Sample Exam #1

1. Use graphical and numerical methods to approximate the given limits.

   (a)
   $$\lim_{x \to 0} x^3 - 3x^2 + x - 5$$

   (b)
   $$\lim_{x \to 2} \frac{x^2 + 7x + 10}{x^2 - 4x + 4}$$

   (c) $\lim_{x \to 3} f(x)$ where
   $$f(x) = \begin{cases} x^2 - x + 1 & x \leq 3 \\ 2x + 1 & x > 3 \end{cases}$$

2. For $f(x) = \dfrac{1}{x+1}$ and $a = 2$, approximate the limit of the difference quotient,

   $$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

   using $h = \pm 0.1, \pm 0.01$.

3. Given $\lim_{x \to 9} f(x) = 6$, $\lim_{x \to 9} g(x) = 3$, $\lim_{x \to 6} f(x) = 9$, $\lim_{x \to 6} g(x) = 3$, evaluate the following limits. If it is not possible to know, state so.

   (a)
   $$\lim_{x \to 9} \left(\frac{f(x) - 2g(x)}{g(x)} \right)$$

   (b)
   $$\lim_{x \to 6} f(g(x))$$

4. Evaluate the given limits.

   (a)
   $$\lim_{x \to \pi} \left(\frac{x - 3}{x - 5} \right)^7$$

   (b)
   $$\lim_{x \to \pi} \left(\frac{x^2 + 3x + 5}{5x^2 - 2x - 3} \right)$$

   (c)
   $$\lim_{x \to -1} \left(\frac{x^2 + 9x + 8}{x^2 - 6x - 7} \right)$$

5. Evaluate the given limits of the piecewise defined function given.

$$f(x) = \begin{cases} \cos(x) & x < \pi \\ \sin(x) & x \geq \pi \end{cases}$$

(a) $\lim_{x \to \pi^-} f(x)$

(b) $\lim_{x \to \pi^+} f(x)$

(c) $\lim_{x \to \pi} f(x)$

(d) $f(\pi)$

6. Determine if f is continuous at the indicated values. If not, explain why.

(a)
$$f(x) = \begin{cases} x^3 - x & x < 1 \\ x - 2 & x \geq 1 \end{cases}$$

    i. $x = 0$

    ii. $x = 1$

(b)
$$f(x) = \begin{cases} \dfrac{x^2 - 64}{x^2 - 11x + 24} & x \neq 8 \\ 5 & x = 8 \end{cases}$$

    i. $x = 0$

    ii. $x = 8$

7. Give the intervals on which the given function is continuous.

(a) $h(k) = \sqrt{1 - k} + \sqrt{k + 1}$

(b) $g(t) = \dfrac{1}{\sqrt{1 - t^2}}$

(c) $f(k) = \sqrt{1 - e^k}$

8. Identify the horizontal and vertical asymptotes, if any, of the given function.

(a) $f(x) = \dfrac{2x^2 - 2x - 4}{x^2 + x - 20}$

(b) $f(x) = \dfrac{x^2 + x - 12}{7x^3 - 14x^2 - 21x}$

(c) $f(x) = \dfrac{x^2 - 9}{9x + 27}$

9. Evaluate the given limits.

(a) $\lim_{x\to\infty} \dfrac{x^3 + 2x^2 + 1}{x - 5}$

(b) $\lim_{x\to-\infty} \dfrac{x^3 + 2x^2 + 1}{x^2 - 5}$

(c) $\lim_{x\to-\infty} \dfrac{3x^2 - 7x + 9}{7x^3 - 4}$