Sample Exam 4

1. (a) Find the equation of the tangent line (linearization) to $f(x) = \dfrac{1}{\sqrt{x}}$ at $x = 4$.

   (b) Find the differential dy of $f(x) = \dfrac{1}{\sqrt{x}}$ and evaluate it for $x = 4$ and $dx = 0.3$.

2. Approximate $\sqrt[3]{27.1}$ by linearization. Is your estimate an under or over estimate?

3. A closed cylindrical can is to hold 1 liter (1000 cm^3) of liquid. How should we choose the height and radius to minimize the amount of material needed to manufacture the can?

4. A church window consisting of a rectangle topped by a semicircle is to have a perimeter p. Find the radius of the semicirce if the area of the window is to be maximum.

5. Assume that oil spilled from a ruptured tanker spreads in a circluar pattern whose radius increases at a constant rate of 2 ft/s. How fast is the area of the spill increasing when the radius of the spill is 60 ft/s?

6. Sand pouring from a chute forms a conical pile whose height is always equal to the diameter. If the height increases at a constant rate of 5 ft/min, at what rate is sand pouring from the chute when the pile is 10 ft high?

7. Evaluate:

   (a) $\int (3x^6 - 2x^2 + 7x + 1)dx$

   (b) $\int \left(\dfrac{t^2 - 2t^4}{t^4}\right)dt$

   (c) $\int \left(\dfrac{7}{y^{3/4}} - \sqrt[3]{y} + 4\sqrt{y}\right)dy$

   (d) $\int_{-1}^{2} (x + 2)dx$

   (e) $\int_{-2}^{2} (4e^x + 3)dx$

   (f) $\int_{-\pi/3}^{\pi/3} \sin(x)dx$

8. A particle moves on a coordinate line so that its velocity at time t is $v(t) = t^2 - 2t$ m/s.

   (a) Find the displacement of particle during the time interval $0 \leq t \leq 3$.

   (b) Find the distance traveled by them particle during the time interval $0 \leq t \leq 3$.

9. Find total area between the curve $y = 1 - x^2$ and the x-axis over the integral $[0, 2]$.