New York City College of Technology

MAT 1475 - Prof. Ghezzi

Exam 2 - Version A - Total Points: ~~100~~
105

NAME: Key

Instructions: Write your solutions in the space provided after each question. To receive (partial) credit you must **show all your work in a clear and organized manner**.

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1. Find the derivatives of the following functions (2 points each, no partial credit):

   (a) $f(x) = 2x^8$.

   $f'(x) = 16x^7$

   (b) $f(x) = 4^x$.

   $f'(x) = 4^x \ln 4$

   (c) $f(x) = \ln 5$.

   $f'(x) = 0$

   (d) $f(x) = \sec(x)$.

   $f'(x) = \sec(x)\tan(x)$

   (e) $f(x) = \sin^{-1}(x)$.

   $f'(x) = \frac{1}{\sqrt{1-x^2}}$

   (f) $f(x) = \sin(4x)$.

   $f'(x) = 4\cos(4x)$

   (g) $f(x) = 3 + \cos(x)$.

   $f'(x) = -\sin(x)$

   (h) $f(x) = \sqrt{x}$. $= x^{1/2}$

   $f'(x) = \frac{1}{2\sqrt{x}}$

   (i) $f(x) = x + 11$.

   $f'(x) = 1$

   (j) $f(x) = \frac{1}{x^3}$. $= x^{-3}$

   $f'(x) = -3x^{-4} = \boxed{\frac{-3}{x^4}}$

2. (10 points) Use logarithmic differentiation to find the derivative of $y = x^{(x+1)}$.

$\ln y = \ln x^{(x+1)}$

$\ln y = (x+1)\ln x$

$(\ln y)' = [(x+1)\ln x]'$ here we use product rule

$\frac{1}{y} y' = (1)\ln x + (x+1)\frac{1}{x}$

$\frac{1}{y} y' = \ln x + 1 + \frac{1}{x}$

$y' = y\left[\ln x + 1 + \frac{1}{x}\right]$

$\boxed{y' = x^{(x+1)}\left[\ln x + 1 + \frac{1}{x}\right]}$

3. Find the derivatives of the following functions.

(a) (5 points) $f(x) = \tan^{-1}(3x)$.

$f'(x) = \frac{1}{1+(3x)^2}(3x)' = \boxed{\frac{3}{1+9x^2}}$

(b) (5 points) $f(x) = \tan(3x)$.

$f'(x) = \sec^2(3x) \cdot 3 = \boxed{3\sec^2(3x)}$

(c) (5 points) $f(x) = e^{x^3+7x}$.

$f'(x) = \boxed{e^{x^3+7x}(3x^2+7)}$

(d) (5 points) $f(x) = \ln(x^6 + 10)$.

$f'(x) = \boxed{\frac{6x^5}{x^6+10}}$

(e) (6 points) $f(x) = \dfrac{1}{\sqrt[4]{x^3}}$. Write your final answer in radical form.

$$f(x) = x^{-3/4}$$

$$f'(x) = -\frac{3}{4}x^{-3/4-1} = -\frac{3}{4}x^{-7/4} = -\frac{3}{4}\frac{1}{x^{7/4}} = \boxed{-\frac{3}{4\sqrt[4]{x^7}}}$$

(f) (7 points) $f(x) = (5x^7 - 3x^4)^3$.

$$f'(x) = 3(5x^7 - 3x^4)^2(35x^6 - 12x^3)$$ by chain rule

(g) (7 points) $f(x) = x^4 e^{\cos x}$.

$$f'(x) = (x^4)' e^{\cos x} + x^4 (e^{\cos x})'$$

product rule

$$= 4x^3 e^{\cos x} + x^4 e^{\cos x}(-\sin x) = e^{\cos x}\left[4x^3 - x^4 \sin x\right]$$

(h) (10 points) $f(x) = \dfrac{x^2+8}{x^3-4x}$. Simplify your answer.

$$f'(x) = \frac{(x^2+8)'(x^3-4x) - (x^2+8)(x^3-4x)'}{(x^3-4x)^2}$$

quotient rule

$$= \frac{2x(x^3-4x) - (x^2+8)(3x^2-4)}{(x^3-4x)^2}$$

$$= \frac{2x^4 - 8x^2 - 3x^4 + 4x^2 - 24x^2 + 32}{(x^3-4x)^2} = \boxed{\frac{-x^4 - 28x^2 + 32}{(x^3-4x)^2}}$$

4. (15 points) a) Find y' by implicit differentiation given that $2y^3 + 5xy + x = -10$.

$(2y^3 + 5xy + x)' = (-10)'$

$6y^2y' + (5x)'y + 5xy' + 1 = 0$

$6y^2y' + 5y + 5xy' + 1 = 0$

$6y^2y' + 5xy' = -5y - 1$

$y'(6y^2 + 5x) = -5y - 1$

$\boxed{y' = \dfrac{-5y-1}{6y^2+5x}}$

b) Find the slope of the tangent line to $2y^3 + 5xy + x = -10$ at the point $(-2, 1)$.

Plug in $x=-2$ $y=1$

$M = \dfrac{-5-1}{6+5(-2)} = \dfrac{-6}{6-10} = \dfrac{-6}{-4} = \boxed{\dfrac{3}{2}}$

5. (10 points) Find the equation of the line that is tangent to the curve $y = e^x \cos x$ at the point $(0, 1)$.

