2.3 Sets - Worksheet

1. Determine whether f is a function from $\mathbb{Z}$ to $\mathbb{R}$ if
   (a) $f(n) = \pm n$
   (b) $f(n) = \sqrt{n^2 + 1}$
   (c) $f(n) = \dfrac{1}{n^2 - 4}$
2. Find these values
   (a) $\lceil 1.1 \rceil$
   (b) $\lceil -0.1 \rceil$
   (c) $\lfloor -0.1 \rfloor$
   (d) $\left\lfloor \dfrac{1}{2} + \left\lceil \dfrac{1}{2} \right\rceil \right\rfloor$
3. Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one.
   (a) $f(n) = n - 1$
   (b) $f(n) = n^2 + 1$
   (c) $f(n) = n^3$
   (d) $f(n) = \left\lceil \dfrac{n}{2} \right\rceil$
4. Determine whether $f : Z \times Z \to Z$ is onto if
   (a) $f(m, n) = 2m - n$
   (b) $f(m, n) = m^2 - n^2$
5. Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.
   (a) $f(x) = -3x + 4$
   (b) $f(x) = -3x^2 + 7$
   (c) $f(x) = \dfrac{x + 1}{x + 2}$
   (d) $f(x) = x^5 + 1$
6. Find $f \circ g$ and $g \circ f$ where $f(x) = x^2 + 1$ and $g(x) = x + 2$ are functions from $\mathbb{R}$ to $\mathbb{R}$.
7. Show that the function $f(x) = ax + b$ from $\mathbb{R}$ to $\mathbb{R}$ is invertible, where a and b are constants, with $a \neq 0$, and find the inverse of f, f^{-1}, and compute $f^{-1}(1)$.
8. How many bytes are required to encode n bits of data where n equals

(a) 4?

(b) 10?

(c) 500?

(d) 3000?