New York City College of Technology
MAT 1475 - Prof. Ghezzi
Exam 3 - Version A - Total Points: 105

NAME: Key

Instructions: Write your solutions in the space provided after each question. You may use the back of each page for any scratch work that you need to do. To receive (partial) credit you must **show all your work in a clear and organized manner.**

1. (10 points) The radius of a circular oil slick increases at a rate of 8 meters/minute. How fast is the area of the oil slick increasing when the radius is 30 meters? (Hint: Use the formula $A = \pi R^2$).

$A(t)$ = area at time t
$R(t)$ = radius at time t

$dR/dt = 8$

$A = \pi R^2$

$\frac{dA}{dt} = \pi (2R) \frac{dR}{dt}$ plug in $R = 30$ $\frac{dR}{dt} = 8$

$\frac{dA}{dt} = \pi (60) 8 = \boxed{480\pi \text{ meters}^2/\text{minute}}$ exact answer

$\simeq \boxed{1507.96 \text{ meters}^2/\text{minute}}$

2. (10 points) Find the extreme values of the function $f(x) = 2\sin(x)$ on $[\pi/4, \pi]$.

We use the closed interval method

$f'(x) = 2\cos x = 0$ $x = \frac{\pi}{2}$ (only solution in $[\frac{\pi}{4}, \pi]$)

$f(\frac{\pi}{2}) = 2\sin(\frac{\pi}{2}) = 2$

$f(\frac{\pi}{4}) = 2\frac{\sqrt{2}}{2} = \sqrt{2}$ $(\simeq 1.41)$

$f(\pi) = 2\sin(\pi) = 0$

Absolute minimum = $f(\pi) = \boxed{0}$

Absolute maximum = $f(\frac{\pi}{2}) = \boxed{2}$

3. (30 points) Given $f(x) = x^3 - 3x^2 - 9x + 5$:

a) Find the intervals of increase or decrease. Show all your work.

b) Find local (relative) maxima and mimima. Show all your work.

c) Find the intervals of concavity. Show all your work.

d) Find the points of inflection. Show all your work.

e) Find the y-intercept.

f) Use the information from a)–e) to sketch the graph. On your graph clearly label local maxima, local minima, inflection points, y-intercept.

WARNING: Due to the nature of this problem only limited partial credit is available. Make sure you work carefully and check your work with a graphing calculator.

$f'(x) = 3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$x = 3 \quad x = -1$ critical numbers

Sign chart for $f'(x) = 3x^2 - 6x - 9$

$+ + + + + +$ -1 $- - - - - -$ 3 $+ + + + +$

$f'(-2) =$ $12 + 12 - 9 = 15 > 0$; $f'(0) = -9 < 0$; $f'(4) =$ $48 - 24 - 9 = 15 > 0$

a) f is increasing in $(-\infty, -1) \cup (3, \infty)$
f is decreasing in $(-1, 3)$

b) By the first derivative test there is a local max at $x = -1$ and a local min at $x = 3$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 5 = -1 - 3 + 9 + 5 = 10$

