Antiderivatives and The Definite Integral - Handout/Worksheet

1. Definition: A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all $x \in I$.

2. Theorem: If F is a antiderivative of f on an interval I, then the most general antiderivative of f on I is $F(x) + C$ where C is an arbitrary constant.

3. Find the most general antiderivative of each of the following functions.

   (a) $f(x) = \cos(x)$

   (b) $f(x) = x^n$ for $n \geq 0$

   (c) $f(x) = x^{-4}$

   (d) $f(x) = 6\sqrt{x} - \sqrt[6]{x}$

4. The process of finding antiderivatives is called **antidifferentiation** or **integration**. Thus, if

$$\frac{d}{dx}[F(x)] = f(x)$$

then integrating (or antidifferentiating) $f(x)$ produces the antiderivatives $F(x) + C$.

5. Since differentiation and integration are inverses

$$\frac{d}{dx}\left[\int f(x)\ dx\right] = f(x)$$

6. Alternatively, the integral $\int_a^b f(x)\ dx$ can be interpreted as the signed area of the region between the graph and x-axis over $[a, b]$

7. Theorem: If f and g are integrable over $[a, b]$,

   (a) $\int_a^b cf(x)\ dx = c\int_a^b f(x)\ dx$

   (b) $\int_a^b [f(x) \pm g(x)]\ dx = \int_a^b f(x)dx \pm \int_a^b g(x)\ dx$

8. Draw a graph of the signed area represented by the integral and compute it using geometry.

   (a) $\int_0^5 (3 - x)\ dx$

   (b) $\int_0^5 \sqrt{25 - x^2}\ dx$