Sample Final Exam

1. Not valid: p false, q false, r true.

2. Valid

3. See class notes.

4. Not one-to-one, not onto.

5. **procedure** $greaterthanfive(a_1, ..., a_n : integers)$
   $answer := 0$
   for $i := 1$ **to** n
   if $a_i > 5$ **then** $answer := answer + 1$
   return $answer$

6. $f(n) \leq 3n^2 + 8n^2 + 7n^2 = 18n^2$ if $n \geq 1$; therefore $C = 18$ and $k = 1$ can be used.

7. Use the Euclidean algorithm to find $gcd(34, 21)$. 1

8. $5 + 11k$

9. $P(1) : 1 = \frac{1 \cdot 2}{2}$, which is true since $1 = 1$.

$$P(k) \to P(k+1) : 1 + 4 + \cdots + (3(k+1) - 2) = \frac{k(3k-1)}{2} + (3k+1)$$
$$= \frac{k(3k-1) + 2(3k+1)}{2} = \frac{3k^2 + 5k + 2}{2}$$
$$= \frac{(3k+2)(k+1)}{2} = \frac{(k+1)(3(k+1)-1)}{2}.$$

10. Must show both directions: 1) Let n be an integer, then if n is even then $5n+4$ is even. Proof: Since n is even it can be written in the form $n = 2k$ where k is some integer. Now, $5n + 4 = 5(2k) + 4 = 10k + 4 = 2(5k + 2)$. $5k + 2$ is just some integer, call it s. So $5n + 4$ can be written as $2s$ and so it is even. 2) If $5n + 4$ even, then n is even. Proof: Instead of proving this statement directly, we can prove the contrapositive of it, which states: If n is odd, then $5n + 4$ is odd. Since n is odd, it can be written in the form $n = 2k + 1$ for some integer k. Now, $5n + 4 = 5(2k + 1) + 1 = 10k + 5$. $10k + 5 = 5(2k + 1)$, since we know $2k + 1$ is odd, then 5 times $2k + 1$ is also odd. So $5n + 4$ is odd.

    The proof is now complete.

11. 1091