$f(3) = 3^3 - 3(3)^2 - 9(3) + 5 = 27 - 27 - 27 + 5 = -22$

$(-1, 10)$ local max
$(3, -22)$ local min

$f''(x) = 6x - 6 = 0 \quad x = 1$

sign chart for $f''(x) = 6x - 6$

C.D $- - - -$ 1 $+ + \ + +$ C.U

$f''(0) = -6 < 0$; $f''(2) = 6 > 0$

c) f is concave down in $(-\infty, 1)$
f is concave up in $(1, \infty)$

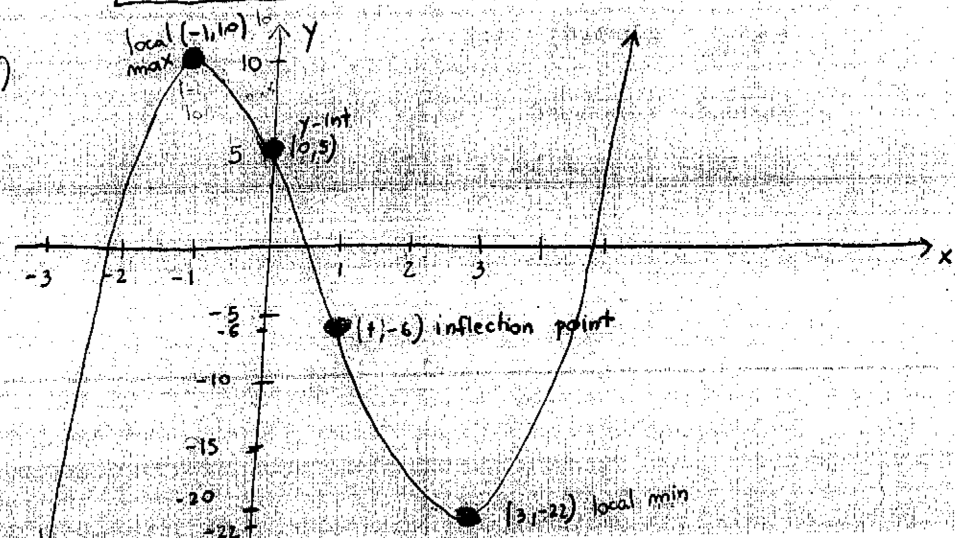
Since f changes concavity at $x = 1$, there is an inflection point at $x = 1$

$f(1) = 1 - 3 - 9 + 5 = -6$

d) $(1, -6)$ inflection point

e) y-intercept $(0, 5)$

f)



4. (30 points) Given $f(x) = \frac{4x-8}{x+2}$.

a) Find the domain of f. Write your answer in interval notation.

b) Find the x-intercept and the y-intercept.

c) Find the asymptotes **by setting up and calculating the appropriate limits**. Show all your work.

d) Find the intervals of increase or decrease. Show all your work.

e) Find local (relative) maxima and mimima. Show all your work.

f) Use the information from a)–e) to sketch the graph. On your graph clearly label local maxima, local minima, asymptotes, intercepts.

WARNING: Due to the nature of this problem only limited partial credit is available. Make sure you work carefully and check your work with a graphing calculator.

a) $x+2=0 \quad x=-2 \qquad \boxed{(-\infty,-2)\cup(-2,\infty)}$

b) $x=0 \quad y=f(0)=-4 \qquad \boxed{(0,-4)\ y\text{-int}}$

$y=0 \quad 4x-8=0 \quad x=2 \qquad \boxed{(2,0)\ x\text{-int}}$

c) $\lim_{x\to -2} \frac{4x-8}{x+2} = \frac{-16}{0}$ infinite limit so $\boxed{x=-2 \text{ is a vertical asymptote}}$

$\lim_{x\to\pm\infty} \frac{4x-8}{x+2} \underset{\frac{\infty}{\infty}}{\overset{H}{=}} \lim_{x\to\pm\infty} \frac{4}{1} = 4$ so $\boxed{y=4 \text{ is a horizontal asymptote}}$

d) $f'(x) = \frac{(4x-8)'(x+2)-(4x-8)(x+2)'}{(x+2)^2} = \frac{4(x+2)-(4x-8)}{(x+2)^2} = \frac{\cancel{4x}+8-\cancel{4x}+8}{(x+2)^2} = \frac{16}{(x+2)^2}$

