**Class #6 - Monday September 16**
**Paired Data: Scatterplots and the Correlation Coefficient**

**Textbook readings**:

- Ross, Sec 2.5: Paired Data & Sec 3.7: Correlation Coefficient
- Phillips, Chapter 6: Correlation

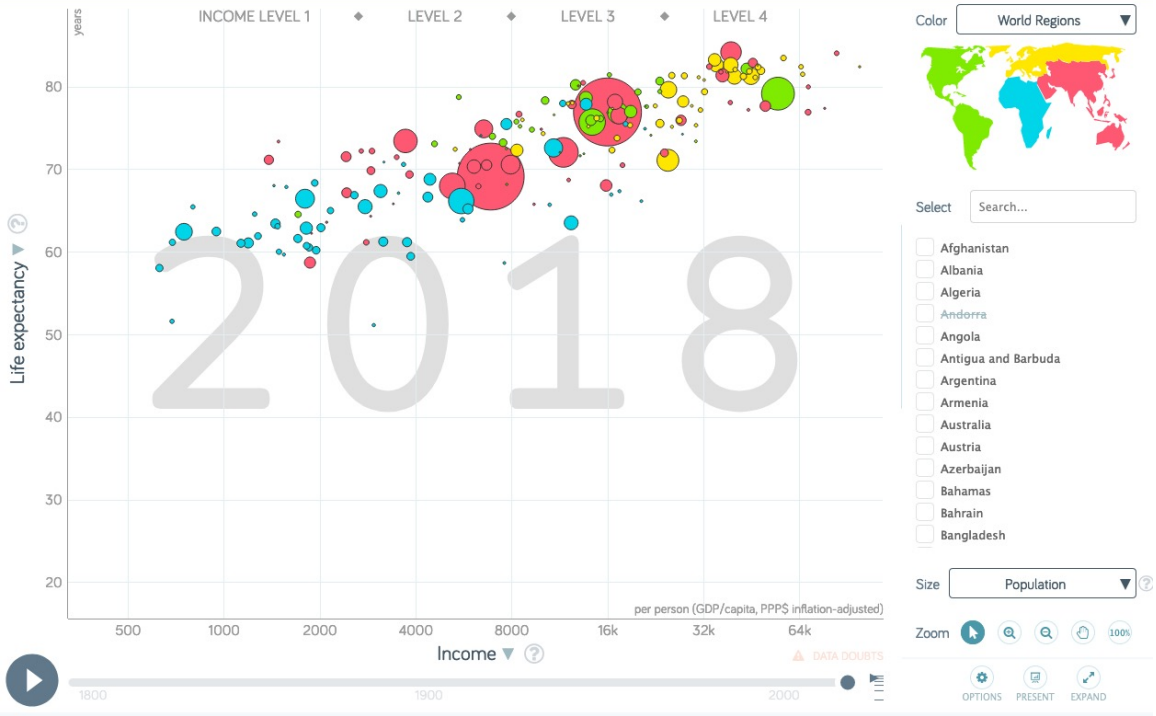
**Concepts:**

- paired data sets
- scatterplots
- correlation: positive correlation vs. negative correlation
- correlation coefficient

**Intro:** So far we have looked at data sets for a single variable (or "attribute") for each member of the sample, e.g., $x_i$ = weight of individual $i$. Now suppose the data set contains values for a second variable for each individual.

**Example:** Suppose we have a sample of 23 individuals, and we measure each individual's height and weight. Then we have **a set of paired data**:

| Individual # | Weight (lbs) | Height (inches) |
|---|---|---|
| 1 | 132 | 59 |
| 2 | 114 | 58 |
| 3 | 187 | 78 |
| 4 | 214 | 84 |
| 5 | 158 | 71 |
| 6 | 212 | 72 |
| 7 | 175 | 69 |
| 8 | 147 | 59 |
| 9 | 173 | 69 |
| 10 | 182 | 72 |
| 11 | 194 | 78 |
| 12 | 215 | 80 |
| 13 | 180 | 74 |
| 14 | 147 | 68 |
| 15 | 168 | 76 |
| 16 | 157 | 70 |
| 17 | 168 | 73 |
| 18 | 179 | 68 |
| 19 | 216 | 74 |
| 20 | 200 | 77 |
| 21 | 194 | 75 |
| 22 | 147 | 68 |
| 23 | 156 | 67 |



