New York City College of Technology
MAT 1475 - Prof. Ghezzi
Exam 1 - Version A - Total Points: 100

NAME: Key

Instructions: Write your solutions in the space provided after each question. You may use the back of each page for any scratch work that you need to do. To receive (partial) credit you must **show all your work in a clear and organized manner.**

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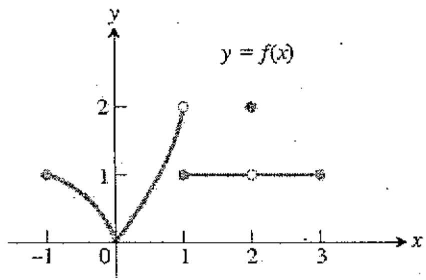
1. Use the graph below to answer the following questions.



a) (3 points-no partial credit) $\lim_{x\to 2^-} f(x) =$ 1

b) (3 points-no partial credit) $\lim_{x\to 2^+} f(x) =$ 1

c) (3 points-no partial credit) $\lim_{x\to 2} f(x) =$ 1

d) (3 points-no partial credit) $f(2) =$ 2

e) (3 points) Is f continuous at $x = 2$? Justify your answer precisely.

No because $\lim_{x\to 2} f(x) \neq f(2)$

f) (3 points-no partial credit) $\lim_{x\to 0} f(x) =$ 0

g) (3 points) Is f continuous at $x = 0$? Justify your answer precisely.

Yes, because $\lim_{x\to 0} f(x) = f(0)$ (both 0)

2. (4 points) Given that $\lim_{x\to 3} f(x) = 5$ and $\lim_{x\to 3} g(x) = -2$, find $\lim_{x\to 3}(2g(x) - f(x)g(x))$.

$2(-2) - 5(-2) = -4 + 10 = \boxed{6}$

3. (7 points)

$$f(x) = \begin{cases} x^2 & x \leq 2 \\ -2x + C & x \geq 2 \end{cases}$$

For which value of C is f continuous at $x = 2$?

$4 = -2(2) + C$

$\boxed{C = 8}$

4. (4 points each) Evaluate the following limits algebraically. Show your work. If the limit is not a number write "does not exist".

a) $\lim\limits_{x \to 2} x^3 - 3x$

$8 - 6 = \boxed{2}$

b) $\lim\limits_{x \to 0} \dfrac{x-2}{x^2-4}$ $= \dfrac{-2}{-4} = \boxed{\dfrac{1}{2}}$

c) $\lim\limits_{x \to \infty} \dfrac{x-2}{x^2-4}$ $= \boxed{0}$ (degree of denominator > degree of numerator)

d) $\lim\limits_{x \to 2} \dfrac{x-2}{x^2-4}$ $\dfrac{0}{0}$ $= \lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \boxed{\dfrac{1}{4}}$

e) $\lim\limits_{x \to -2} \dfrac{x-2}{x^2-4}$ $= \dfrac{-4}{0} =$ $\boxed{\text{does not exist}}$

f) $\lim\limits_{x \to 3} \dfrac{x-3}{x+3}$ $= \dfrac{0}{6} = \boxed{0}$

g) $\lim\limits_{x \to -5} 11$ $= \boxed{11}$

5. (20 points) a) Find the derivative of $f(x) = 4x^2 - 3x + 8$ using the **definition** (the formula with limit).

