Sample Exam #1

1. What is the negation of this proposition? "If you pay your membership dues, then if you come to the club, you can enter free."

2. Write the contrapositive, converse and inverse of the following proposition: "If the number is positive, then its square is positive."

3. Suppose you have three cards: one red on both sides (red/red), one green on both sides (green/green), and one red on one side and green on the other side (red/green). The three cards are placed in a row on a table. Explain how to determine the identity of all three cards by selecting one card and turning it over.

4. Prove that $\neg[r \vee (q \wedge (\neg r \rightarrow \neg p))] \equiv \neg r \wedge (p \vee \neg q)$ by using a truth table.

5. Suppose you want to prove a theorem of the form $p \rightarrow (q \vee r)$. Prove that this is equivalent to showing that $(p \wedge \neg q) \rightarrow r$.

6. Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

7. Let $Q(x, y)$ be the statement $x + y = x - y$ where the universe for x and y is the set of all real numbers. Determine the truth value of:

   (a) $Q(5, -2)$

   (b) $Q(4.7, 0)$

   (c) Determine the set of all pairs of numbers, x and y, such that $Q(x, y)$ is true.

8. Write the following statement in English, using the predicates
   $C(x)$: "x is a Computer Science major"
   $M(y)$: "y is a math course"
   $T(x, y)$: "x is taking y"
   where x represents students and y represents courses:

$$\forall x \exists y (C(x) \rightarrow M(y) \wedge T(x, y)).$$

9. Determine whether this argument is valid:
   It is not sunny this afternoon and it is colder than yesterday.
   If we go swimming, then it is sunny.
   If we do not go swimming, then we will take a canoe trip.
   If we take a canoe trip, then we will be home by sunset.
   Therefore we will be home by sunset.

10. Suppose we have the two propositions (with symbols to represent them): It is raining (r) or I work in the yard (w) It is not raining ($\neg r$) or I go to the library (l). What conclusion can we draw from these propositions?

11. Determine the truth value of each of these statements if the doman consists of all integers.

    (a) $\forall n(n^2 \geq 0)$
    (b) $\exists n(n^2 = 2)$
    (c) $\forall n(n^2 \geq n)$
    (d) $\exists n(n^2 < 0)$

12. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

    (a) $\forall x(x^2 \geq x)$
    (b) $\forall x(x > 0 \vee x < 0)$
    (c) $\forall x(x = 1)$

13. Let $L(x, y)$ be the statement x loves y, where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements.

    (a) Everybody loves Jerry.
    (b) Everybody loves somebody.
    (c) There is somebody whom everybody loves.
    (d) Nobody loves everybody.
    (e) There is somebody whom Lydia does not love.