3$ $\quad \boxed{(-\infty,3)\cup(3,\infty)}$

b) $x=0 \quad f(0)=-3$
$3x+9=0 \quad x=-3$ $\quad \boxed{\begin{array}{l}(0,-3)\ y\text{-int}\\(-3,0)\ x\text{-int}\end{array}}$

c) $\lim\limits_{x\to 3} \dfrac{3x+9}{x-3} = \dfrac{18}{0}$ infinite limit so $\boxed{x=3 \text{ is a vertical asymptote}}$

$\lim\limits_{x\to\pm\infty} \dfrac{3x+9}{x-3} \underset{\frac{\infty}{\infty}}{\overset{H}{=}} \lim\limits_{x\to\pm\infty} \dfrac{3}{1} = 3$ $\quad \boxed{y=3 \text{ is a horizontal asymptote}}$

d) $f'(x) = \dfrac{(3x+9)'(x-3)-(3x+9)(x-3)'}{(x-3)^2} = \dfrac{3(x-3)-(3x+9)}{(x-3)^2} = \dfrac{\cancel{3x}-9-\cancel{3x}-9}{(x-3)^2} = \dfrac{-18}{(x-3)^2}$

Notice that $f'(x)$ is always negative in its domain so

$\boxed{f(x) \text{ is decreasing in } (-\infty,3)\cup(3,\infty)}$

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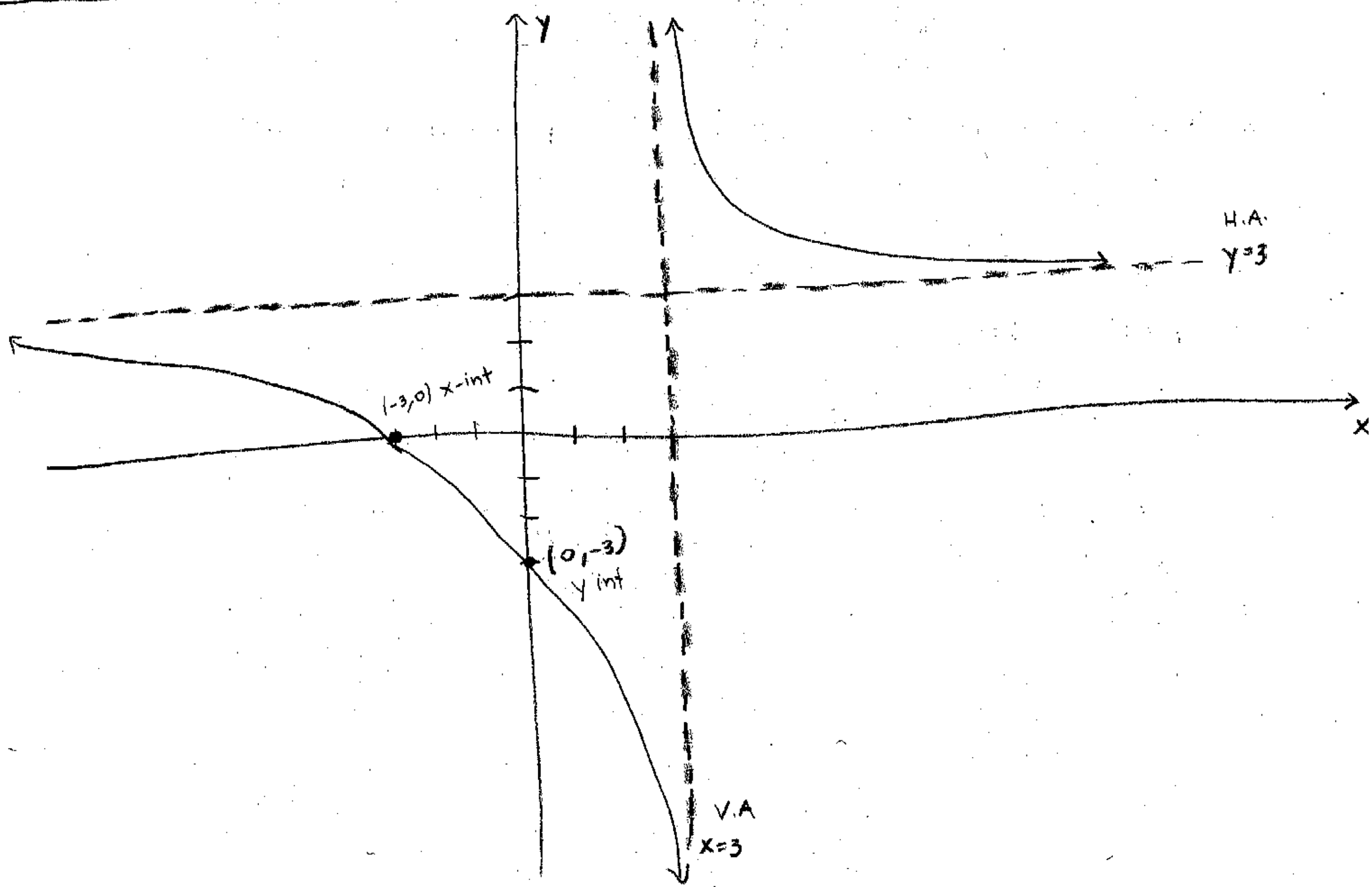
e) $\boxed{\text{No local max and no local min}}$ since $f'(x)$ is never zero

f)



5. (10 points) Find $\lim_{x\to 0} \frac{e^{-6x} - e^{4x}}{8x}$. $\frac{1-1}{0} = \frac{0}{0}$

$\underset{H}{=} \lim_{x\to 0} \frac{-6e^{-6x} - 4e^{4x}}{8} \underset{\text{plug in } x=0}{=} \frac{-6-4}{8} = -\frac{10}{8} = \boxed{-\frac{5}{4}}$ $(= -1.25)$

6. (10 points) Find $\lim_{x\to 0} \frac{2\cos(3x) - 2}{x^2}$. $\frac{2\cos(0) - 2}{0} = \frac{2-2}{0} = \frac{0}{0}$

$\underset{H}{=} \lim_{x\to 0} \frac{\cancel{2}(-3\sin(3x))}{\cancel{2}x} \underset{\frac{0}{0}\, H}{=} \lim_{x\to 0} \frac{-9\cos(3x)}{1} \underset{x=0}{=} \boxed{-9}$

7. (Extra credit 5 points) Check if the Mean Value Theorem can be applied to $f(x) = x^2 - 3$ on the interval $[2, 4]$. If so find a value c in $(2, 4)$ guaranteed by the Mean Value Theorem.

$f(x)$ is a polynomial, so it is continuous and differentiable in $(-\infty, \infty)$

So MVT applies. We find c such that

$f'(c) = \frac{f(b) - f(a)}{b-a}$ where $a=2$ $b=4$

$f(2) = 2^2 - 3 = 1$ $f(4) = 16 - 3 = 13$

$f'(x) = \frac{f(4) - f(2)}{4-2} = \frac{13-1}{2} = \frac{12}{2} = 6$

$2x = 6$ $x = 3$

$\boxed{c = 3}$