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{4(x+h)^2 - 3(x+h) + 8 - (4x^2 - 3x + 8)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4x^2} + 8xh + 4h^2 - \cancel{3x} - 3h + \cancel{8} - \cancel{4x^2} + \cancel{3x} - \cancel{8}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(8x + 4h - 3)}{\cancel{h}} = \lim_{h\to 0} 8x + 4h - 3 = 8x - 3$$

$$\boxed{f'(x) = 8x - 3}$$

b) Find the equation of the tangent line to $f(x) = 4x^2 - 3x + 8$ at the point $(1, 9)$.

slope $= f'(1) = 8(1) - 3 = 5$

$y = 5x + b$

$9 = 5 + b \quad b = 4$

$$\boxed{y = 5x + 4}$$

6. (20 points) Given the function $f(x) = \dfrac{x^2 - 36}{x^2 - 5x - 6}$.

a) Find the intervals where f is continuous. Write the answer in interval notation.

b) Identify the horizontal and vertical asymptotes of the function **by calculating the appropriate limits.**

c) Sketch the graph of the function. Graph and label the asymptotes. Use the graphing paper.

a) $x^2 - 5x - 6 = 0$ $\boxed{(-\infty,-1) \cup (-1,6) \cup (6,\infty)}$

$(x-6)(x+1) = 0$

$x = 6 \quad x = -1$

b) H A $\displaystyle\lim_{x \to \infty} \frac{x^2 - 36}{x^2 - 5x - 6} = 1$ $\boxed{y = 1 \text{ is H.A.}}$

(numerator and denominator have the same degree)

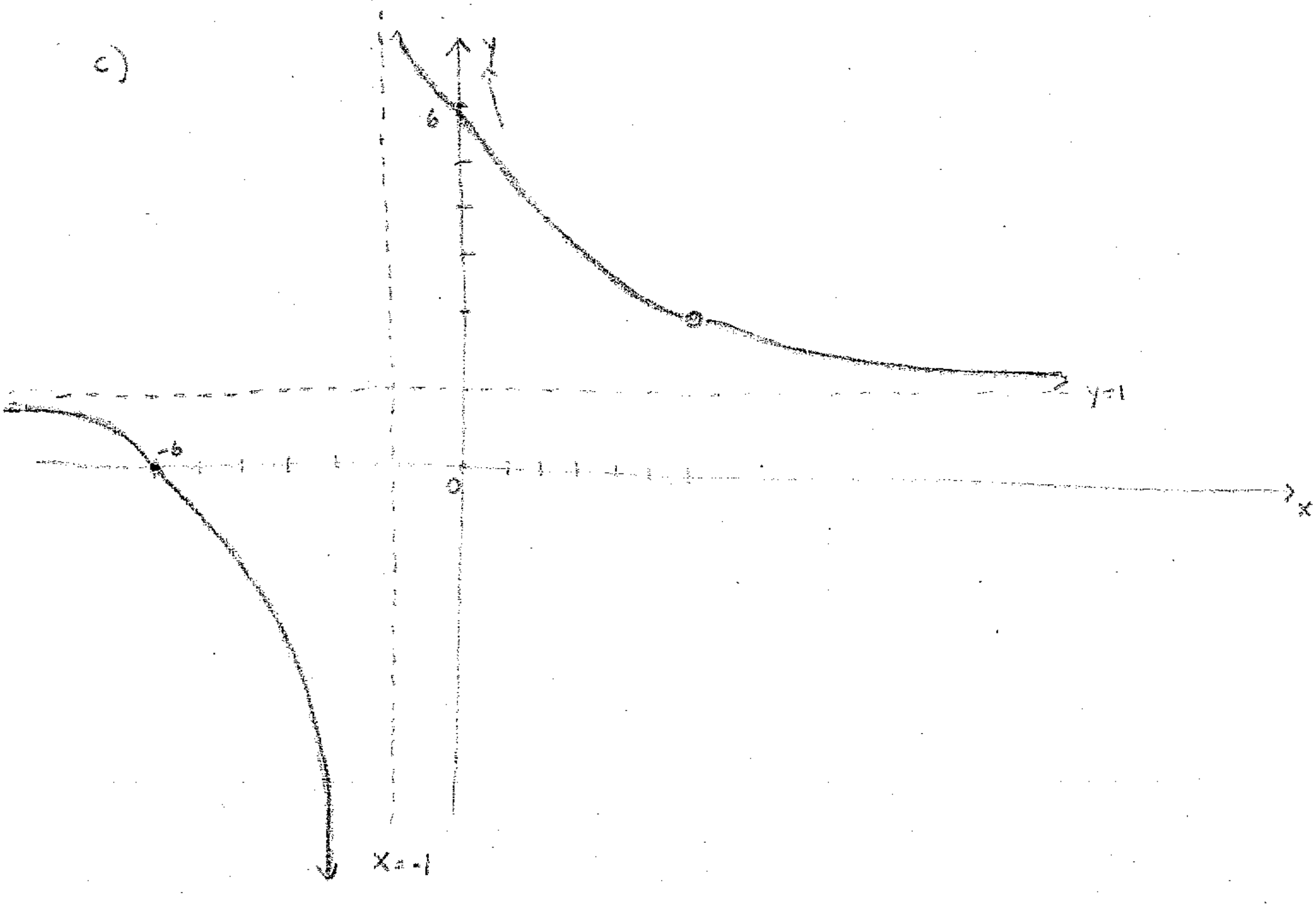
SAME for $\displaystyle\lim_{x \to -\infty} f(x) = 1$