$y' = (e^x)' \cos x + e^x (\cos x)' = e^x \cos x - e^x \sin x = e^x [\cos x - \sin x]$

product rule

At $x=0$ $y'(0) = e^0 [\cos(0) - \sin(0)] = 1 [1-0] = 1$

slope $=1$

$y = x + b$ plug in $(0,1)$

$1 = 0 + b$ $b=1$

$\boxed{y = x+1}$

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Exam 2 - Version B - Total Points: 105

NAME: Key

Instructions: Write your solutions in the space provided after each question. To receive (partial) credit you must **show all your work in a clear and organized manner.**

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1. Find the derivatives of the following functions (2 points each, no partial credit):

   (a) $f(x) = 3x^6$.

   $f'(x) = 18x^5$

   (b) $f(x) = 5^x$.

   $f'(x) = 5^x \ln 5$

   (c) $f(x) = \sqrt{x}$. $f(x) = x^{1/2}$

   $f'(x) = \frac{1}{2} x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$

   (d) $f(x) = \csc(x)$.

   $f'(x) = -\csc(x)\cot(x)$

   (e) $f(x) = \tan^{-1}(x)$.

   $f'(x) = \frac{1}{1+x^2}$

   (f) $f(x) = \ln 2$.

   $f'(x) = 0$

   (g) $f(x) = \cos(3x)$.

   $f'(x) = -3\sin(3x)$

   (h) $f(x) = 1 + \sin(x)$.

   $f'(x) = \cos(x)$

   (i) $f(x) = x - 4$.

   $f'(x) = 1$

   (j) $f(x) = \frac{1}{x^2}$. $f(x) = x^{-2}$

   $f'(x) = -2x^{-3} = \boxed{\frac{-2}{x^3}}$

2. (10 points) Use logarithmic differentiation to find the derivative of $y = x^{\cos(x)}$.

$y = x^{\cos(x)}$

$\ln y = \ln x^{\cos(x)}$

$\ln y = \cos x \ln x$

$(\ln y)' = (\cos x \ln x)'$

$\frac{1}{y} y' = (\cos x)' \ln x + \cos x (\ln x)'$

product rule

$\frac{1}{y} y' = -\sin x \ln x + \cos x \frac{1}{x}$

$y' = y \left[-\sin x \ln x + \frac{\cos x}{x}\right]$

$\boxed{y' = x^{\cos(x)} \left[-\sin x \ln x + \frac{\cos x}{x}\right]}$

3. Find the derivatives of the following functions.

(a) (5 points) $f(x) = \ln(3x^4 + 9)$.

$f'(x) = \frac{12x^3}{3x^4 + 9}$

(b) (5 points) $f(x) = \sin^{-1}(3x)$.

$f'(x) = \frac{3}{\sqrt{1-(3x)^2}} = \frac{3}{\sqrt{1-9x^2}}$

(c) (5 points) $f(x) = e^{5x^2+6x}$.

$f'(x) = e^{5x^2+6x}(10x+6)$

(d) (5 points) $f(x) = \tan(4x)$.

$f'(x) = 4\sec^2(4x)$

(e) (6 points) $f(x) = \frac{1}{\sqrt[5]{x^4}}$. Write your final answer in radical form.

$f(x) = x^{-4/5}$ so

$f'(x) = -\frac{4}{5} x^{-4/5 - 1} = -\frac{4}{5} x^{-9/5} = -\frac{4}{5x^{9/5}} = \boxed{\frac{-4}{5\sqrt[5]{x^9}}}$

(f) (7 points) $f(x) = x^5 e^{\sin x}$.

$f'(x) = (x^5)' e^{\sin x} + x^5 (e^{\sin x})' = 5x^4 e^{\sin x} + x^5 e^{\sin x} \cos x$

product rule

$= \boxed{e^{\sin x}\left[5x^4 + x^5 \cos x\right]}$

(g) (7 points) $f(x) = (7x^3 - 2x^5)^4$.

$f'(x) = 4(7x^3 - 2x^5)^3 (21x^2 - 10x^4)$

(h) (10 points) $f(x) = \frac{x^3 + 10}{x^3 - 3x}$. Simplify your answer.

$f'(x) = \frac{(x^3+10)'(x^3-3x) - (x^3+10)(x^3-3x)'}{(x^3-3x)^2}$

quotient rule

$= \frac{3x^2(x^3-3x) - (x^3+10)(3x^2-3)}{(x^3-3x)^2} = \frac{\cancel{3x^5} - 9x^3 - \cancel{3x^5} + 3x^3 - 30x^2 + 30}{(x^3-3x)^2}$

$= \boxed{\frac{-6x^3 - 30x^2 + 30}{(x^3-3x)^2}}$

4. (15 points) a) Find y' by implicit differentiation given that $x^3 + 2x^2y + y^2 = 7$.

$(x^3 + 2x^2y + y^2)' = 0$

$3x^2 + (2x^2)'y + (2x^2)y' + 2yy' = 0$

$3x^2 + 4xy + 2x^2y' + 2yy' = 0$

$y'(2x^2 + 2y) = -4xy - 3x^2$

$\boxed{y' = \frac{-4xy - 3x^2}{2x^2 + 2y}}$

b) Find the slope of the tangent line to $x^3 + 2x^2y + y^2 = 7$ at the point $(-1, 2)$.

$M = \frac{-4(-1)\cdot 2 - 3(-1)^2}{2(-1)^2 + 2(2)} = \frac{8-3}{2+4} = \boxed{+\frac{5}{6}}$

plug in y'

$x = -1$

$y = 2$

5. (10 points) Find the equation of the line that is tangent to the curve $y = x^2e^{x-2}$ at the point $(2, 4)$.

$y = x^2 e^{x-2}$

$y' = (2x)e^{x-2} + x^2 e^{x-2}$

product rule

slope $= y'(2) = 4e^0 + 4e^0 = 8$

$y = 8x + b$ plug in $(2, 4)$

$4 = 8(2) + b$

$4 = 16 + b$ $b = -12$

$\boxed{y = 8x - 12}$