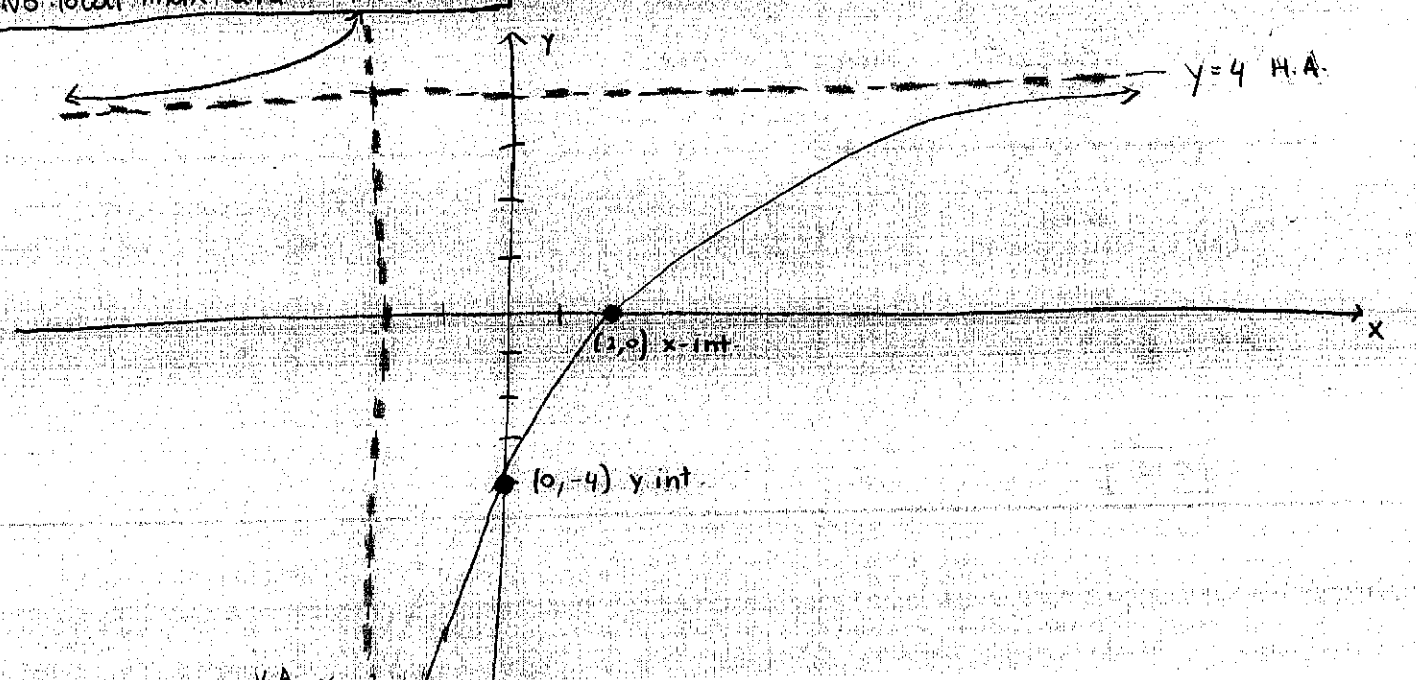
$+ + + + + \quad -2 \quad + + + + + +$

Notice that $f'(x)$ is always positive in its domain so

$\boxed{f(x) \text{ is increasing in } (-\infty,-2)\cup(-2,\infty)}$

e) $\boxed{\text{No local max and no local min}}$ since $f'(x)$ is never zero

f)



5. (10 points) Find $\lim_{x\to 0} \frac{3\cos(5x)-3}{x^2}$ $\quad \frac{3\cos(0)-3}{0} = \frac{3-3}{0} = \frac{0}{0}$

$$\overset{H}{=} \lim_{x\to 0} \frac{3\,(-\sin(5x) * 5)}{2x} = \lim_{x\to 0} \frac{-15\sin(5x)}{2x} \overset{=}{\frac{0}{0}} H$$

$$= \lim_{x\to 0} \frac{-15\,(\cos(5x)) * 5}{2} = \boxed{\frac{-75}{2}} = -37.5$$

plug in $x=0$

6. (10 points) Find $\lim_{x\to 0} \frac{e^{4x}-e^{-8x}}{10x}$ $\quad \frac{1-1}{0} = \frac{0}{0}$

$$\overset{=}{H} \lim_{x\to 0} \frac{4e^{4x} - (-8e^{-8x})}{10} = \lim_{x\to 0} \frac{4e^{4x}+8e^{-8x}}{10} = \frac{4+8}{10} = \frac{12}{10} = \boxed{\frac{6}{5}} = 1.2$$

plug in $x=0$

7. (Extra credit 5 points) Check if the Mean Value Theorem can be applied to $f(x) = x^2 + 2$ on the interval $[1,3]$. If so find a value c in $(1,3)$ guaranteed by the Mean Value Theorem.

$f(x)$ is a polynomial, so it is continuous and differentiable in $(-\infty,\infty)$.
So MVT applies. We find c such that

$f'(c) = \frac{f(b)-f(a)}{b-a}$ where $a=1$ $b=3$

$f(3) = 3^2+2 = 11$ $\quad f(1) = 3$

$f'(x) = \frac{11-3}{3-1}$ $\quad f'(x) = 2x$

$2x = \frac{8}{2} = 4$

$x = 2$

$\boxed{c=2}$