V.A at $x = 6$ $\quad \dfrac{0}{0}$

$\displaystyle\lim_{x \to 6} f(x) = \lim_{x \to 6} \frac{\cancel{(x-6)}(x+6)}{\cancel{(x-6)}(x+1)} = \frac{12}{7}$ There is no V.A at $x=6$ (hole)

at $x = -1$ $\displaystyle\lim_{x \to -1} f(x) = \frac{-35}{0}$ infinite limit $\boxed{x = -1 \text{ is V.A.}}$

c)



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Exam 1 - Version B - Total Points: 100

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Instructions: Write your solutions in the space provided after each question. You may use the back of each page for any scratch work that you need to do. To receive (partial) credit you must **show all your work in a clear and organized manner**.

1. (4 points each) Evaluate the following limits algebraically. Show your work. If the limit is not a number write "does not exist".

a) $\lim_{x\to -2} x^3 - 4x$ $\quad (-2)^3 - 4(-2) = -8+8 = \boxed{0}$

b) $\lim_{x\to 0} \frac{x+3}{x^2-9}$ $\quad = \frac{3}{-9} = \boxed{-\frac{1}{3}}$

c) $\lim_{x\to \infty} \frac{x+3}{x^2-9}$ $\quad = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{3}{x^2}}{1-\frac{9}{x^2}} = \frac{0}{1} = \boxed{0}$

(or. it is 0 because the degree of the numerator is less than the degree of the denominator)

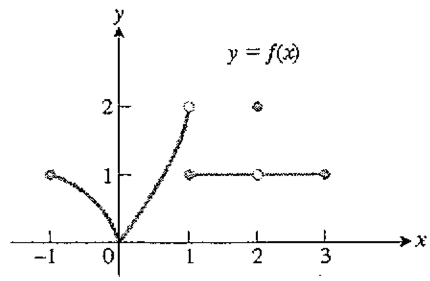
d) $\lim_{x\to 3} \frac{x+3}{x^2-9}$ $\quad = \frac{6}{0} =$ DNE

e) $\lim_{x\to -3} \frac{x+3}{x^2-9}$ $\quad \underset{\frac{0}{0}}{=} \lim_{x\to -3} \frac{\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \lim_{x\to -3} \frac{1}{x-3} = \frac{1}{-3-3} = \boxed{-\frac{1}{6}}$

f) $\lim_{x\to 5} \frac{x-5}{x+5}$ $\quad = \frac{0}{10} = \boxed{0}$

g) $\lim_{x\to 6} -7$ $\quad = \boxed{-7}$

2. Use the graph below to answer the following questions.



a) (3 points-no partial credit) $\lim_{x \to 1^-} f(x) =$ 2

b) (3 points-no partial credit) $\lim_{x \to 1^+} f(x) =$ 1

c) (3 points-no partial credit) $\lim_{x \to 1} f(x) =$ DNE

d) (3 points-no partial credit) $f(1) =$ 1

e) (3 points) Is f continuous at $x = 1$? Justify your answer precisely.