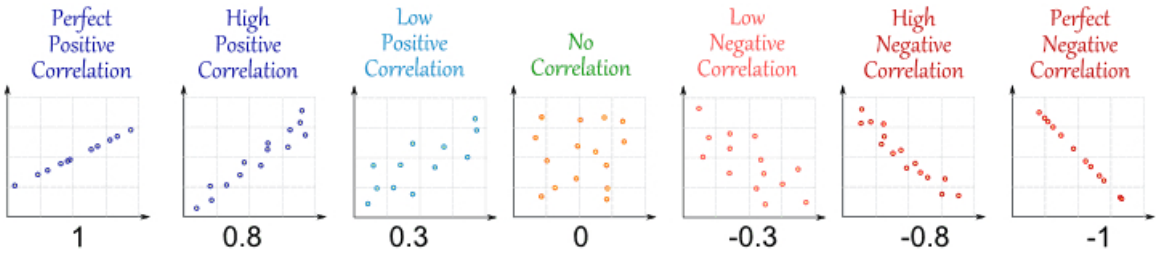

- So far, we have discussed methods of looking at the distribution of heights or the distribution of weights *separately*
  - frequency tables and histograms
  - summary statistics: mean, median, standard deviation
- But these concepts won't tell us anything about the *relationship* between height and weight

The first thing we can do is visualize the paired data values with a **scatterplot**, where we plot each data point as an ordered pair on an $xy$-coordinate system.

**Exercise:** Enter the data above into a spreadsheet and create the scatterplot: (1) Select the two columns of data; (2) use "Insert → Chart ..." (either from the menus or the chart icon in the toolbar); (3) select "Scatter" for chart type; (4) customize the chart title, axis labels, etc.

**Additional Examples:** See the following for or scatterplots showing life expectancy vs income data:

- `http://www.gapminder.org/tools` (shown in chart above)
- `http://www.nytimes.com/interactive/2014/03/15/business/higher-income-longer-liv` `html`




**Correlation Coefficient:** For a sample of paired data $(x_i, y_i)$, the sample correlation coefficient is a statistic that measures the degree to which the variables $x$ and $y$ are correlated, i.e.,

- as $x$-values increase, do the $y$-values also tend to increase? (positive correlation)
- as $x$-values increase, do the $y$-values tend to decrease? (negative correlation)

**Definition:** The correlation coeffcient $r$ is defined as

$$r = \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y} = \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\Sigma_{i=1}^{n}(y_i - \bar{y})^2}}$$

where $s_x$ and $s_y$ are the standard deviations of the $\{x_i\}$ and $\{y_i\}$, respectively.

- the terms $(x_i - \bar{x})(y_i - \bar{y})$ in the sum in the numerator are positive when the deviations $(x_i - \bar{x})$ & $(y_i - \bar{y})$ have the same sign, i.e., when relatively large $x_i$ are paired with relatively large $y_i$ and small $x_i$ correspond to small $y_i$ (read Ross, Sec 3.7, pp121-123)
- on the other hand, $(x_i - \bar{x})(y_i - \bar{y})$ is negative if the deviations $(x_i - \bar{x})$ & $(y_i - \bar{y})$ have opposite signs, i.e., large $x_i$ are paired with small $y_i$ and small $x_i$ with large $y_i$
- the denominator serves to "normalize" or "standardize" the value of $r$, so that $r$ is between -1 and 1
- spreadsheet function: `=correl(data_x, data_y)`

**Exercise:** Would you expect height and weight to be positively correlated, negatively correlated, or uncorrelated? What does your scatterplot indicate?

Calculate the correlation coefficient for the example dataset, using the spreadsheet function `=correl(data_x, data_y)`.