No, because $\lim_{x \to 1} f(x)$ DNE

f) (3 points-no partial credit) $\lim_{x \to 0} f(x) =$ 0

g) (3 points) Is f continuous at $x = 0$? Justify your answer precisely.

yes, because $f(0) = \lim_{x \to 0} f(x)$ (they are both 0)

3. (4 points) Given that $\lim_{x \to 4} f(x) = -3$ and $\lim_{x \to 4} g(x) = 5$, find $\lim_{x \to 4} (f(x)g(x) + 2f(x))$.

$(-3)(5) + 2(-3) = -15 - 6 = \boxed{-21}$

4. (7 points)

$$f(x) = \begin{cases} 4x + C & x < 3 \\ x^3 - 6x & x \geq 3 \end{cases}$$

For which value of C is f continuous at $x = 3$?

$\lim_{x \to 3^-} f(x) = 4(3) + C = 12 + C$

$\lim_{x \to 3^+} f(x) = 3^3 - 18 = 27 - 18 = 9$

$12 + C = 9$

$\boxed{C = -3}$

5. (20 points) a) Find the derivative of $f(x) = 3x^2 - 2x + 1$ using the **definition** (the formula with limit).

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2 - 2(x+h) + 1 - (3x^2-2x+1)}{h} =$$

$$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2) - \cancel{2x} - 2h + \cancel{1} - 3x^2 + \cancel{2x} - \cancel{1}}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - 2h - \cancel{3x^2}}{h} = \lim_{h\to 0} \frac{\cancel{h}(6x+3h-2)}{\cancel{h}}$$

$$= \lim_{h\to 0} (6x+3h-2) = 6x-2$$

$$\boxed{f'(x) = 6x-2}$$

b) Find the equation of the tangent line to $f(x) = 3x^2 - 2x + 1$ at the point $(2, 9)$.

slope $= f'(2) = 6(2)-2 = 10$

$y = 10x + b$

$9 = 10(2) + b$

$9 = 20 + b$

$b = -11$

$$\boxed{y = 10x - 11}$$

6. (20 points) Given the function $f(x) = \dfrac{x^2 - 25}{x^2 + 4x - 5}$:

a) Find the intervals where f is continuous. Write the answer in interval notation.

b) Identify the horizontal and vertical asymptotes of the function **by calculating the appropriate limits.**

c) Sketch the graph of the function. Graph and label the asymptotes. Use the graphing paper.

a) f is continuous when the denominator is not zero

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5 \quad x = 1$

$\boxed{(-\infty, -5) \cup (-5, 1) \cup (1, \infty)}$

b) H.A (limits at infinity)

$\lim_{x \to \infty} \dfrac{x^2 - 25}{x^2 + 4x - 5} = 1 \quad \left(\dfrac{1x^2}{1x^2} \right.$ since numerator and denominator have the same degree$)$

Similarly $\lim_{x \to -\infty} \dfrac{x^2 - 25}{x^2 + 4x - 5} = 1$ so $\boxed{y = 1 \text{ is H.A.}}$

V.A (infinite limits)

$\lim_{x \to -5} \dfrac{x^2 - 25}{x^2 + 4x - 5} \underset{\frac{0}{0}}{=} \lim_{x \to -5} \dfrac{\cancel{(x+5)}(x-5)}{\cancel{(x+5)}(x-1)} = \dfrac{-10}{-6} = \dfrac{5}{3}$

This is not an infinite limit, so $x = -5$ is NOT a V.A. (there is a hole at $x = -5$)

$\lim_{x \to 1} \dfrac{x^2 - 25}{x^2 + 4x - 5} = \dfrac{-24}{0}$ is an infinite limit so $\boxed{x = 1 \text{ is V.A.}}$

c)